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PREFACE

This issue of the QUARTERLY TRANSACTIONS, Number 2 of Volume 51, contains the balance of the 1932 Winter Convention papers, the first of which were published in Number 1 of this volume. This completes the publication of the Winter Convention program with the exception of two papers, *viz.*: *Surge Proof Transformers* and *Effect of Transient Voltages on Power Transformer Design—IV*. The discussions on these two papers were received too late to permit their inclusion in this volume. They will appear in the Number 3 QUARTERLY TRANSACTIONS in September, 1932.

Equivalent Circuits—I

BY FRANK M. STARR*
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Synopsis.—*Equivalent circuits have been found valuable in the solution of certain problems in that they simplify the analysis. Furthermore, equivalent circuits provide a means of replacing magnetic coupling with simple impedance links thereby permitting the use of the calculating board for solution.*

In Part I of this paper, the general equivalent mesh for the n-winding transformer is derived, and other less rigorous transformer circuits are discussed and compared.

In Part II the mesh equivalent for the general network having $(m + l)$ *points of entry is derived and its application is explained.*

The generalizations of Parts I and II provide a background for the particular problem treated in Part III in which the study of system networks involving groups of parallel transmission lines is considered. The usually difficult problem of accurately determining ground fault currents in such a system is readily solved by applying the special equivalent circuits developed in Part III to a calculating board set-up of the system. These circuits are particularly valuable in the solution of any problem involving an analysis of the zero phase impedance diagram of a system.

The methods and circuits developed in this paper are perfectly general and have many evident applications other than those treated.

* * * * *

INTRODUCTION

VARIOUS authors have used equivalent circuits from time to time in analyzing certain particular problems.[1] The present paper presents general methods of attack suitable for applying equivalent circuits to a great variety of problems. The particular problems treated here concern transformers and networks.

In general, any problem which can be analyzed with an equivalent circuit can be analyzed without it. However, the equivalent circuit may be useful for any or all of the following reasons:

1. Since the equivalent circuit represents only terminal conditions, it is frequently simpler than the actual machine or circuit.
2. It facilitates and reduces calculation in some cases, and even though the amount of calculation has not been reduced, it will have been speeded up and systematized.
3. The equivalent circuit aids in visualizing the problem.
4. Since the equivalent circuit is made up of simple impedance links, it may readily be set up on the calculating board,† thereby lending itself to the solution of complicated networks.

The particular problem which prompted the present study was that of representing a system of coupled circuits by an equivalent composed of simple impedance links. This problem arose primarily through the difficulty experienced in adequately representing on the calculating board groups of parallel transmission lines in which zero phase currents were flowing. In order to represent such a group, it is desirable to replace it with an equivalent circuit in which mutual impedances between the lines are simulated by simple impedance links.

Closely related to the problem of coupled circuits, and probably a broader one, is the general network problem. In the study of transmission and distribution networks, it is often desirable to simplify and reduce them. One method of doing this (the method generalized in this paper) is to replace the actual system with an equivalent network having the minimum number of links required to satisfy terminal conditions.

In order to treat the above mentioned problems adequately, the present paper is divided into *Part I* in which the general mesh linkage to represent *n* electrically isolated but magnetically coupled circuits is developed, and the corresponding star-mesh equivalent is derived; *Part II* in which the equivalent mesh linkage to represent a general network having any number of points of entry is derived; and *Part III* in which, by means of the generalizations developed in Parts I and II, a number of special equivalent circuits representing groups of two, three, and four parallel transmission lines under various conditions are derived. The method of adapting these circuits to a calculating board set-up of the zero phase diagram of a system is fully explained.

The two generalized equivalents discussed in Parts I and II are quite broad and should find many applications other than those cited in this paper.

It should be pointed out here that only linear circuits and networks are dealt with in the present paper. It should also be noted that the circuits developed here are, in general, equivalent only at a single frequency, although certain particular ones are equivalent at all frequencies.‡

Part I

THE TRANSFORMER PROBLEM

In practise, the multi-circuit problem frequently presents itself, that is, the problem of a number of circuits each having a self impedance and a mutual impedance with every other circuit. Notable examples are the

---

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†The calculating board is referred to in numerous places throughout the paper. It is understood that, in general, an a-c. board will be required since both resistance and reactance are involved in the circuits developed here. If resistance may be neglected, the d-c. board will be adequate.

1. For numbered references see Bibliography.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

‡Circuits independent of frequency are shown in Figs. 8, 9, 11, 12, 13, and 14.

multi-winding transformer, a group of parallel transmission lines, the multi-winding generator, etc.[3] The usual problem in such systems is to determine the current in each branch having given the voltage impressed across each circuit.

The multi-circuit problem may always be solved analytically. However, it frequently happens that to do so involves a prohibitive amount of labor, in which case it is desirable to set up the system on the calculating board for solution. If the board is to be used, the system of coupled circuits must be replaced by an equivalent linkage, since magnetic coupling of adequate flexibility usually cannot be simulated on the calculating board.

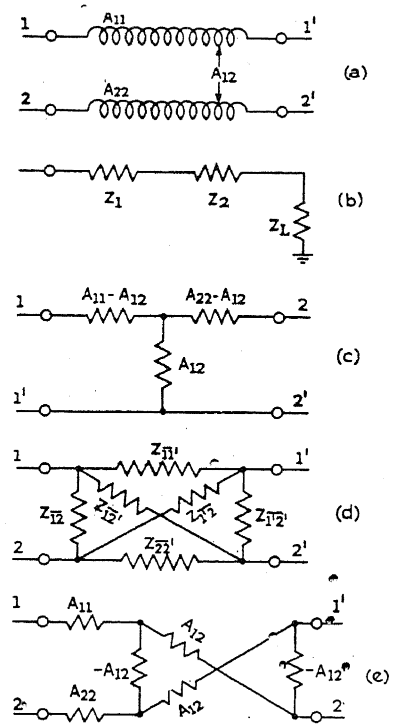

FIG. 1—VARIOUS EQUIVALENT CIRCUITS FOR THE TWO-WINDING TRANSFORMER

a. The two-winding transformer

b. Equivalent circuit used for calculating regulation, magnetizing current being neglected. This circuit gives no voltage transformation

c. Equivalent circuit which gives voltage transformation and does not neglect magnetizing current. This circuit must be viewed from one winding only, since it falls down when viewed as a 4-point network

d. A rigorous equivalent for the two-winding transformer which cannot be distinguished from the actual transformer under any imposed terminal conditions

e. An equivalent equally as rigorous as that of Fig. 1*d*, but much simpler

Before discussing the general equivalent circuit for the n-winding transformer, which is derived in Appendix A, it seems pertinent to consider other well known equivalents for coupled circuits, and to point out their limitations as compared with the present development.

In order to study the various types of circuits, the two-winding transformer shown in Fig. 1a will be analyzed. Here, A_{11} is the primary self or magnetizing impedance, A_{22} is the secondary impedance, and A_{12} is the mutual impedance between the windings.

In Fig. 1b is shown a conventional equivalent for the two-winding transformer. This circuit consists of the primary leakage impedance, Z_1 (resistance plus leakage reactance), the secondary leakage impedance, Z_2, and the load impedance, Z_L, all three of which must be expressed on a common voltage base since the same current flows in each. If α is the ratio of primary to secondary turns, then the values of Z_1 and Z_2 referred to the primary are in terms of the characteristic A coefficients,

$$Z_1 = A_{11} - A_{12}\alpha \qquad (1)$$

$$Z_2 = A_{22}\alpha^2 - A_{12}\alpha \qquad (2)$$

Note in particular that this circuit neglects magnetizing current, and that it does not simulate the voltage transformation of the actual transformer. The circuit of Fig. 1b is quite useful in calculating regulation since magnetizing current in a loaded transformer can usually be neglected. This equivalent is universally used to represent the two-winding transformer in the single line diagram of a polyphase system. Similar circuits have been developed for the three-winding transformer* the double-winding generator, etc.

Now consider the circuit shown in Fig. 1c, which likewise represents a two-winding transformer with loaded secondary. Unlike the circuit of Fig. 1b, this circuit gives actual voltage transformation. Moreover, it does not neglect magnetizing current. The magnetizing impedances of the primary and secondary are respectively A_{11} and A_{22}. The short-circuit impedance of the primary is

$$Z = \frac{A_{11}A_{22} - A_{12}^2}{A_{22}} \qquad (3)$$

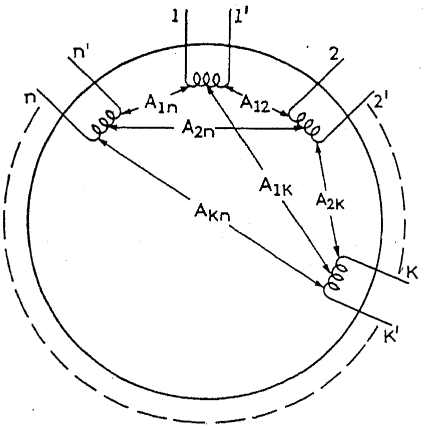

FIG. 2—A SYSTEM OF *n* ELECTRICALLY ISOLATED BUT MAGNETICALLY COUPLED CIRCUITS

Such a system may be replaced by an equivalent mesh of $n(2n - 1)$ links connecting each of the $2n$ points of entry in all possible ways. For example of equivalent mesh when $n = 2$, see Fig. 1d

---

*Mr. A. Boyajian demonstrated the value of this type of circuit in analyzing the three-winding transformer in his paper, *Theory of Three-Circuit Transformers*, TRANS. A.I.E.E., Feb. 1924. In the same paper, he writes the general equations for setting up a similar type of circuit for the n-winding transformer.

Thus, it is seen that the circuit of Fig. 1c more nearly represents the actual transformer. However, as a network having four points of entry (*i.e.*, 1, 1′, 2, and 2′), the equivalent of Fig. 1c is still not strictly correct. For example, the impedance between points 1 and 2, $Z_{1\text{-}2}$ is infinite on the actual transformer, whereas on the equivalent circuit, this impedance has a finite value.

The General Equivalent Circuit. A true equivalent circuit for a transformer is one which cannot be distinguished from the actual transformer by any observations which can be made at the terminals. Evidently the circuits (*b*) and (*c*) of Fig. 1 fall down in this respect.

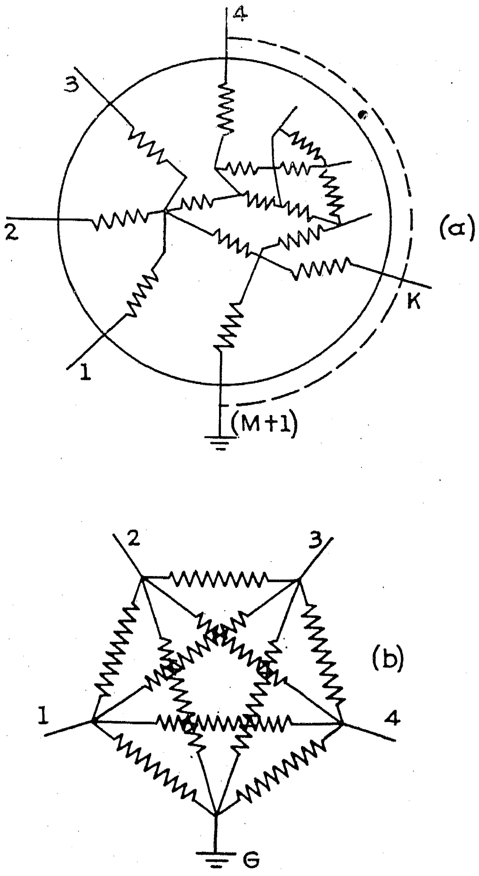

FIG. 3

a. Diagrammatic representation of the general network composed of many erratic links and having $(m+1)$ points of entry. A general network of this type may be simplified by replacing it with an equivalent mesh whose links are easily evaluated in terms of simple impedance measurements made at the terminals

b. Example of the equivalent mesh for a network having five points of entry, the fifth point being considered as a ground point

In Appendix A, there is derived a general equivalent circuit for the n-winding transformer having $2n$ points of entry. This circuit is a mesh composed of $n\,(2n-1)$ impedance links, one connecting each point of entry with every other point of entry. It is shown that the impedance of the link connecting the two points of entry of some winding j, *i. e.*, points j and j', is the driving point impedance from point j; and the link connecting two points of entry of two different windings, say points j and k' of windings j and k, is equal to the transfer impedance between points j and k'.

In Appendix A, it is further shown how these link impedances may be measured directly or calculated from known transformer constants, the A coefficients.

The general equivalent circuit for the two-winding transformer* is shown in Fig. 1d, and is seen to consist of a six-link mesh, the values of which are[4–8]

$$Z_{\overline{11'}} = \frac{A_{11}A_{22} - A_{12}{}^2}{A_{22}} \tag{4}$$

$$Z_{\overline{22'}} = \frac{A_{11}A_{22} - A_{12}{}^2}{A_{11}} \tag{5}$$

$$Z_{\overline{12}} = \frac{A_{11}A_{22} - A_{12}{}^2}{A_{12}} \tag{6}$$

$$= Z_{\overline{1'2'}} = -Z_{\overline{12'}} = -Z_{\overline{1'2}}$$

Although there are six links, it is evident from the above equations that only three of them are independent, since four links have values differing in sign only.

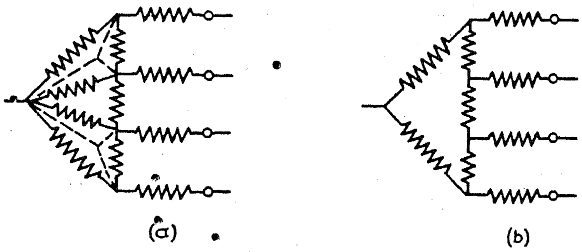

FIG. 4—THESE DIAGRAMS ARE GIVEN TO SHOW THAT AN EQUIVALENT MAY NOT ALWAYS BE UNIQUE

The circuit (*a*) has 11 links only, 9 of which are unique and independent since, by the indicated Y-delta transformations, it may be reduced to circuit (*b*) which has only 9 links. Note that the number of points of entry has not changed

A circuit which is much simpler than that of Fig. 1d, but equally as rigorous and having identical properties, is that shown in Fig. 1e. This circuit is seen to be a combination mesh and star arrangement, and may be arrived at by simple reasoning from the mesh of Fig. 1d. Since the self-impedance of any winding is independent of the other windings, its value need not appear in any of the mutual links if it be lumped as a radial link at the terminals of the circuit. Thus, setting A_{11} and A_{22} equal to zero in equations (4), (5), and (6), and inserting radial links having impedances equal to A_{11} and A_{22} in terminals 1 and 2 respectively, one will obtain the circuit of Fig. 1e. This latter circuit is much more useful than that of Fig. 1d since the mutual links are purely reactive, the resistances appearing only in the terminal links. This arrangement avoids the possibility of negative resistance appearing in any link.

In an entirely similar manner, the mesh equivalent of any number of coupled circuits may be simplified and rendered more useful by reducing to a combination star-

---

*The circuit for the two-winding transformer was given by Geo. A. Campbell in his paper, *Cisoidal Oscillations*, PROC. A.I.E.E., 1911. Equations for the n-winding transformer, however, have never been published.

mesh arrangement. A rigorous proof for this transformation is given in Appendix A.

Part II

THE NETWORK PROBLEM

In the study of the behavior of a large transmission and distribution system, the transmission lines and connected apparatus are commonly represented by an appropriate single line diagram, all units of connected equipment being replaced with suitable impedance links. In most cases, no trouble is experienced in mapping out the single-line diagram, at least none for the positive and negative phase diagrams,* since standard methods are available for doing this.[9,10]

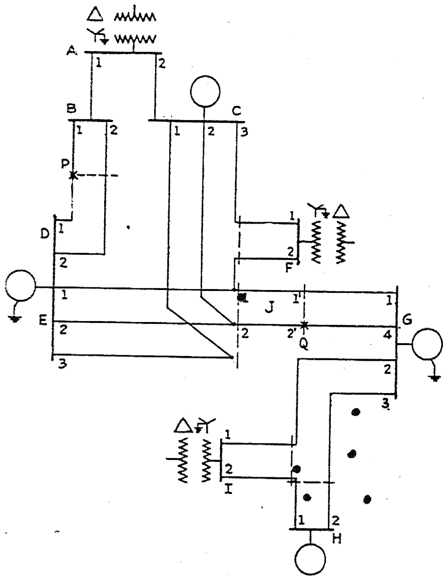

FIG. 5—SINGLE-LINE DIAGRAM OF A SYSTEM OF TRANSMISSION LINES

This system is made up of several groups of parallel lines. In Figs. 8 to 14 is given a number of equivalent circuits to represent such groups when the zero-phase diagram is to be set up on the calculating board

In setting up the single line diagram on the calculating board, for purposes of simplification, it may be desirable to replace either the whole system or some section of it with an equivalent impedance mesh which has the same number of points of entry but a greatly reduced number of links.[12] For example, Fig. 3a shows a complicated network of impedance links having $(m + 1)$ points of entry. It is shown in Appendix B how such a network may be replaced with an equivalent mesh having $\frac{m\,(m + 1)}{2}$ links, one connecting each point of entry with every other point of entry. The equivalent mesh for a five-point network is shown in Fig. 3b (note that $m = 4$ and that the $m + 1$ point is considered as a ground point for purposes of analysis).

The values of the links of the equivalent mesh which is to replace the general network may be determined either by direct calculation from the network constants or by measurements made at the terminals of the network. It is shown in Appendix B that the impedance of the link connecting any two points of entry such as j and k is equal to the transfer impedance between the points j and k, *i. e.*,

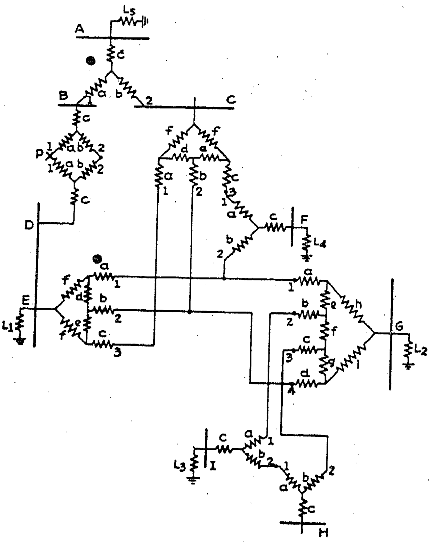

FIG. 6—EQUIVALENT CIRCUIT FOR THE ZERO-PHASE DIAGRAM OF THE SYSTEM SHOWN IN FIG. 5 FOR A GROUND FAULT AT POINT *P*

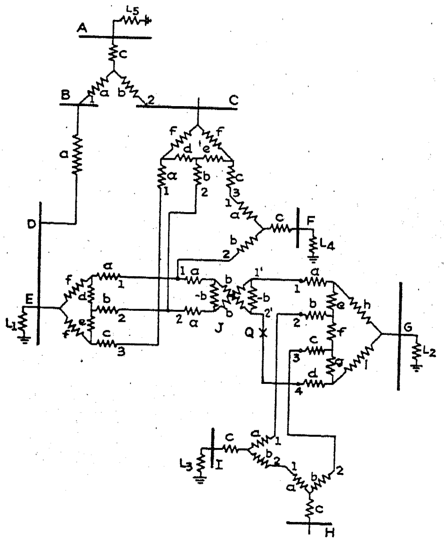

FIG. 7—EQUIVALENT CIRCUIT FOR THE ZERO-PHASE DIAGRAM OF THE SYSTEM SHOWN IN FIG. 5 FOR A GROUND FAULT AT POINT *Q*

$$Z_{\overline{jk}} = Z_{jk} \qquad (7)$$

It is further shown that the impedances of the links of the equivalent mesh may be calculated by determinants in terms of the through impedances of the network (by through impedance between any two points such as j and k is meant the impedance between these points with all other points open). In terms of the notation defined in Appendix E

---

*The zero-phase single-line diagram of a system is not so easily set up if the system involves groups of parallel transmission lines. This particular problem is discussed fully in Part III of the paper.

$$Z_{jk} = \frac{\Delta_{m-c}}{Q_{c-jk}} \tag{8}$$

where the characteristic coefficients of the determinants Δ_{m-c} and Q_{c-jk} are

$$C_{jj} = Z_{j-0} \tag{9}$$

$$C_{kk} = Z_{k-0} \tag{10}$$

$$C_{jk} = \frac{Z_{j-0} + Z_{k-0} - Z_{j-k}}{2} \tag{11}$$

The methods developed in Appendix B apply to any general network provided the impedances are linear and provided there are no internal voltages. Note that, although a network is usually composed of simple impedance links, the general network as it is considered here may involve magnetic coupling between any of its elements. In fact, the general network mesh may be applied directly in obtaining an equivalent mesh derived in Appendix A for the n-winding transformer is much simpler and more appropriate to the transformer problem than is the general network mesh.

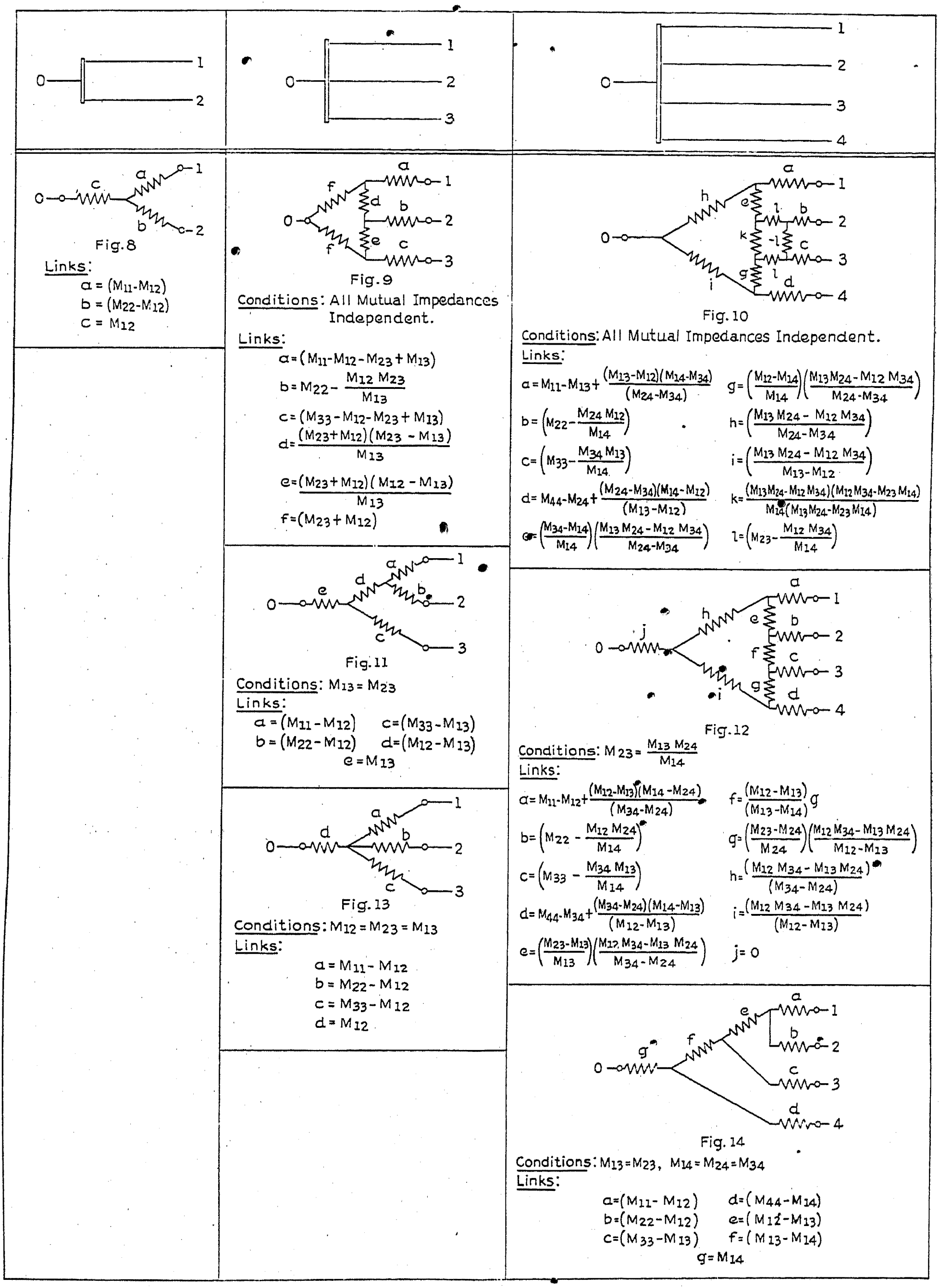

Fig. 8

Links:

$a = (M_{11} - M_{12})$

$b = (M_{22} - M_{12})$

$c = M_{12}$

Fig. 9

Conditions: All Mutual Impedances Independent.

Links:

$a = (M_{11} - M_{12} - M_{23} + M_{13})$

$b = M_{22} - \frac{M_{12} M_{23}}{M_{13}}$

$c = (M_{33} - M_{12} - M_{23} + M_{13})$

$d = \frac{(M_{23} + M_{12})(M_{23} - M_{13})}{M_{13}}$

$e = \frac{(M_{23} + M_{12})(M_{12} - M_{13})}{M_{13}}$

$f = (M_{23} + M_{12})$

Fig. 10

Conditions: All Mutual Impedances Independent.

Links:

$a = M_{11} - M_{13} + \frac{(M_{13} - M_{12})(M_{14} - M_{34})}{(M_{24} - M_{34})}$

$b = \left(M_{22} - \frac{M_{24} M_{12}}{M_{14}}\right)$

$c = \left(M_{33} - \frac{M_{34} M_{13}}{M_{14}}\right)$

$d = M_{44} - M_{24} + \frac{(M_{24} - M_{34})(M_{14} - M_{12})}{(M_{13} - M_{12})}$

$e = \left(\frac{M_{34} - M_{14}}{M_{14}}\right)\left(\frac{M_{13} M_{24} - M_{12} M_{34}}{M_{24} - M_{34}}\right)$

$g = \left(\frac{M_{12} - M_{14}}{M_{14}}\right)\left(\frac{M_{13} M_{24} - M_{12} M_{34}}{M_{24} - M_{34}}\right)$

$h = \left(\frac{M_{13} M_{24} - M_{12} M_{34}}{M_{24} - M_{34}}\right)$

$i = \left(\frac{M_{13} M_{24} - M_{12} M_{34}}{M_{13} - M_{12}}\right)$

$k = \frac{(M_{13} M_{24} - M_{12} M_{34})(M_{12} M_{34} - M_{23} M_{14})}{M_{14}(M_{13} M_{24} - M_{23} M_{14})}$

$l = \left(M_{23} - \frac{M_{12} M_{34}}{M_{14}}\right)$

Fig. 11

Conditions: $M_{13} = M_{23}$

Links:

$a = (M_{11} - M_{12})$ $c = (M_{33} - M_{13})$

$b = (M_{22} - M_{12})$ $d = (M_{12} - M_{13})$

$e = M_{13}$

Fig. 12

Conditions: $M_{23} = \frac{M_{13} M_{24}}{M_{14}}$

Links:

$a = M_{11} - M_{12} + \frac{(M_{12} - M_{13})(M_{14} - M_{24})}{(M_{34} - M_{24})}$

$b = \left(M_{22} - \frac{M_{12} M_{24}}{M_{14}}\right)$

$c = \left(M_{33} - \frac{M_{34} M_{13}}{M_{14}}\right)$

$d = M_{44} - M_{34} + \frac{(M_{34} - M_{24})(M_{14} - M_{13})}{(M_{12} - M_{13})}$

$e = \left(\frac{M_{23} - M_{13}}{M_{13}}\right)\left(\frac{M_{12} M_{34} - M_{13} M_{24}}{M_{34} - M_{24}}\right)$

$f = \frac{(M_{12} - M_{13})}{(M_{13} - M_{14})} g$

$g = \left(\frac{M_{23} - M_{24}}{M_{24}}\right)\left(\frac{M_{12} M_{34} - M_{13} M_{24}}{M_{12} - M_{13}}\right)$

$h = \frac{(M_{12} M_{34} - M_{13} M_{24})}{(M_{34} - M_{24})}$

$i = \frac{(M_{12} M_{34} - M_{13} M_{24})}{(M_{12} - M_{13})}$

$j = 0$

Fig. 13

Conditions: $M_{12} = M_{23} = M_{13}$

Links:

$a = M_{11} - M_{12}$

$b = M_{22} - M_{12}$

$c = M_{33} - M_{12}$

$d = M_{12}$

Fig. 14

Conditions: $M_{13} = M_{23}$, $M_{14} = M_{24} = M_{34}$

Links:

$a = (M_{11} - M_{12})$ $d = (M_{44} - M_{14})$

$b = (M_{22} - M_{12})$ $e = (M_{12} - M_{13})$

$c = (M_{33} - M_{13})$ $f = (M_{13} - M_{14})$

$g = M_{14}$

FIGS. 8–14

Part III

THE ZERO PHASE SYSTEM DIAGRAM

As pointed out above, the zero phase diagram of a system which involves parallel transmission lines does not lend itself so readily to a calculating board set up. This is owing to the fact that, when zero phase currents are flowing in the parallel lines, the mutual magnetic coupling between them cannot be neglected; neither can it be adequately represented with the usual calculating board equipment.*

A typical example of a system involving groups of parallel lines is shown in Fig. 5. The analytical determination of the currents in every branch of this system due to a ground fault is practically impossible, but if each group of parallel lines is replaced with an equivalent linkage, the calculating board may be applied to obtain an exact solution.

A group of circuits representing two, three, and four parallel lines under various conditions is shown in Figs. 8 to 14 inclusive. The derivation of these circuits is discussed in Appendix C. Since more than four parallel lines seldom exist for any great distance, equivalents for groups of more than four lines have not been derived. However, should a case arise in which more than four parallel lines do occur, appropriate circuits may be readily developed by applying the rules outlined in Appendix C.

In regard to this special group of equivalents for parallel lines, it should be noted that:

1. The values of impedance links are short usable expressions in terms of the self and mutual impedances of the lines, which are constants usually known.
2. Negative impedance links may usually be avoided in the application of these circuits (the circuit of Fig. 10 being excepted) thereby permitting the use of the d-c. calculating board where resistance may be neglected.
3. The self-impedance of each line appears only in the radial link at the terminal, the mutual links being purely reactive. This arrangement insures that negative resistance will always be avoided.

To illustrate the utility of these special circuits, the zero phase diagram of the system of Fig. 5 is set up for the solution of ground faults in Appendix D. The equivalent network is fully developed for two different fault conditions.

ACKNOWLEDGMENT

The author gratefully acknowledges the aid of Miss Edith Clarke in critically reviewing the manuscript.

Appendix A

THE GENERAL EQUIVALENT CIRCUIT FOR THE *N*-WINDING TRANSFORMER

Consider the system of mutually coupled circuits shown in Fig. 2, and having $2n$ points of entry. The canonical equations which completely define such a system are[17]

$$\begin{aligned} V_1 &= A_{11} I_1 + A_{12} I_2 + \ldots\ldots A_{1n} I_n \\ &\vdots \\ V_k &= A_{k1} I_1 + A_{k2} I_2 + \ldots\ldots A_{kn} I_n \qquad \text{(1a)} \\ &\vdots \\ V_n &= A_{n1} I_1 + A_{n2} I_2 + \ldots\ldots A_{nn} I_n \end{aligned}$$

in which V_k is the voltage impressed across the terminals k and k' of any circuit k, and I_k is the current flowing from k to k'. If A_{jk} is the characteristic impedance coefficient, there is in a conservative system

$$A_{jk} = A_{kj} \qquad \text{(2a)}$$

Note that coefficients of the form A_{jk} represent mutual impedances (or reactances in this case) of the system, whereas coefficients of the form A_{jj} represent self or magnetizing impedances.

Solving equation (**1a**) for the currents

$$\begin{aligned} I_1 &= B_{11} V_1 + B_{12} V_2 + \ldots\ldots B_{1n} V_n \\ &\vdots \\ I_k &= B_{k1} V_1 + B_{k2} V_2 + \ldots\ldots B_{kn} V_n \qquad \text{(3a)} \\ &\vdots \\ I_n &= B_{n1} V_1 + B_{n2} V_2 + \ldots\ldots B_{nn} V_n \end{aligned}$$

in which[16]

$$B_{jk} = B_{kj} = \frac{Q_{A-jk}}{\Delta_{n-A}} (-1)^{j+k} \qquad \text{(4a)}$$

where

$$\Delta_{n-A} = \begin{vmatrix} A_{11}\, A_{12} \ldots\ldots A_{1n} \\ \vdots \\ A_{k1}\, A_{k2} \ldots\ldots A_{kn} \\ \vdots \\ A_{n1}\, A_{n2} \ldots\ldots A_{nn} \end{vmatrix} \qquad \text{(5a)}$$

and Q_{A-jk} is the cofactor of A_{jk} in Δ_{n-A}, (*i. e.*, the minor of Δ_{n-A} with the j'th column and k'th row deleted), the sign of which is $(-1)^{j+k}$.

There are n^2 coefficients in equation (**1a**), $\dfrac{n(n+1)}{2}$ of which are independent. Therefore an equivalent linkage which represents the system must have $2n$ points of entry and at least $\frac{n\,(n+1)}{2}$ independent links. A convenient equivalent linkage which satisfies these conditions is the straight mesh consisting of a system of links, one connecting each point of entry with every other point of entry (for example of mesh when $n = 2$, see Fig. 1d).

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*It is conceivable that actual transformers might be used for this purpose, but the multiplicity of windings with the highly flexible couplings required make such a method impracticable.

Evaluation of the Links of the Equivalent Mesh. In order to evaluate the links of the equivalent mesh, let all points of entry of the actual system be grounded except point k of circuit k. Then all impressed voltages except V_k will be zero and equation (**3a**) becomes

$$\begin{aligned} I_1 &= B_{1k}\,V_k \\ &\;\vdots \\ I_k &= B_{kk}\,V_k \\ &\;\vdots \\ I_n &= B_{nk}\,V_k \end{aligned} \tag{6a}$$

If the condition of grounding is likewise applied to the equivalent mesh, there results the analagous group of relations

$$\begin{aligned} I_1 &= \frac{-V_k}{Z_{\overline{k1}}} = \frac{V_k}{Z_{\overline{k1'}}} \\ &\;\vdots \\ I_k &= \frac{V_k}{Z_{\overline{kk}}} \\ &\;\vdots \\ I_n &= \frac{-V_k}{Z_{\overline{kn}}} = \frac{V_k}{Z_{\overline{kn'}}} \end{aligned} \tag{7a}$$

where $Z_{\overline{jk}}$ is the impedance of the link connecting any two points of entry j and k. The relations of equation (**7a**) follow from the fact that the link connecting point k with the grounded point k' carries current I_k, and the link connecting point k with any point such as j or j' carries current I_j. This must be true since all links between pairs of grounded points can carry no current.

From equations (**6a**) and (**7a**), the following general relations are evident:

$$Z_{\overline{kk'}} = \frac{1}{B_{kk}} \tag{8a}$$

$$Z_{\overline{jk'}} = \frac{1}{B_{\overline{jk}}} = Z_{\overline{j'k}} = -\,Z_{\overline{j'k'}} = -\,Z_{\overline{jk}} \tag{9a}$$

It may be observed that links of the equivalent mesh connecting the two points of entry of the same winding are driving point impedances, and links connecting points of entry of different windings are transfer impedances, all of which may be measured directly as indicated in equation (**7a**), or calculated as indicated in equation (**4a**).

The Combination Star-Mesh Equivalent. The general equivalent developed above for the n-winding transformer may be reduced to a much simpler combination star-mesh equivalent, the derivation and proof of which follow.

Let equation (**4a**) be rewritten as

$$\begin{aligned} V_1 - A_{11}\,I_1 &= 0 + A_{12}\,I_2 + \ldots + A_{1n}\,I_n \\ &\;\vdots \\ V_k - A_{kk}\,I_k &= A_{k1}\,I_1 + A_{k2}\,I_2 + \ldots + A_{kn}\,I_n \\ &\;\vdots \\ V_n - A_{nn}\,I_n &= A_{n1}\,I_1 + A_{n2}\,I_2 + \ldots + 0 \end{aligned} \tag{10a}$$

Considering the left-hand members of equation (**10a**) as the impressed voltages, and solving for currents as before, one obtains

$$\begin{aligned} I_1 &= B'_{11}\,(V_1 - A_{11}\,I_1) + B'_{12}\,(V_2 - A_{22}\,I_2) + \ldots \\ &\qquad + B'_{1n}\,(V_n - A_{nn}\,I_n) \\ &\;\vdots \\ I_k &= B'_{k1}\,(V_1 - A_{11}\,I_1) + B'_{k2}\,(V_2 - A_{22}\,I_2) + \ldots \\ &\qquad + B'_{kn}\,(V_n - A_{nn}\,I_n) \\ &\;\vdots \\ I_n &= B'_{n1}\,(V_1 - A_{11}\,I_1) + B'_{n2}\,(V_2 - A_{22}\,I_2) + \ldots \\ &\qquad + B'_{nn}\,(V_n - A_{nn}\,I_n) \end{aligned}$$

The B coefficients are primed in this case to distinguish them from those of equation (**3a**). In the same manner that equation (**3a**) was interpreted to represent a 2 n-terminal mesh (such as that of Fig. 4b for $n = 3$), equation (**11a**) may be interpreted to represent a similar mesh except that the impressed voltage on circuit k is $(V_k - A_{kk}I_{kk})$ instead of V_k. The impedances of the links of the new mesh will be

$$Z'_{\overline{jj'}} = \frac{1}{B'_{jj}} \tag{12a}$$

$$Z'_{\overline{kk'}} = \frac{1}{B'_{kk}} \tag{13a}$$

$$Z'_{\overline{jk'}} = \frac{1}{B'_{jk}} = Z'_{\overline{j'k}} = -\,Z'_{\overline{j'k'}} = -\,Z'_{\overline{jk}} \tag{14a}$$

Since I_k flows into point k, and since $A_{kk}I_{kk}$ is a voltage drop due to I_k alone, then this drop may be replaced with a series impedance link equal to A_{kk} in the terminal of point k. It is evident that the combination star-mesh is exactly equivalent to the pure mesh since a voltage V_k is impressed at terminal k and a current I_k flows into terminal k in both circuits. Note that the radial impedance link may be placed in either terminal k or k', or may be distributed in both.

Appendix B

THE EQUIVALENT MESH LINKAGE FOR A GENERAL NETWORK HAVING ANY NUMBER OF POINTS OF ENTRY

Consider the general network shown in Fig. 3a, which it is desirable to replace with an equivalent mesh linkage. Assume this network to have $(m + 1)$ points of entry, where m is any number. Let the $(m + 1)$ point of entry be the ground point or point of zero potential. If a point of zero potential does not exist, this terminal may be arbitrarily specified as such in order to provide a reference point for the voltages impressed at the other m terminals.

The canonical equations which define the network at its terminals are[17]

$$
\begin{aligned}
V_{10} &= C_{11}I_1 + C_{12}I_2 + \ldots + C_{1m}I_m \\
&\;\vdots \\
V_{k0} &= C_{k1}I_1 + C_{k2}I_2 + \ldots + C_{km}I_m \qquad \textbf{(1b)} \\
&\;\vdots \\
V_{m0} &= C_{m1}I_1 + C_{m2}I_2 + \ldots + C_{mm}I_m
\end{aligned}
$$

where V_{k0} is the voltage impressed from any point k to ground, I_k is the current flowing into point k, and C_{jk} is the characteristic impedance coefficient whose value is to be determined.

The solution of equation (**1b**) for currents gives

$$
\begin{aligned}
I_1 &= D_{11}V_{10} + D_{12}V_{20} + \ldots + D_{1m}V_{m0} \\
&\;\vdots \\
I_k &= D_{k1}V_{10} + D_{k2}V_{20} + \ldots + D_{km}V_{m0} \qquad \textbf{(2b)} \\
&\;\vdots \\
I_m &= D_{m1}V_{10} + D_{m2}V_{20} + \ldots + D_{mm}V_{m0}
\end{aligned}
$$

Expressed by means of determinants as functions of the C coefficients, the D coefficients are

$$D_{jk} = D_{kj} = \frac{Q_{c-jk}}{\Delta_{m-c}}(-1)^{j+k} \qquad \textbf{(3b)}$$

where

$$\Delta_{m-c} = \begin{vmatrix} C_{11} & C_{12} & \ldots & C_{1m} \\ \vdots & & & \\ C_{k1} & C_{k2} & \ldots & C_{km} \\ \vdots & & & \\ C_{m1} & C_{m2} & \ldots & C_{mm} \end{vmatrix} \qquad \textbf{(4b)}$$

The relations between the C and D coefficients of equations (**1b**) and (**2b**) are exactly analagous to the relations between the A and B coefficients of equations (**1a**) and (**3a**). In equation (**3b**) above, Q_{C-jk} is the cofactor of C_{jk} in Δ_{m-C}. Since a mesh composed of $\frac{m(m+1)}{2}$ links connecting $(m + 1)$ points of entry in all possible ways may be just as completely defined by equations (**1b**) and (**2b**) as the network itself, it follows that the network may be replaced by an equivalent mesh having $(m + 1)$ points of entry and $\frac{m(m+1)}{2}$ links. The equivalent mesh representing a network having five points of entry is shown in Fig. 3a.

Referring to Figs. 3a and 3b, let a voltage V_k be impressed at point k of the network and let all other points of entry be grounded. Then equation (**2b**) becomes

$$
\begin{aligned}
\frac{V_{k0}}{I_1} &= \frac{1}{D_{1k}} \\
&\;\vdots \\
\frac{V_{k0}}{I_k} &= \frac{1}{D_{kk}} \qquad \textbf{(5b)} \\
&\;\vdots \\
\frac{V_{k0}}{I_m} &= \frac{1}{D_{mk}}
\end{aligned}
$$

Now let the same conditions obtain with the equivalent mesh (the five-point mesh of Fig. 3b may be used for reference). Current entering ground from any grounded point j will be negative in sign and must flow through the link $Z_{\overline{jk}}$ since links connecting all grounded points can carry no current. In general then

$$\frac{V_{k0}}{I_j} = -Z_{\overline{jk}} = -Z_{jk} = \frac{1}{D_{jk}} \qquad \textbf{(6b)}$$

The link connecting points k and o is not the driving point impedance from k to ground as might be supposed by analogy with equation (**8a**), but rather the transfer impedance between points k and o. Thus,

$$\frac{V_{k0}}{I_0} = Z_{\overline{k0}} = \frac{1}{\sum_{j=1}^{j=m} D_{jk}} \qquad \textbf{(7b)}$$

where

$$I_0 = \sum_{j=1}^{j=m} I_j$$

The method of measuring the D coefficients is evident from equation (**5b**).

Evaluation of the C Coefficients. The C coefficients of equation (**1b**) may be evaluated by measuring voltages, whereas the D's were evaluated by measuring currents. In equation (**1b**), let all currents except I_k equal zero (*i. e.*, let all points of entry except k be opened up). Then,

$$V_{k0} = C_{kk}I_k \qquad \textbf{(8b)}$$

$$V_{j0} = C_{jk}I_k \qquad \textbf{(9b)}$$

where V_{j0} is the voltage appearing between point j and ground when current I_k flows. By impressing a voltage to ground on each of the terminals in succession, and measuring the voltage to ground appearing on each of the open terminals, equations (**8b**) and (**9b**) will suffice to determine all C coefficients. If it is not convenient to measure voltages, the following method is available.

Referring to equation (**1b**), let all currents equal zero except I_j and I_k where j and k are any two points of entry. Also let the condition $I_j + I_k = 0$ be imposed. Then

$$V_{j0} = I_jC_{jj} + I_kC_{jk} \qquad \textbf{(10b)}$$

$$V_{k0} = I_jC_{jk} + I_kC_{kk} \qquad \textbf{(11b)}$$

which when solved for currents give

$$\begin{aligned}\frac{V_{j0} - V_{k0}}{I} &= C_{jj} + C_{kk} - 2\,C_{jk} \\ &= Z_{j-k}\end{aligned} \qquad \textbf{(12b)}$$

where Z_{j-k} is the through impedance between terminals j and k.

Referring again to equation (**1b**), let V_{k0} be impressed to ground at k with all other points open. Then the through impedance from any point k to ground is

$$\frac{V_{k0}}{I_k} = Z_{k-0} = C_{kk} \qquad \textbf{(13b)}$$

Also

$$Z_{j-0} = C_{jj} \qquad \textbf{(14b)}$$

From equations (**12b**), (**13b**), and (**14b**)

$$\frac{Z_{j-0} + Z_{k-0} - Z_{j-k}}{2} = C_{jk} \qquad \textbf{(15b)}$$

Equations (**13b**), (**14b**), and (**15b**) give the fundamental relations between the C coefficients and the through impedance measurements (or calculations) of the network.

Appendix C

EQUIVALENT CIRCUITS FOR TWO, THREE, AND FOUR PARALLEL TRANSMISSION LINES BUSSED AT ONE END

For purposes of analysis, a group of parallel lines bussed at one end may be considered as a network. Thus, two parallel lines constitute a three-point network, three parallel lines constitute a four-point network, etc. Since the lines are bussed, the through impedance between any two points of entry of such a network will be finite, and therefore equations (**13b**), (**14b**), and (**15b**) may be applied.

As pointed out in Appendix B, a network having $(m + 1)$ points of entry might be represented by the general mesh equivalent. However, a network may be represented by other equivalents having the same number of independent links provided the arrangement of links is unique and non-reducible. Circuits of this type are shown in Figs. 8 to 14 inclusive, and are seen to be combination star-mesh arrangements. The reasons for adopting these particular linkage arrangements will be evident from the following derivations.

TWO PARALLEL LINES BUSSED AT ONE END, FIG. 8

Consider the case of two parallel lines bussed at one end and having self-impedances of M_{11} and M_{22},[*] and having a mutual impedance between them of M_{12}. The system is a three-point network, and may be represented by two voltage equations. If the bus be considered as the ground or zero point, there is from equations (**13b**), (**14b**), and (**15b**), noting that the M coefficients replace the C coefficients in this case,

$$Z_{1-0} = M_{11} \qquad \textbf{(1c)}$$

$$Z_{2-0} = M_{22} \qquad \textbf{(2c)}$$

$$\frac{Z_{1-0} + Z_{2-0} - Z_{1-2}}{2} = M_{12} \qquad \textbf{(3c)}$$

Since there are $\frac{m\,(m + 1)}{2}$ or 3 independent coefficients which describe the system, there must be a minimum of three links in the equivalent. Either a Y or a delta satisfies the necessary conditions, but a Y is simpler and will be used here. Referring to Fig. 8,[†] one may write

$$Z_{1-0} = a + c \qquad \textbf{(4c)}$$

$$Z_{2-0} = b + c \qquad \textbf{(5c)}$$

$$Z_{1-2} = a + b \qquad \textbf{(6c)}$$

---

[*] By self-impedance is meant a term of the form $(r + jx)$ where x is a reactance corresponding to self-inductance. The mutual coefficient M_{jk} is purely reactive. In some cases, resistance may be neglected, in which cases all coefficients are reactive.

[†] This circuit was derived by Edith Clarke in *Simultaneous Faults*, (see Bibliography 8).

From equations (**1c**) to (**6c**), it follows that

$$a = M_{11} - M_{12} \qquad (7c)$$

$$b = M_{22} - M_{12} \qquad (8c)$$

$$c = M_{12} \qquad (9c)$$

FOUR PARALLEL LINES BUSSED AT ONE END, FIG. 12

Here the method of attack is exactly the same as in the previous case. In this case,

$$m = 4, \quad \frac{m\,(m+1)}{2} = 10,$$

and the minimum number of links is ten. The radial links a, b, c, and d identify the self impedance coefficients, and the links in the loop identify the mutuals.

Link j has been included to illustrate a principle. Note in the simultaneous equations below that j appears in exactly the same manner in each equation. Hence, it will be automatically eliminated in the solution and cannot be considered as an independent link. The circuit of Fig. 12 does not have, then, the requisite ten links but has only nine, one mutual link being deficient. Since six mutual impedances are being expressed in terms of five links, it is necessary that one of the mutuals be related to the others in some manner. In general, one may choose which mutual is to be independent.

From Fig. 12, the ten circuit equations for this case are

$$Z_{1-0} = a + j + \frac{(e + f + g + i)\,h}{D} \qquad (10c)$$

$$Z_{2-0} = b + j + \frac{(f + g + i)\,(h + e)}{D} \qquad (11c)$$

$$Z_{3-0} = c + j + \frac{(h + e + f)\,(g + i)}{D} \qquad (12c)$$

$$Z_{4-0} = d + j + \frac{(h + e + f + g)\,i}{D} \qquad (13c)$$

$$Z_{1-2} = a + b + \frac{(f + g + i + h)\,e}{D} \qquad (14c)$$

$$Z_{1-3} = a + c + \frac{(g + i + h)\,(e + f)}{D} \qquad (15c)$$

$$Z_{1-4} = a + d + \frac{(e + f + g)\,(h + i)}{D} \qquad (16c)$$

$$Z_{2-3} = b + c + \frac{(g + i + h + e)\,f}{D} \qquad (17c)$$

$$Z_{2-4} = b + d + \frac{(h + e + i)\,(f + g)}{D} \qquad (18c)$$

$$Z_{3-4} = c + d + \frac{(h + e + f + i)\,g}{D} \qquad (19c)$$

where

$$D = e + f + g + h + i \qquad (20c)$$

Applying the general equations (**13b**), (**14b**), and (**15b**) simultaneously as before, and solving equations (**10c**) to (**19c**) simultaneously, one obtains for the impedances of the ten links,

$$a = M_{11} - M_{12} + \frac{(M_{11} - M_{13})\,(M_{14} - M_{24})}{(M_{34} - M_{24})} \qquad (21c)$$

$$b = M_{22} - \frac{M_{12}\,M_{24}}{M_{14}} \qquad (22c)$$

$$c = M_{33} - \frac{M_{34}\,M_{13}}{M_{14}} \qquad (23c)$$

$$d = M_{44} - M_{34} + \frac{(M_{34} - M_{24})\,(M_{14} - M_{13})}{(M_{12} - M_{13})} \qquad (24c)$$

$$e = \frac{(M_{23} - M_{13})}{M_{13}} \left(\frac{M_{12}\,M_{34} - M_{13}\,M_{24}}{M_{34} - M_{24}} \right) \qquad (25c)$$

$$f = \frac{(M_{12} - M_{13})}{(M_{13} - M_{14})}\, g \qquad (26c)$$

$$g = \left(\frac{M_{23} - M_{24}}{M_{24}} \right) \left(\frac{M_{12}\,M_{34} - M_{13}\,M_{24}}{M_{12} - M_{13}} \right) \qquad (27c)$$

$$h = \left(\frac{M_{12}\,M_{34} - M_{13}\,M_{24}}{M_{34} - M_{24}} \right) \qquad (28c)$$

$$i = \left(\frac{M_{12}\,M_{34} - M_{13}\,M_{24}}{M_{12} - M_{13}} \right) \qquad (29c)$$

$$j = 0 \qquad (30c)$$

and the condition that

$$M_{23} = \frac{M_{13}\,M_{24}}{M_{14}}$$

If the parallel lines are grouped at equal spacings in the same horizontal plane, the above relation is closely approximated, and the circuit may be used without great error. However, it is shown in Appendix D that when this circuit is used as a part of a major network, it may be applied without any error by shifting the unbalanced part of M_{23} to another part of the system.

The circuit which provides ten fully independent links for the case of four parallel lines is shown in Fig. 10. Preference is to be given to the circuit of Fig. 12 however, when it can be used, because of its simplicity and because of the absence of negative links.

Rules for Setting Up Equivalent Circuits. From the foregoing analysis and discussion, the following rules for setting up an arbitrary equivalent circuit to represent any network having $(m + 1)$ points of entry and having finite impedances between each pair of terminals, may be inferred:

1. The equivalent circuit must have the same number of points of entry as the network.

2. Assign the $(m+1)$ point of the network as the ground point to which all voltages may be referred.

3. Set up an equivalent linkage having a minimum of $\frac{m(m+1)}{2}$ links, remembering that:

a. $\frac{m(m-1)}{2}$ of these links must be mutual with more than one circuit.

b. It must not be possible to reduce the equivalent, by a Y-delta transformation or similar artifice, to one having fewer links (for example, see Fig. 4).

c. There must be no link common to all of the m circuits in the same way (for example, see link j Fig. 12).

d. More than the required number of links, if used judiciously, often simplify the result (for example, see link c Fig. 9).

e. If less than the required number of links are used, there will be as many coefficients which are not independent as there are deficient links.

4. Evaluate the M coefficients by measuring or calculating the through impedances of the network and applying equations (**13b**), (**14b**), and (**15b**).

5. Set up the values of the through impedances of the equivalent circuit in terms of the unknown links.

6. Solve simultaneously the $m(m+1)$ relations of items 4 and 5 above.

Appendix D

APPLICATION OF THE SPECIAL CIRCUITS OF APPENDIX C TO THE ZERO PHASE DIAGRAM OF A TRANSMISSION SYSTEM

To illustrate the utility of the special circuits shown in Figs. 8 to 14 inclusive for representing groups of parallel transmission lines in the zero phase diagram of a system, the following typical problem and solution are given.

Problem. Map out the zero phase impedance diagram, in a form suitable for setting up on the calculating board, of the transmission system shown in Fig. 5.*

Solution. It may be noted that this system includes eight buses. Four of them are connected to rotating machines two of which are grounded, and three are connected to transformers all of which are grounded. The zero phase diagram of the system of Fig. 5 is shown in Fig. 6 for a fault at point P, and in Fig. 7 for a fault at point Q.

The method of obtaining the equivalent diagrams in Figs. 6 and 7 is evident. The group of magnetically coupled parallel lines emanating from any one bus (each bus is designated with a capital letter) has been replaced with an appropriate equivalent circuit. Note that each equivalent includes line constants up to the dotted lines shown in Fig. 5. Further note in Fig. 5 that D and E are the same bus but are considered as separate circuits since the lines form no mutual with those from D. Circuit J is not associated with a bus, but merely consists of two parallel lines with four points of entry. It is necessary to segregate this portion of lines 1 and 4 emanating from G for the case of a fault at Q. For the case of the fault at P, the constants of circuit J are included with those of circuit G.

Bus G has four lines, all of which are mutually coupled for a certain distance. Space does not permit the inclusion of the circuit constants which were used in the solution of this particular system. However, although the relation $M_{23} = M_{13} M_{24}/M_{14}$ is not quite satisfied by the actual system constants, (the actual value of M_{23} being slightly greater than that obtained from the above relation), the circuit of Fig. 12 was applied. The additional impedance mutual between lines 2 and 3, which could not be included in this circuit, was added to the mutual link between lines 1 and 2 emanating from bus I. Of course this shifting of mutuals is permissible only provided it is not intended to examine voltage conditions at some point in the equivalent circuit between buses G and I.

In setting up the equivalent system shown in Fig. 6 for a fault at point P, it was found that in no case did any of the circuits involve negative links. The significance of this fact is that, in case resistance may be neglected, the lay out may be adapted to a d-c. calculating board. Note that the circuit shown in Fig. 6 may be used without change for a number of fault conditions such as any bus fault for example.

The zero phase impedances in the machines and transformers are included in the L links shown in Figs. 6 and 7. Incidentally, it presumably is evident that any lumped reactance in a line, such as a transformer reactance, may be added directly to the self reactance of the line and the total used as a coefficient in setting up the equivalent circuit.

Appendix E

NOTATION

A_{jj} —Characteristic magnetizing impedance of any winding j in the multi-winding transformer.

A_{jk} —Characteristic mutual impedance between any two windings, j and k, of the multi-winding transformer.

B_{jj} —Characteristic admittance of any winding j of a transformer with all other windings short-circuited (also driving point impedance from point j).

B_{jk} —Characteristic admittance of the multi-winding transformer corresponding to a voltage impressed on winding j and a current in winding k with all windings except j short-circuited, and *vice versa*.

C_{jj} —Characteristic impedance between any point j of a network and ground.

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*This layout is that of an actual system which was received for solution.

C_{jk} —Characteristic mutual impedance between any two circuits of a network j and k—the voltage appearing between point of entry j and ground when unit current flows from point of entry k to ground, and *vice versa*.

D_{jj} —Characteristic admittance of a network measured from point j to ground with all other points of entry grounded (driving point admittance).

D_{jk} —Characteristic admittance of a network corresponding to a voltage impressed from point of entry j to ground and a current flowing to ground from point of entry k, when all points except j are grounded (transfer admittance).

M_{jj} —Self-impedance of any transmission line, j.

M_{jk} —Mutual reactance between any two transmission lines, j and k, *i. e.*, the voltage induced in one due to unit current in the other.

Δ_{n-A} —N'th order determinant in A.

Δ_{m-0} —M'th order determinant in C.

Q_{A-jk}—Cofactor of A_{jk} in A

$Z_{\overline{jk}}$ —Impedance of a link connecting any two points, j and k, in general mesh.

Z_{j-k} —Through impedance measured between points j and k of a system or network (assuming all other points of entry open).

Z_{jk} —Transfer impedance between points j and k of a network or system.

Z_{jj} —Driving point impedance to ground from any point j of a network or system.

V_k —Voltage impressed across any winding k of a multi-winding transformer.

V_{k0} —Voltage impressed from any point k to ground in a general network.

I_k —Current flowing in any winding k of a multi-winding transformer, or the current entering any point of entry k of the general network.

m —Any integral number referring to the number of points of entry of a general network (excluding the ground pcint).

n —Any integral number referring to the number of windings of the multi-winding transformer.

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Discussion

For discussion of this paper see page 321.

Transient Oscillations of Mutually Coupled Windings

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Synopsis.—The paper is a mathematical study of the oscillations occurring in a pair of mutually coupled windings—the primary and secondary of a transformer—when one of the terminals is subjected to the impact of a traveling wave. Solutions for grounded and open terminal conditions are obtained, but the general solution for any impedances at the terminals is not attempted, except for the initial distribution.

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INTRODUCTION

In the past, the mathematical analysis of transient oscillations in transformer windings has been based upon a single winding having self and mutual inductances between its turns, capacitances along the stack and to ground, and in one instance resistances to represent the losses.[1] Such a circuit ignores the presence of secondary circuits, but strangely enough, proves quite adequate to describe the internal high-frequency transients of the winding under consideration, not only qualitatively, but quantitatively as well. However, cathode ray oscillograms do show some difference in the characteristics of the oscillations, depending upon the terminal connections of the secondary circuits. This is particularly true in the case of an isolated neutral primary winding, where a change from an open-circuited to a short-circuited secondary may bring about considerable changes in the frequencies of oscillations. This shift of frequencies has been loosely attributed to a change in the effective inductance of the circuit, brought about by closing the secondary circuit; just as the natural frequency of a simple lumped inductance-capacitance series circuit is changed if the inductance is reduced by short-circuiting a coupled winding. In addition to the above limitation of the single winding theory of transformer oscillations, there was nothing in the equations relative to the internal oscillations of the secondary circuit.

The idealized circuit of a two-winding transformer is shown in Fig. 1. The primary winding, on which the traveling wave is assumed to impinge, is designated as winding No. 1 and all associated voltages and currents have the corresponding subscript as e_1, i_{c1}, i_{k1}, i_{L1}. The secondary winding is designated as winding No. 2. The terminals are shown connected to impedances Z_1, Z_2, Z_3, and the incoming surge impedance Z. Each winding has capacitance C and K to ground and along the stack respectively, and an effective self-inductance L. The two windings are coupled magnetically by an effective mutual inductance M, and electrostatically by capacitance C_3. The various circuit constants and variables are specifically defined in the List of Symbols. It is pertinent to remark at this point that the effective inductances of the circuit include the partial interlinkages, on certain assumptions, and are not simple uniformly distributed constants. The necessity for these more complicated assumptions is responsible for greatly increasing the complexity of the problem.

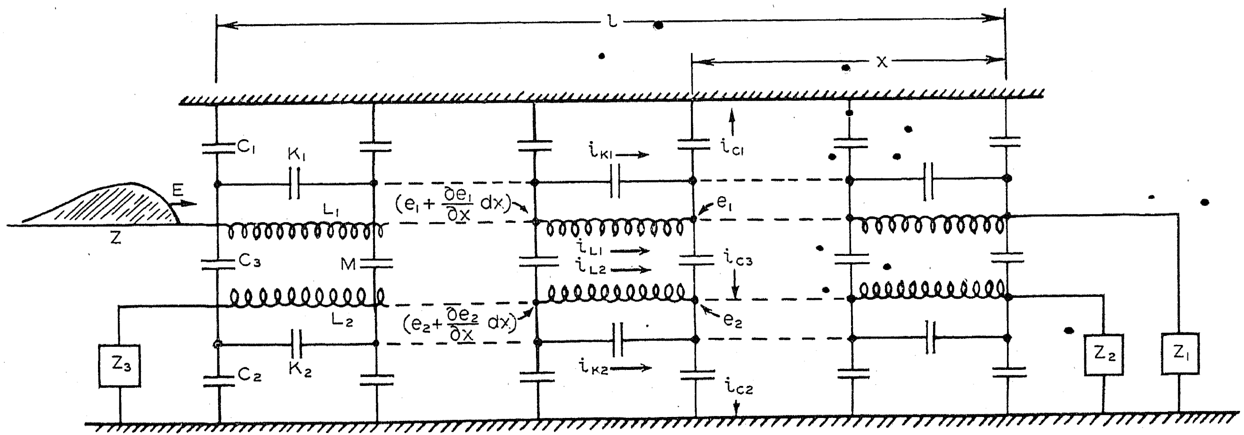

Fig. 1—Complete Idealized Circuit of a Transformer to High-Frequency Transients

The application of Kirchhoff's laws to the elements of the circuit shown in Fig. 1, permits the partial differential equations of the circuit to be written. A solution is then found such that the terminal and boundary conditions are satisfied.

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The initial distribution caused by the impact of an abrupt wave front depends only on the capacitance elements of the circuit; because the currents through the capacitive paths are infinite for an infinite rate of change of voltage, whereas the currents in the inductive paths are all zero at that first instant. Thus Fig. 2 shows the circuit which is effective for calculating the initial distribution.

The idealized circuit of Fig. 1 is free of losses, and therefore stabilization does not occur (theoretically) unless the terminal impedances have losses. For zero or infinite values of the terminal impedances the oscillations never do subside but persist forever. However, any actual circuit possesses some losses, and consequently the oscillations eventually die out and the currents reach permanent values, so that there is no change of flux and no induced voltages. The distribution of voltage in the primary winding is then uniform (ignoring the conductance from the winding to ground), and the potential of the secondary is constant throughout the winding. Thus while the realization of these final distributions is contingent upon the presence of losses, either in the circuit itself or in the terminal impedances, nevertheless there are fixed axes of oscillation about which the transient oscillations of the no-loss circuit take place.

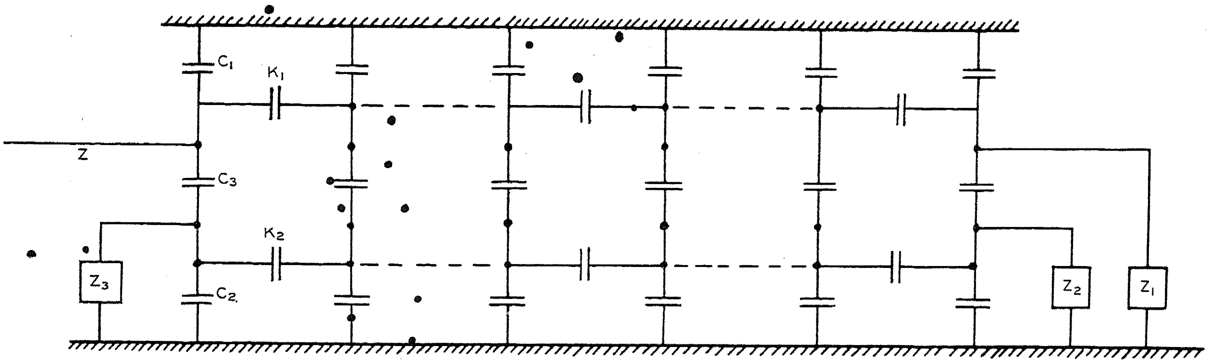

Fig. 2—Circuit Determining the Initial Distribution

The complete solutions are not attempted, except for zero and infinite values of the terminal impedance; that is, for grounded and isolated terminals. The solutions given are also restricted to rectangular applied waves. The response to waves of arbitrary shape can always be found in the usual way by an application of Duhamel's Theorem.[1] The solutions are in the form

$$e = \text{(final distribution)} + \Sigma \text{ (harmonic space and time oscillations)}$$

The time harmonics are always *cosine* functions, and since Duhamel's Theorem is operative only on the *time* functions of the solution, the response to different arbitrary wave shapes may be written down by inspection from the corresponding solutions given in a previous paper,[1] where the time functions were also harmonic *cosine* oscillations. By way of illustration, explicit solutions are worked out in detail for three different terminal conditions.

CONCLUSIONS AND FINDINGS

As far as a description of the transient in the primary winding is concerned, both qualitatively and quantitatively, it is not necessary to consider the influence of the secondary winding beyond keeping in mind that its terminal conditions will change the relative importance of the natural frequencies of oscillation. But these changes in frequency can be fairly adequately accounted for by changing the effective inductance in the equations.

The transient in the two-winding circuit consists of an infinite series of space and time harmonics, oscillating about the final distribution as an axes of oscillation. The amplitudes of these harmonic oscillations depend upon the initial and final distribution, and are in fact given by the Fourier analysis of the difference between them.

The space harmonics which appear in the solution are decided by the Fourier series chosen to represent the initial distribution throughout the length of the winding. The fundamental wavelength upon which the Fourier analysis is made is dictated by convenience in any specific case, and is therefore more or less arbitrary. Thus for certain terminal conditions, a full range *cosine* series may prove the most appropriate, while in another case a half range *sine* series may lead to the greatest simplification.

To each space harmonic there correspond two distinct time harmonics; that is, each space harmonic consists of two components vibrating at widely different rates. The amplitudes of these two time harmonics are usually quite different, but it is possible, by a change of terminal conditions, to change their relative importance.

The harmonic frequencies of the secondary circuit are the same as those of the primary winding, but the amplitudes bear a ratio to the corresponding amplitudes in the primary which depends upon the order of the space harmonic. In other words, there is no fixed proportionality between the oscillations in the two circuits.

The oscillations corresponding to an infinite rectangular applied wave, start simultaneously throughout the windings, and the time harmonics are all *cosine* functions. The response to applied wave shapes of arbitrary shape can be obtained by an application of Duhamel's Theorem.

List of Symbols

C_1 = capacitance of primary winding to ground.
C_2 = capacitance of secondary winding to ground.
C_3 = capacitance between primary and secondary windings.
K_1 = capacitance between ends of primary.
K_2 = capacitance between ends of secondary.
L_1 = effective self-inductance of primary (see eq. (**12**)).
L_2 = effective self-inductance of secondary (see eq. (**12**)).
M = effective mutual inductance between windings (see eq. (**12**)).
Z = surge impedance of line at primary.
$Z_1\,(p)$ = grounding impedance of primary neutral.
$Z_2\,(p)$ = terminal impedance of secondary, neutral end.
$Z_3\,(p)$ = terminal impedance of secondary, line end.
E = terminal voltage of primary due to traveling wave.
$i_{c1}, i_{k1}, i_{L1}, i_{c2}, i_{k2}, i_{L2}, i_{c3}$ = currents through C_1, K_1, etc.
e_1, e_2 = instantaneous voltages at any point x of primary and secondary respectively.
x = fraction of total winding from neutral end.
l = 1 = total length of winding.
t = time.
p = $\partial/\partial t$ = partial time derivative.

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Appendix I

The General Differential Equation

The idealized circuit of a transformer with two windings, each having ground and series capacitances and self-inductance, and inductively coupled electrostatically and electromagnetically, is illustrated in Fig. 1. The fundamental circuit equations are (see List of Symbols)

$$i_{k1} = K_1 \frac{\partial^2 e_1}{\partial x\, \partial t} \tag{1}$$

$$i_{k2} = K_2 \frac{\partial^2 e_2}{\partial x\, \partial t} \tag{2}$$

$$i_{c1} = C_1 \frac{\partial e_1}{\partial t} = \frac{\partial i_{k1}}{\partial x} + \frac{\partial i_{L1}}{\partial x} - i_{c3} \tag{3}$$

$$i_{c2} = C_2 \frac{\partial e_2}{\partial t} = \frac{\partial i_{k2}}{\partial x} + \frac{i_{L2}}{\partial x} + i_{c3} \tag{4}$$

$$i_{c3} = C_3 \frac{\partial}{\partial t} (e_1 - e_2) \tag{5}$$

On the assumption of a linearly graded mutual inductance between points of the same winding, and assuming further that the comparative effectiveness of an ampere-turn in one winding in producing flux density at a point in the other winding, is a fixed fraction σ, there is

$$\varphi_1 = \varphi_{m1} + 2\,(m\,l\,t)\,l \int_0^x \int_x^1 \frac{.4\,\pi}{h} (n_1\, i_{L1} + \sigma\, n_2\, i_{L2})\, d\,y \tag{6}$$

$$\varphi_2 = \varphi_{m2} + 2\,(m\,l\,t)\,l \int_0^x \int_x^1 \frac{.4\,\pi}{h} (\sigma\, n_1\, i_{L1} + n_2\, i_{L2})\, d\,y \tag{7}$$

These fluxes induce voltage gradients

$$\frac{\partial e_1}{\partial x} = \frac{n_1}{10^8} \frac{\partial \varphi_1}{\partial t} \tag{8}$$

$$\frac{\partial e_2}{\partial x} = \frac{n_2}{10^8} \frac{\partial \varphi_2}{\partial t} \tag{9}$$

Then by (**6**) with (**8**), and (**7**) with (**9**) there is

$$\frac{\partial^4 e_1}{\partial x^4} = - \frac{0.8\,\pi\,(m\,l\,t)\,n_1\,l}{10^8\,h} \frac{\partial^2}{\partial t\, \partial x} (n_1\, i_{L1} + \sigma\, n_2\, i_{L2})$$

$$= - \frac{\partial^2}{\partial t\, \partial x} (L_1\, i_{L1} + M\, i_{L2}) \tag{10}$$

$$\frac{\partial^4 e_2}{\partial x^4} = - \frac{\partial^2}{\partial t\, \partial x} (M\, i_{L1} + L_2\, i_{L2}) \tag{11}$$

In which

$$\left.\begin{aligned} L_1 &= \frac{0.8\,\pi\,n_1^2\,(m\,l\,t)\,l}{10^8\,h} \\ L_2 &= \frac{0.8\,\pi\,n_2^2\,(m\,l\,t)\,l}{10^8\,h} \\ M &= \frac{0.8\,\pi\,n_1\,n_2\,(m l t)\,\sigma\,l}{10^8\,h} \end{aligned}\right\} \qquad (12)$$

By (**1**), (**2**), (**3**), and (**4**)

$$\frac{\partial^2}{\partial t\,\partial x}\,i_{L1} = (C_1 + C_3)\,\frac{\partial^2 e_1}{\partial t^2} - C_3\,\frac{\partial^2 e_2}{\partial t^2} - K_1\,\frac{\partial^4 e_1}{\partial x^2\,\partial t^2} \qquad (13)$$

$$\frac{\partial^2}{\partial t\,\partial x}\,i_{L2} = (C_2 + C_3)\,\frac{\partial^2 e_2}{\partial t^2} - C_3\,\frac{\partial^2 e_1}{\partial t^2} - K_2\,\frac{\partial^4 e_2}{\partial x^2\,\partial t^2} \qquad (14)$$

Substituting (**13**) and (**14**) into (**10**) and (**11**) respectively, there is

$$\frac{\partial^4 e_1}{\partial x^4} - L_1\,K_1\,\frac{\partial^4 e_1}{\partial x^2\,\partial t^2} - M\,K_2\,\frac{\partial^4 e_2}{\partial x^2\,\partial t^2} + (L_1\,C_1 + L_1\,C_3 - M\,C_3)\,\frac{\partial^2 e_1}{\partial t^2} + (M\,C_2 + M\,C_3 - L_1\,C_3)\,\frac{\partial^2 e_2}{\partial t^2} = 0 \qquad (15)$$

$$\frac{\partial^4 e_2}{\partial x^4} - L_2\,K_2\,\frac{\partial^4 e_2}{\partial x^2\,\partial t^2} - M\,K_1\,\frac{\partial^4 e_1}{\partial x^2\,\partial t^2} + (L_2\,C_2 + L_2\,C_3 - M\,C_3)\,\frac{\partial^2 e_2}{\partial t^2} + (M\,C_1 + M\,C_3 - L_2\,C_3)\,\frac{\partial^2 e_1}{\partial t^2} = 0 \qquad (16)$$

Solving the simultaneous equations (**15**) and (**16**), for either $e = e_1$ or $e = e_2$, there is

$$\frac{\partial^8 e}{\partial x^8} - (L_1\,K_1 + L_2\,K_2)\,\frac{\partial^8 e}{\partial^6 x\,\partial t^2} + K_1\,K_2\,(L_1\,L_2 - M^2)\,\frac{\partial^8 e}{\partial^4 x\,\partial t^4} + [L_1\,C_1 + L_2\,C_2 + (L_1 + L_2 - 2\,M)\,C_3]\,\frac{\partial^6 e}{\partial^4 x\,\partial t^2} - (L_1\,L_2 - M_2)(K_1\,C_2 + K_1\,C_3 + K_2\,C_1 + K_2\,C_3)\,\frac{\partial^6 e}{\partial^2 x\,\partial t^4} + (L_1\,L_2 - M_2)\,(C_1\,C_2 + C_2\,C_3 + C_1\,C_3)\,\frac{\partial^4 e}{\partial t^4} = 0 \qquad (17)$$

If $\sigma = 1$ then $(L_1\,L_2 - M_2) = 0$ and the above differential equation simplifies to

$$\frac{\partial^4 e}{\partial x^4} - (L_1\,K_1 + L_2\,K_2)\,\frac{\partial^4 e}{\partial^2 x\,\partial t^2} + [L_1\,C_1 + L_2\,C_2 + (L_1 + L_2 - 2\,M)\,C_3]\,\frac{\partial^2 e}{\partial t^2} = 0 \qquad (18)$$

which has the same form as the differential equation for a single circuit, but the coefficients are, of course, different.

THE INITIAL DISTRIBUTION

The differential equation for the initial distribution can be obtained from the general differential equation (**17**), by substituting $p = \infty$ in accordance with the method of operational calculus. Then

$$\frac{\partial^4 e}{\partial x^4} - \left(\frac{K_1\,(C_2 + C_3) + K_2\,(C_1 + C_3)}{K_1\,K_2}\right)\frac{\partial^2 e}{\partial x^2} + \left(\frac{C_1\,C_2 + C_2\,C_3 + C_3\,C_1}{K_1\,K_2}\right) e = 0 \qquad (19)$$

Or putting $i_{L1} = i_{L2} = 0$ in (**3**) and (**4**), and using (**1**) and (**2**), there is

$$K_1\,\frac{\partial^3 e_1}{\partial x^2\,\partial t} = i_{c1} + i_{c3} = C_1\,\frac{\partial e_1}{\partial t} + C_3\,\frac{\partial}{\partial t}\,(e_1 - e_2) \qquad (20)$$

$$K_2\,\frac{\partial^3 e_2}{\partial x^2\,\partial t} = i_{c2} - i_{c3} = C_2\,\frac{\partial e_2}{\partial t} - C_3\,\frac{\partial}{\partial t}\,(e_1 - e_2)$$

The solution to (**19**) is

$$e_1 = P\,\epsilon^{\alpha x} + Q\,\epsilon^{-\alpha x} + R\,\epsilon^{\beta x} + S\,\epsilon^{-\beta x} \qquad (22)$$

where

$$\alpha^2 = \frac{K_1\,(C_2 + C_3) + K_2\,(C_1 + C_3) + \sqrt{[K_1\,(C_2 + C_3) - K_2\,(C_1 + C_3)]^2 + 4\,K_1\,K_2\,C_3^2}}{2\,K_1\,K_2}$$

$$\beta^2 = \frac{K_1\,(C_2 + C_3) + K_2\,(C_1 + C_3) - \sqrt{[K_1\,(C_2 + C_3) - K_2\,(C_1 + C_3)]^2 + 4\,K_1\,K_2\,C_3^2}}{2\,K_1\,K_2} \qquad (23)$$

By (**20**) and (**22**)

$$e_2 = P_2\,\epsilon^{\alpha x} + Q_2\,\epsilon^{-\alpha x} + R_2\,\epsilon^{\beta x} + S_2\,\epsilon^{-\beta x} = \frac{C_1 + C_3}{C_3}\,e_1 - \frac{K_1}{C_3}\,\frac{\partial^2 e_1}{\partial x^2} \qquad (24)$$

So that the set of integration constants are related by

$$\left.\begin{aligned} P_2 &= P\,(C_1 + C_3 - K_1\,\alpha^2)/C_3 = m\,P \\ Q_2 &= Q\,(C_1 + C_3 - K_1\,\alpha^2)/C_3 = m\,Q \\ R_2 &= R\,(C_1 + C_3 - K_1\,\beta^2)/C_3 = n\,R \\ S_2 &= S\,(C_1 + C_3 - K_1\,\beta^2)/C_3 = n\,S \end{aligned}\right\} \qquad (25)$$

Thus the solutions for the initial distributions of e_1 and e_2 have the same functional form, and the integration constants of their separate solutions are related, so that there are only four integration constants that must be determined from the boundary conditions. Referring to Fig. 2, and designating by $Z(\infty)$ the values of the terminal impedances at the first or initial instant, there is:

$$e_1 = E \text{ at } x = 1 \tag{26}$$

$$e_1 = Z_1(p)\, i_{k1} = p\, Z_1(p)\, K_1 \frac{\partial e_1}{\partial x} \text{ at } x = 0 \text{ and } p = \infty \tag{27}$$

$$e_2 = Z_2(p)\, i_{k2} = p\, Z_2(p)\, K_2 \frac{\partial e_2}{\partial x} \text{ at } x = 0 \text{ and } p = \infty \tag{28}$$

$$e_2 = -Z_3(p)\, i_{k2} = -p\, Z_3(p)\, K_2 \frac{\partial e_2}{\partial x} \text{ at } x = 1 \text{ and } p = \infty \tag{29}$$

The employment of these four conditions will yield four simultaneous equations in P, Q, R, S, which will just suffice for their unique determination.

THE FINAL DISTRIBUTION

The realization of a final steady state distribution at $t = \infty$ for an infinite rectangular applied wave, is contingent upon the presence of losses, either in the transformer or in the terminal impedances, and the characteristics of such a final state will depend upon the nature of the losses. But in the no-loss circuit with zero or infinite terminal impedances, the axes of oscillation are determined by the electrostatic and electromagnetic fields necessary to establish the terminal voltages of the windings. Possible solutions to the general differential equation (17) independent of t and which satisfy the terminal conditions are:

$$\left.\begin{aligned} e_1 &= e_n + (E - e_n)\, x \\ e_2 &= E_2' + x\, E_2 \\ &= \frac{C_3 (E + e_n) - (C_2 + C_3 + 2\, C_3')E_2}{2\,(C_2 + C_3 + C_2' + C_3')} + x\, E_2 \end{aligned}\right\} \tag{30}$$

in which E is the primary terminal voltage, e_n is the primary neutral voltage ($e_n = 0$ for grounded neutral, $e_n = E$ for isolated neutral), E_2' is the secondary potential due to its position in the electrostatic field, and C_2' and C_3' are the capacitances of the secondary terminal impedances Z_2 and Z_3 respectively. A comparison of (30) with (87) and (88) of Appendix II identifies $(E - e_n)\, x$ and $x\, E_2$ as the voltage components due to a magnetic field rising without limit at a uniform rate. The value of E_2' is determined from the condition that the total dielectric flux from the primary to the secondary must be equal to that from the secondary to ground. Thus

$$\int_0^1 C_3 \left[e_n + (E - e_n)\, x - E_2' - x\, E_2\right] d\,x$$
$$= C_2' E_2' + C_2 \int_0^1 (E_2' + x\, E_2)\, d\,x + C_3' (E_2 + E_2')$$

The idealized circuit of Fig. 1 does not include any losses, so that the complete solution will not reduce to (30) at $t = \infty$; but will nevertheless represent sustained oscillations about (30) as axes.

SOLUTION OF THE DIFFERENTIAL EQUATION

The solution must satisfy the initial and final distributions, the terminal conditions, and the differential equation. As a tentative solution suitable for zero and infinite values of the terminal impedances, assume

$$e_1 = e_n + (E - e_n)\, x + \Sigma\, A\, \epsilon^{(ax+bt)} \tag{31}$$

$$e_2 = \Sigma\, A_2\, \epsilon^{(ax+bt)} + E_2\, x + E_2' \tag{32}$$

in which A and A_2 are integration constants. Substituting (31) or (32) into the differential equation (17) and simplifying, there results

$$\begin{aligned} &(L_1 L_2 - M^2)[K_1 K_2\, a^4 - (K_1 C_2 + K_2 C_1 + K_1 C_3 + K_2 C_3)a^2 \\ &\quad + (C_1\ C_2 + C_2\ C_3 + C_1\ C_3)]\ b^4 + [-(L_1\ K_1 + L_2\ K_2)a^6 \\ &+ (L_1\ C_1 + L_2\ C_2 + L_1\ C_3 + L_2\ C_3 - 2\ M\ C_3)a^4]b^2 + a^8 = 0 \end{aligned} \tag{33}$$

This equation is a quadratic in b^2, of which the coefficients are all positive if a is a pure imaginary $\pm j\, \lambda$; and because $L_1 L_2 \geq M^2$ and $(L_1 + L_2) \geq 2\, M$. Moreover, upon expanding the terms under the radical of the solution for b^2, it becomes evident that the radical is a positive real number, and accordingly b^2 has two real negative values. Corresponding to these two negative values, there is

$$b = \begin{cases} \pm j\, \omega \\ \pm j\, \Omega \end{cases} \tag{34}$$

This result is a necessary consequence of the fact that the circuit is a conservative non-dissipative network, and therefore exponential decrement factors cannot appear in its solution. The solutions may now be written in the form

$$\begin{aligned} e_1 = e_n + (E - e_n)x + \Sigma[&(A \cos \omega\, t + A' \cos \Omega\, t) \sin \lambda\, x \\ &+ (B \sin \omega\, t + B' \sin \Omega\, t) \sin \lambda\, x \\ &+ (C \cos \omega\, t + C' \cos \Omega\, t) \cos \lambda\, x \\ &+ (D \sin \omega\, t + D' \sin \Omega\, t) \cos \lambda\, x] \end{aligned} \tag{35}$$

$$\begin{aligned} e_2 = \Sigma\, [&(A_2 \cos \omega\, t + A_2' \cos \Omega\, t) \sin \lambda\, x \\ &+ (B_2 \sin \omega\, t + B_2' \sin \Omega\, t) \sin \lambda\, x \\ &+ (C_2 \cos \omega\, t + C_2' \cos \Omega\, t) \cos \lambda\, x \\ &+ (D_2 \sin \omega\, t + D_2' \sin \Omega\, t) \cos \lambda\, x] + E_2 x + E_2' \end{aligned} \tag{36}$$

Thus, there are *two* time harmonics ω and Ω associated with each space harmonic λ. If the coupling between the two windings is perfect, $L_1 L_2 = M^2$ then there will be only one time harmonic corresponding to a given space harmonic. The more nearly that $M^2 \rightarrow L_1 L_2$ the more closely $\Omega \rightarrow 0$. Now in a transformer ω and Ω are usually quite different (say 2:1) but the amplitude factors of the two frequencies are also quite different for given terminal conditions of the secondary. However, a change in the secondary terminal conditions (such as switching from open circuit to short circuit) may cause a complete reversal in the relative importance of the two frequency sets.

In (**35**) and (**36**) there are nineteen constants to be determined. Of these, λ is determined by a comparison with the initial distribution, ω and Ω then follow by (**33**) and (**34**), and the integration constants for the secondary circuit may be expressed in terms of those for the primary circuit through equations (**15**) and (**16**). The remaining constants are determined from the initial distributions and the terminal conditions. Substituting (**35**) and (**36**) into (**15**), and separately equating the coefficients of like trigonometric terms, there results

$$\frac{A_2}{A} = \frac{B_2}{B} = \frac{C_2}{C} = \frac{D_2}{D}$$

$$= \frac{\lambda^4 - \omega^2 (\lambda^2 L_1 K_1 + L_1 C_1 + L_1 C_3 - M C_3)}{\omega^2 (\lambda^2 M K_2 + M C_2 + M C_3 - L_1 C_3)} = r_s \tag{37}$$

$$\frac{A_2'}{A'} = \frac{B_2'}{B'} = \frac{C_2'}{C'} = \frac{D_2'}{D'}$$

$$= \frac{\lambda^4 - \Omega^2 (\lambda^2 L_1 K_1 + L_1 C_1 + L_1 C_3 - M C_3)}{\Omega^2 (\lambda^2 M K_2 + M C_2 + M C_3 - L_1 C_3)} = r_s' \tag{38}$$

Or by (**16**) there is

$$\frac{A_2}{A} = \frac{B_2}{B} = \frac{C_2}{C} = \frac{D_2}{D}$$

$$= \frac{\omega^2 (\lambda^2 M K_1 + M C_1 + M C_3 - L_2 C_3)}{\lambda^4 - \omega^2 (\lambda^2 L_2 K_2 + L_2 C_2 + L_2 C_3 - M C_3)} = r_s \tag{39}$$

$$\frac{A_2'}{A'} = \frac{B_2'}{B'} = \frac{C_2'}{C'} = \frac{D_2'}{D'}$$

$$= \frac{\Omega^2 (\lambda^2 M K_1 + M C_1 + M C_3 - L_2 C_3)}{\lambda^4 - \Omega^2 (\lambda^2 L_2 K_2 + L_2 C_2 + L_2 C_3 - M C_3)} = r_s' \tag{40}$$

That (**37**) is equal to (**39**) and that (**38**) is equal to (**40**) is evident by (**33**). Therefore it is immaterial whether (**37**) and (**38**) or (**39**) and (**40**) be used in subsequent substitutions.

Equating (**35**) at $t = 0$ to (**22**) and (**36**) at $t = 0$ to (**24**), making use of (**25**), (**37**) and (**38**), there is

$$\Sigma\,[(A + A')\sin\lambda x + (C + C')\cos\lambda x]$$
$$= (P\,\epsilon^{\alpha x} + Q\,\epsilon^{-\alpha x} + R\,\epsilon^{\beta x} + S\,\epsilon^{-\beta x}) - [e_n + (E - e_n)\,x]$$
$$= \sum_1^{\infty}\left(X_s \sin\frac{s\,\pi\,x}{c} + Y_s \cos\frac{s\,\pi\,x}{c}\right) \tag{41}$$

$$\Sigma\,[(r_s A + r_s' A')\sin\lambda x + (r_s C + r_s' C')\cos\lambda x]$$
$$= (m P\,\epsilon^{\alpha x} + m Q\,\epsilon^{-\alpha x} + n R\,\epsilon^{\beta x} + n\,S\,\epsilon^{-\beta x}) - E_2 x - E_2'$$
$$= \sum_1^{\infty}\left(U_s \sin\frac{s\,\pi\,x}{c} + V_s \cos\frac{s\,\pi\,x}{c}\right) \tag{42}$$

where X_s Y_s, U_s and V_s are the coefficients in the Fourier analysis of the *difference* between the *initial* and the *final* distributions, and c is the arbitrary half wavelength upon which the Fourier analysis is made. Of course, the choice of c as well as the decision to expand, for instance, as a half-range *sine* or *cosine* series, is dictated by convenience in simplifying any particular special case of boundary conditions.

By direct comparison of the coefficients of like trigonometric terms in (**41**) and (**42**) there is

$$\left.\begin{aligned} A + A' &= X_s \\ C + C' &= Y_s \\ r_s A + r_s' A' &= U_s \\ r_s C + r_s' C' &= V_s \\ \lambda &= \frac{s\,\pi}{c} \end{aligned}\right\} \tag{43}$$

and solving these equations, there is

$$\left.\begin{aligned} A' &= (r_s X_s - U_s)\,/(r_s - r_s') \\ A &= (r_s' X_s - U_s)/(r_s' - r_s) \\ C' &= (r_s Y_s - V_s)\,/(r_s - r_s') \\ C &= (r_s' Y_s - V_s)/(r_s' - r_s) \end{aligned}\right\} \tag{44}$$

The only conditions which remain to be satisfied are

$$\left.\begin{aligned} i_{L1} = i_{L2} &= 0 \text{ at } t = 0 \\ (i_{L1} + i_{K1})\,Z_1 &= e_1 \text{ at } x = 0 \\ (i_{L2} + i_{K2})\,Z_2 &= e_2 \text{ at } x = 0 \\ -(i_{L2} + i_{K2})\,Z_3 &= e_2 \text{ at } x = 1 \end{aligned}\right\} \tag{45}$$

From (**3**) and (**4**) respectively and making use of (**1**), (**4**) and (**5**), there are

$$\frac{\partial i_{L1}}{\partial x} = C_1 \frac{\partial e_1}{\partial t} + i_{c3} - \frac{\partial i_{K1}}{\partial x}$$

$$= \left(C_1 + C_3 - K_1 \frac{\partial^2}{\partial x^2}\right)\frac{\partial e_1}{\partial t} - C_3 \frac{\partial e_1}{\partial x} \tag{46}$$

$$\frac{\partial i_{L2}}{\partial x} = C_2 \frac{\partial e_2}{\partial t} - i_{c3} - \frac{\partial i_{K2}}{\partial x}$$

$$= \left(C_2 + C_3 - K_2 \frac{\partial^2}{\partial x^2}\right)\frac{\partial e_2}{\partial t} - C_3 \frac{\partial e_1}{\partial x} \tag{47}$$

$$(i_{L1} + i_{K1}) = \int\left[(C_1 + C_3)\frac{\partial e_1}{\partial t} - C_3 \frac{\partial e_2}{\partial t}\right] d\,x \tag{48}$$

$$(i_{L2} + i_{K2}) = \int\left[(C_2 + C_3)\frac{\partial e_2}{\partial t} - C_3 \frac{\partial e_1}{\partial t}\right] d\,x \tag{49}$$

Now if (**35**) and (**36**) are substituted in the above four equations, and each of the terminal conditions of (**45**) are imposed, it will be evident that for zero and infinite values of the terminal impedance Z_1, Z_2 and Z_3 that conditions (45) are satisfied for

$$B = B' = D = D' = 0 \tag{50}$$

In making such a comparison, a word of warning is advisable. If (35) and (36) are substituted in (46) and $(\partial i_{L1}/\partial x)$ placed equal to zero in conformity with the first condition of (45), and the coefficients of like trigonometric terms are separately equated to zero, there results

$$\frac{B'}{B} = \frac{D'}{D} = -\frac{\omega}{\Omega}\left\{\frac{(C_1 + C_3 + K_1\lambda^2) - r\,C_3}{(C_1 + C_3 + K_1\lambda^2) - r'\,C_3}\right\} \tag{51}$$

Now if the same procedure is followed for (47) there results

$$\frac{B'}{B} = \frac{D'}{D} = -\frac{\omega}{\Omega}\left\{\frac{(C_2 + C_3 + K_2\lambda^2)\,r - C_3}{(C_2 + C_3 + K_2\lambda^2)\,r' - C_3}\right\} \tag{52}$$

But (51) and (52) are satisfied by (50) since $r \neq r'$. The complete solutions may therefore be written (for zero or infinite values of the terminal impedances)

$$\begin{aligned} e_1 = e_n + (E - e_n)\,x + \Sigma\,[(A\cos\omega t + A'\cos\Omega t)\sin\lambda x \\ + (C\cos\omega t + C'\cos\Omega t)\cos\lambda x] \end{aligned} \tag{53}$$

$$\begin{aligned} e_2 = \Sigma\,[(r\,A\cos\omega t + r'\,A'\cos\Omega t)\sin\lambda x \\ + r\,C\cos\omega t + r'\,C'\cos\Omega t)\cos\lambda x] + E_2\,x + E_2' \end{aligned} \tag{54}$$

where

$\lambda = s\,\pi/c$	from (43)
A, A', C, C'	from (44)
r, r'	from (37), (38)
$e_n = 0$	for a grounded neutral
$= E$	for an isolated neutral

The principal conclusions which may be drawn from these equations are:

(a) The space harmonics, s, depend upon the Fourier series chosen to represent the initial distributions within the interval $0 \leq x \leq 1$.

(b) To each space harmonic there corresponds two distinct time harmonics ω and Ω whose amplitudes sum up at every instant to give the space harmonic distribution at that instant. In other words, each space harmonic consists of two components vibrating at different rates.

(c) The amplitudes of oscillation depend upon the Fourier analysis of the initial distributions with respect to the final distributions, and therefore depend primarily upon the capacitances and terminal impedances of the circuits.

(d) Any harmonic amplitude of oscillation of the secondary circuit bears a definite ratio to the corresponding oscillation of the primary circuit.

(e) Identically the same frequencies of oscillation appear in both the primary and secondary circuits.

(f) The transient oscillations take place about the final distributions as axes of oscillation.

By way of illustration of the above general equations and procedure, a number of special cases will be worked out in detail.

I. GROUNDED NEUTRAL AND SHORT-CIRCUITED SECONDARY

The Initial Distributions

Since $Z_1 = Z_2 = Z_3 = 0$ for this case, (26) to (29) give

$$\left.\begin{aligned} P\,\epsilon^{\alpha} + Q\,\epsilon^{-\alpha} + R\,\epsilon^{\beta} + S\,\epsilon^{-\beta} &= E \\ P + Q + R + S &= 0 \\ m\,P\,\epsilon^{\alpha} + m\,Q\,\epsilon^{-\alpha} + n\,R\,\epsilon^{\beta} + n\,S\,\epsilon^{-\beta} &= 0 \\ m\,P + m\,Q + n\,R + n\,S &= 0 \end{aligned}\right\} \tag{55}$$

Solving these four simultaneous equations for the integration constants, there are

$$\left.\begin{aligned} P &= n\,E/(n-m)\,2\sinh\alpha \\ Q &= -\,n\,E/(n-m)\,2\sinh\alpha \\ R &= -\,m\,E/(n-m)\,2\sinh\beta \\ S &= m\,E/(n-m)\,2\sinh\beta \end{aligned}\right\} \tag{56}$$

Then by (22) and (24), the initial distributions (expressed as half-range sine series) are

$$\begin{aligned} \underset{t=0}{e_1} &= \frac{E}{n-m}\left[\frac{n\sinh\alpha x}{\sinh\alpha} - \frac{m\sinh\beta x}{\sinh\beta}\right] \\ &= \sum_{s=1}^{\infty}\frac{2E}{m-n}\left[\frac{n\,s\,\pi}{\alpha^2+s^2\pi^2} - \frac{m\,s\,\pi}{\beta^2+s^2\pi^2}\right]\cos s\pi\,.\,\sin s\pi x \end{aligned} \tag{57}$$

$$\begin{aligned} \underset{t=0}{e_2} &= \frac{n\,m\,E}{n-m}\left[\frac{\sinh\alpha x}{\sinh\alpha} - \frac{\sinh\beta x}{\sinh\beta}\right] \\ &= \sum_{s=1}^{\infty}\frac{2\,n\,m\,E}{m-n}\left[\frac{s\,\pi}{\alpha^2+s^2\pi^2} - \frac{s\,\pi}{\beta^2+s^2\pi^2}\right]\cos s\pi\,.\,\sin s\pi x \end{aligned} \tag{58}$$

The Final Distributions

Since the neutral is grounded, $e_n = 0$, and the final distribution may therefore be expressed as a half-range sine series

$$\underset{t=\infty}{e_1} = x\,E = \sum_{s=1}^{\infty}\frac{(-\,2\,E\cos s\pi)}{s\,\pi}\sin s\,\pi x \tag{59}$$

$$\underset{t=\infty}{e_2} = 0 \tag{60}$$

The Complete Solution

The difference between the initial and final distributions, expressed as half-range sine series, give—see (41) and (42)

$$X_s = \frac{2\,E\,s\,\pi}{(m\ -\ n)}\left[\frac{n}{\alpha^2 + s^2\pi^2} - \frac{m}{\beta^2 + s^2\pi^2} + \frac{m-n}{s^2\ \pi^2}\right]\cos s\,\pi \tag{61}$$

$$Y_s = 0 \tag{62}$$

$$U_s = \frac{2\,E\,s\,\pi}{m-n}\left[\frac{n\,m}{\alpha^2 + s^2\,\pi^2} - \frac{n\,m}{\beta^2 + s^2\,\pi^2}\right]\cos s\,\pi \tag{63}$$

$$V_s = 0 \tag{64}$$

$$\lambda\ = s\,\pi \tag{65}$$

Then by (44)

$$A' = \frac{2\,E\,s\,\pi\cos s\,\pi}{(r_s - r_s')\ (m-n)}\left[\frac{n\,(r_s - m)}{\alpha^2 + s^2\,\pi^2} - \frac{m\,(r_s - n)}{\beta^2 + s^2\,\pi^2} + \frac{r_s\,(m-n)}{s^2\,\pi^2}\right] \tag{66}$$

$$A\ = \frac{2\,E\,s\,\pi\cos s\,\pi}{(m-n)\ (r_s' - r_s)}\left[\frac{n\,(r_s' - m)}{\alpha^2 - s^2\,\pi^2} - \frac{m\,(r_s' - n)}{\beta^2 + s^2\,\pi^2} + \frac{r_s'\,(m-n)}{s^2\,\pi^2}\right] \tag{67}$$

$$C\ = C' = 0 \tag{68}$$

The solution, by (53) and (54), then is

$$e_1 = x\,E + \sum_{s=1}^{\infty} \frac{2\,E\,s\,\pi\cos s\,\pi}{(r_s - r_s')\ (m-n)}\left\{\left[\frac{n\,(r_s - m)}{\alpha^2 + s^2\,\pi^2} - \frac{m\,(r_s - n)}{\beta^2 + s^2\,\pi^2} + \frac{r_s\,(m-n)}{s^2\,\pi^2}\right]\cos \Omega_s\,t - \left[\frac{n\,(r_s' - m)}{\alpha^2 + s^2\,\pi^2} - \frac{m\,(r_s' - n)}{\beta^2 + s^2\,\pi^2} + \frac{r_s'\,(m-n)}{s^2\,\pi^2}\right]\cos \omega_s\,t\right\}\sin s\,\pi\,x \tag{69}$$

$$e_2 = \sum_{s=1}^{\infty} \frac{2\,E\,s\,\pi\cos s\,\pi}{(r_s - r_s')\ (m-n)}\left\{\left[\frac{n\,(r_s - m)}{\alpha^2 + s^2\,\pi^2} - \frac{m\,(r_s - n)}{\beta^2 + s^2\,\pi^2} + \frac{r_s\,(m-n)}{s^2\,\pi^2}\right] r_s'\cos \Omega_s\,t - \left[\frac{n\,(r_s' - m)}{\alpha^2 + s^2\,\pi^2} - \frac{m\,(r_s' - n)}{\beta^2 + s^2\,\pi^2} + \frac{r_s'\,(m-n)}{s^2\,\pi^2}\right] r_s\cos \omega_s\,t\right\}\sin s\,\pi\,x \tag{70}$$

The angular velocities ω_s and Ω_s are given by the solution of (33) in terms of $\lambda = s\,\pi$.

II. GROUNDED NEUTRAL AND OPEN-CIRCUITED SECONDARY

THE INITIAL DISTRIBUTIONS

In this case $Z_1 = 0$ and $Z_2 = Z_2 = \infty$. Hence by (26) to (29) there is

$$\left.\begin{array}{lllll} P\,\epsilon^{\alpha} & + Q\,\epsilon^{-\alpha} & + R\,\epsilon^{\beta} & + S\,\epsilon^{-\beta} & = \mathrm{E} \\ P & + Q & + R & + S & = 0 \\ m\,\alpha\,P & - m\,\alpha\,Q & + n\,\beta\,R & - n\,\beta\,S & = 0 \\ m\,\alpha\,P\,\epsilon^{\alpha} & - m\,\alpha\,Q\,\epsilon^{-\alpha} & + n\,\beta\,R\,\epsilon^{\beta} & - n\,\beta\,S\,\epsilon^{-\beta} & = 0 \end{array}\right\} \tag{71}$$

Solving for the integration constants

$$\begin{aligned} P\ &= n\,\beta\,E\,[m\,\alpha\,(\cosh\alpha - \sinh\alpha) \\ &\qquad + (n\,\beta\sinh\beta - m\,\alpha\cosh\beta)]/T \\ Q\ &= n\,\beta\,E\,[m\,\alpha\,(\cosh\alpha + \sinh\alpha) \\ &\qquad - (n\,\beta\sinh\beta + m\,\alpha\cosh\beta)]/T \\ R\ &= m\,\alpha\,E\,[n\,\beta\,(\cosh\beta - \sinh\beta) \\ &\qquad + (m\,\alpha\sinh\alpha - n\,\beta\cosh\alpha)]/T \\ S\ &= m\,\alpha\,E\,[n\,\beta\,(\cosh\beta + \sinh\beta) \\ &\qquad - (m\,\alpha\sinh\alpha - n\,\beta\cosh\alpha)]/T \\ T\ &= 4\,m\,n\,\alpha\,\beta\,(1 - \cosh\alpha\,.\,\cosh\beta) \\ &\qquad + 2\,(m^2\,\alpha^2 + n^2\,\beta^2)\sinh\alpha\,.\,\sinh\beta \end{aligned} \tag{72}$$

The initial distributions then are

$$\underset{t=0}{e_1 = E}\left\{\frac{m\,n\,\alpha\,\beta\,(\cosh\alpha - \cosh\beta)\ (\cosh\alpha\,x - \cosh\beta\,x) + (m\,\alpha\sinh\alpha - n\beta\sinh\beta)(m\,\alpha\sinh\beta\,x - n\beta\sinh\alpha\,x)}{2mn\alpha\beta(1 - \cosh\alpha.\cosh\beta) + (m^2\alpha^2 + n^2\beta^2)\sinh\alpha.\sinh\beta}\right\} \tag{73}$$

$$\underset{t=0}{e_2 = E}\left\{\frac{m\,n\,\alpha\,\beta\,(\cosh\alpha - \cosh\beta)\ (m\cosh\alpha\,x - n\cosh\beta\,x) + m\,n(m\alpha\sinh\alpha - n\,\beta\sinh\beta)\ (\alpha\sinh\beta\,x - \beta\sinh\alpha\,x)}{2mn\alpha\beta(1 - \cosh\alpha.\cosh\beta) + (m^2\alpha^2 + n^2\beta^2)\sinh\alpha.\sinh\beta}\right\} \tag{74}$$

and these can, of course, be expressed as a half-range *sine* series.

THE FINAL DISTRIBUTIONS

That of the primary is the same as in (59) but the secondary takes on the potential, by (30b),

$$\underset{t=\infty}{e_2}\ = \sum_{1}^{\infty}\left(2\,E_2'\sin^2\frac{s\,\pi}{2} - E_2\cos s\,\pi\right)\frac{2}{s\,\pi}\sin s\,\pi\,x \tag{75}$$

$$E_2' = \left[\frac{C_3\,E}{2\,(C_2 + C_3)} - \frac{E_2}{2}\right] \text{ and } E_2 = \frac{M'}{L_1'}\,E$$

The Complete Solution then follows by the same procedure as in the previous case, differing only in the values of the coefficients X_s and Y_s because the initial and final distributions have changed. The frequencies of oscillation are the same as in the previous case—but the relative importance of the amplitudes have been changed.

III. ISOLATED NEUTRAL AND OPEN-CIRCUITED SECONDARY

THE INITIAL DISTRIBUTIONS

In this case $Z_1 = Z_2 = Z_3 = \infty$, and (26) to (29) give

$$\left.\begin{array}{lllll} P\epsilon^{\alpha} & + Q\epsilon^{-\alpha} & + R\epsilon^{\beta} & + S\epsilon^{-\beta} & = E \\ \alpha P & - \alpha Q & + \beta R & - \beta S & = 0 \\ m\alpha P & - m\alpha Q & + n\beta R & - n\beta S & = 0 \\ m\alpha P\epsilon^{\alpha} & - m\alpha Q\epsilon^{-\alpha} & + n\beta R\epsilon^{\beta} & - n\beta S\epsilon^{-\beta} & = 0 \end{array}\right\} \tag{76}$$

Solving for the integration constants:

$$\begin{array}{ll} P & = -(n\beta E\sinh\beta)/T \\ Q & = -(n\beta E\sinh\beta)/T \\ R & = +(m\alpha E\sinh\alpha)/T \\ S & = +(m\alpha E\sinh\alpha)/T \\ T & = 2(m\alpha\sinh\alpha\,.\,\cosh\beta - n\beta\,.\,\cosh\alpha\,.\,\sinh\beta) \end{array} \tag{77}$$

The initial distributions are

$$\underset{t=0}{e_1} = E\left\{\frac{m\alpha\sinh\alpha\,.\,\cosh\beta x - n\beta\sinh\beta\cosh\alpha x}{m\alpha\sinh\alpha\cosh\beta - n\beta\sinh\beta\cosh\alpha}\right\}$$

$$= \sum_{s=1}^{\infty}\frac{-2Es\pi/2\,.\,\sin s\pi/2\,.\,\cos s\pi x/2}{m\alpha\sinh\alpha\,.\,\cosh\beta - n\beta\sinh\beta\,.\,\cosh\alpha}\left[\frac{m\alpha\sinh\alpha}{\beta^2 + s^2\pi^2/4} - \frac{n\beta\sinh\beta}{\alpha^2 + s^2\pi^2/4}\right] \tag{78}$$

$$\underset{t=0}{e_2} = nmE\left\{\frac{\alpha\sinh\alpha\,.\,\cosh\beta x - \beta\sinh\beta\,.\,\cosh\alpha x}{m\alpha\sinh\alpha\,.\,\cosh\beta - n\beta\sinh\beta\,.\,\cosh\alpha}\right\}$$

$$= \sum_{s=1}^{\infty}\frac{-2mnEs\pi/2\,.\,\sin s\pi/2\,.\,\cos s\pi x/2}{m\alpha\sinh\alpha\,.\,\cosh\beta - n\beta\sinh\beta\cosh\alpha}\left[\frac{\alpha\sinh\alpha}{\beta^2 + s^2\pi^2/4} - \frac{\beta\sinh\beta}{\alpha^2 + s^2\pi^2/4}\right] \tag{79}$$

THE FINAL DISTRIBUTIONS

For an isolated neutral the final distribution expressed as a *cosine* series in the interval $0 \leq x \leq 1$ is

$$\underset{t=\infty}{e_1} = E = E\sum_{s=1}^{\infty}\frac{2\sin s\pi/2\,.\,\cos s\pi x/2}{s\pi/2} \tag{80}$$

and

$$\underset{t=\infty}{e_2} = E_2{}' = \frac{C_3E}{C_3 + C_2}$$

$$= \frac{2C_3E}{C_3 + C_2}\sum_{s=1}^{\infty}\frac{\sin s\pi/2\,.\,\cos s\pi x/2}{s\pi/2} \tag{81}$$

THE COMPLETE SOLUTION

The difference between the Fourier series of the initial and final distributions gives, by (41)

$$\left.\begin{array}{ll} X_s & = 0 \\ Y_s & = [(78) - (80)] \\ U_s & = 0 \\ V_s & = [(79) - (81)] \\ \lambda & = (2s - 1)\pi/2 \end{array}\right\} \tag{82}$$

Appendix II

EXACT EQUATIONS FOR THE AXES OF OSCILLATION

This appendix offers a more detailed consideration of the axes of oscillation than was given in Appendix I; and incidentally also derives the simplified equations for calculating the current and voltage transmitted to the secondary external circuit.

Substituting (1) and (2) into (48) and (49) respectively, and integrating with respect to x there is

$$i_{L1} = I_1 + \int\left[(C_1 + C_3)\frac{\partial e_1}{\partial t} - C_3\frac{\partial e_2}{\partial t} - K_1\frac{\partial^3 e_1}{\partial x^2\,\partial t}\right]dx \tag{83}$$

$$i_{L2} = I_2 + \int\left[(C_2 + C_3)\frac{\partial e_2}{\partial t} - C_3\frac{\partial e_1}{\partial t} - K_2\frac{\partial^3 e_2}{\partial x^2\,\partial t}\right]dx \tag{84}$$

In these equations I_1 and I_2 are integration constants with respect to x, and are therefore possible functions of t but not of x. They therefore flow through all parts of their windings, and in conjunction with the final electrostatic field, fix the final potentials of the windings. The indefinite integral yields the space and time harmonics of the oscillation.

Substituting I_1 and I_2 in (6) and (7) there follows by (8) and (9) the potential distributions caused by these currents:

$$\frac{n_1}{10^8}\int_o^x\frac{\partial\phi_1}{\partial t}dx = x\frac{n_1}{10^8}\frac{\partial}{\partial t}\left[\Phi_{m1} + .8\pi(mlt)\frac{l}{h}(n_1I_1 + \sigma n_2I_2)\left(\frac{x}{2} - \frac{x^2}{3}\right)\right] \tag{85}$$

$$\frac{n_2}{10^8}\int_o^x\frac{\partial\phi_2}{\partial t}dx = x\frac{n_2}{10^8}\frac{\partial}{\partial t}\left[\Phi_{m2} + .8\pi(mlt)\frac{l}{h}(\sigma n_1I_1 + n_2I_2)\left(\frac{x}{2} - \frac{x^2}{3}\right)\right] \tag{86}$$

Now the mutual fluxes Φ_{m1} and Φ_{m2} are linear functions of I_1 and I_2 and are large compared with the other terms, so that to an excellent approximation, (85) and (86) are also linear functions of x and may therefore be written in the form,

$$x E_1 = \frac{x}{10^8}\frac{\partial}{\partial t}(n_1 \Phi_{m1}) = x \frac{\partial}{\partial t}(L_1' I_1 + M' I_2) \tag{87}$$

$$x E_2 = \frac{x}{10^8}\frac{\partial}{\partial t}(n_2 \Phi_{m2}) = x \frac{\partial}{\partial t}(M' I_1 + L_2' I_2) \tag{88}$$

Thus comparing with (**30a**) and (**30b**), it is evident that $x E_1 = x\,(E - e_n)$ is the final potential distribution due to the magnetic flux, to which must be added the neutral voltage e_n. Likewise $x\,E_2$ is that part of the secondary final distribution due to the magnetic flux, to which must be added E_2' due to the electrostatic field.

Writing $x = 1$ in (87) and (88), there result the equations for calculating the waves transmitted through the transformer to the secondary circuit.

$$\left.\begin{aligned} E_1 &= p\,(L_1' I_1 + M' I_2) = E - Z_1 I_1 \\ E_2 &= p\,(M' I_1) + L_2' I_2) = -\,(Z_2 + Z_3)\, I_2 \end{aligned}\right\} \tag{89}$$

in which L_1', L_2' and M' are the overall self and mutual inductances of the windings, in the conventional sense.

For a short-circuited secondary

$$\left.\begin{aligned} E_2 &= 0 \\ I_1 &= \frac{E_1 L_2' t}{(L_1' L_2' - M'^2)} \\ I_2 &= \frac{E_1 M' t}{(L_1' L_2' - M'^2)} \end{aligned}\right\} \tag{90}$$

and the currents increase linearly with time at a rate sufficient to consume the primary applied voltage E_1.

For an open-circuited secondary

$$\left.\begin{aligned} I_2 &= 0 \\ I_1 &= E_1\, t/L_1' \\ E_2 &= E_1 M'/L_1' \end{aligned}\right\} \tag{91}$$

More generally, for any impedances Z_2 and Z_3 in the secondary circuit, the transmitted voltage and current may be calculated from (**89**); for the currents I_1 and I_2 very soon greatly exceed in magnitude the oscillatory components of (**83**) and (**84**), and therefore dominate the situation in so far as the secondary external circuit is concerned. Consideration of this aspect of the problem is outside the scope of this paper.

Discussion

For discussion of this paper see page 321.

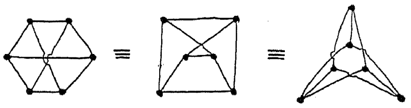
Geometrical Circuits of Electrical Networks

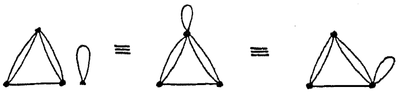
BY RONALD M. FOSTER[1]
Member, A.I.E.E.

***Synopsis.**—If all the electrical properties are abstracted from a given electrical network, there remains a geometrical circuit, completely characterized by the sets of branches terminating at the various vertexes. In this paper, enumerations of geometrical circuits are given, classified according to two different parameters, the nullity (number of branches minus number of vertexes plus number of separate parts) and the rank (number of vertexes minus number of separate parts). There is also a partial list of geometrical circuits which are symmetrical with respect to all the branches and all the vertexes.*

INTRODUCTION

IN many problems in the theory of electrical networks it proves desirable to list all those networks which meet certain electrical conditions. From one point of view this type of problem belongs to the theory of combinations and permutations, but it can be satisfactorily handled on that basis only when the geometrical form of the circuit is specified. One of the first attempts to solve problems of this nature was that of P. A. Macmahon,[2] who, in 1892, investigated the combinations of resistances in series and in parallel, excluding all bridge arrangements such as the wheatstone net. These more complicated circuits, however, are often of greater interest, and it is in enumerating them that difficulties arise. One systematic method of procedure is to start with the geometrical circuits which meet the conditions of the problem, and then assign electrical elements to each branch of the geometrical circuit in all possible ways. At this stage of the investigation we can make use of the well known theory of combinations, taking due account of any symmetries among the branches of the circuit. For a successful application of this method we thus need, to start with, a complete enumeration of all distinct geometrical circuits, classified according to certain specified properties. This method has already been employed in a study of telephone substation and repeater circuits,[3] using as the foundation an enumeration of all distinct geometrical circuits with one, two, three, and four degrees of freedom.


Equivalent by Deformation
(4-1 Used as Example)


Equivalent by Separation
(4-12 Used as Example)

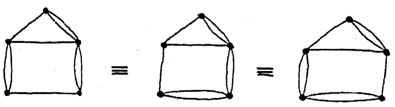

Equivalent by Series Interchange
(5-37 Used as Example)

FIG. 1—EQUIVALENT GEOMETRICAL CIRCUITS

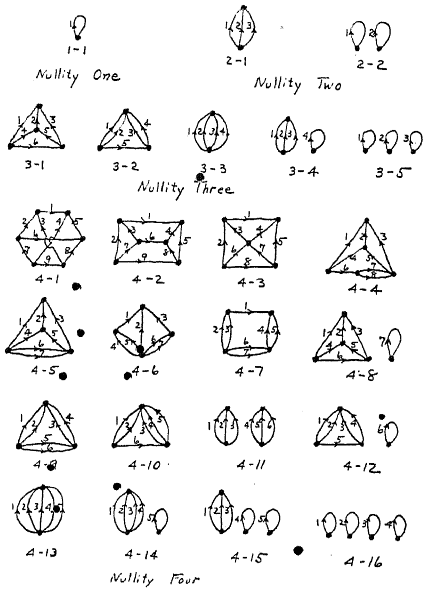


FIG. 2—GEOMETRICAL CIRCUITS OF NULLITY ONE, TWO, THREE, AND FOUR

DISTINCT GEOMETRICAL CIRCUITS

It is assumed that the electrical networks under consideration are composed of two-terminal elements connected together in any fashion. Not all electrical networks are of this type, but many of them may be regarded as such by a suitable designation of elements. The corresponding geometrical circuit consists of branches connected together at branch-points or vertexes, each branch corresponding to a two-terminal electrical element.

Two geometrical circuits are said to be equivalent

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[1] American Tel. & Tel. Co., New York, N. Y.

[2] "The Combinations of Resistances," P. A. Macmahon, *Electrician*, Vol. 28, Apr. 8, 1892, pp. 601-602.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

[3] *Maximum Output Networks for Telephone Substation and Repeater Circuits*, G. A. Campbell and R. M. Foster, TRANS. A.I.E.E., Vol. 39, 1920, pp. 231-280.

if they can be brought into geometrical coincidence by one or more of the following procedures:

I. *By deformation:* That is, if wire models of the two circuits have been made, the circuits are equivalent if the models can be made to coincide by (a) translation and rotation of the model, (b) bending and stretching the wires, and (c) tying or untying knots or other linkages in the wires (that is, the wires can be made to cut themselves and each other like beams of light). This type of equivalence is illustrated by the upper row of diagrams in Fig. 1. Although they differ in appearance all three of these circuits are equivalent. In the mathematical theory of linear graphs, here called geometrical circuits, such equivalent structures are usually termed congruent. The essential feature of a geometrical circuit is thus the relative connection of the branches. If the branches are numbered the circuit is completely described by listing the branches terminating at each vertex.

II. *By separation:* That is, two circuits are equivalent if they can be made to coincide by separating two portions of a circuit which have a single vertex in common, and conversely, by connecting two disconnected portions of a circuit at a single vertex. This is illustrated by the middle row of diagrams in Fig. 1. In the mathematical theory of linear graphs the term equivalent is used to cover concepts I and II as defined here.

III. *By series interchange:* Two circuits are equivalent if they can be made to coincide by the interchange of any branches or groups of branches which have a series connection. This is illustrated by the lower row of diagrams of Fig. 1. This extension of the concept of equivalence is needed to meet the conditions of the usual type of electrical network, where in the case of elements in series the order of those elements in the series is inconsequential. There seems to be no corresponding term for this type of equivalence in the mathematical theory, although the concept might well prove useful, especially in the consideration of dual graphs, since series and parallel are dual concepts, and the interchange of branches in parallel is effected by I.

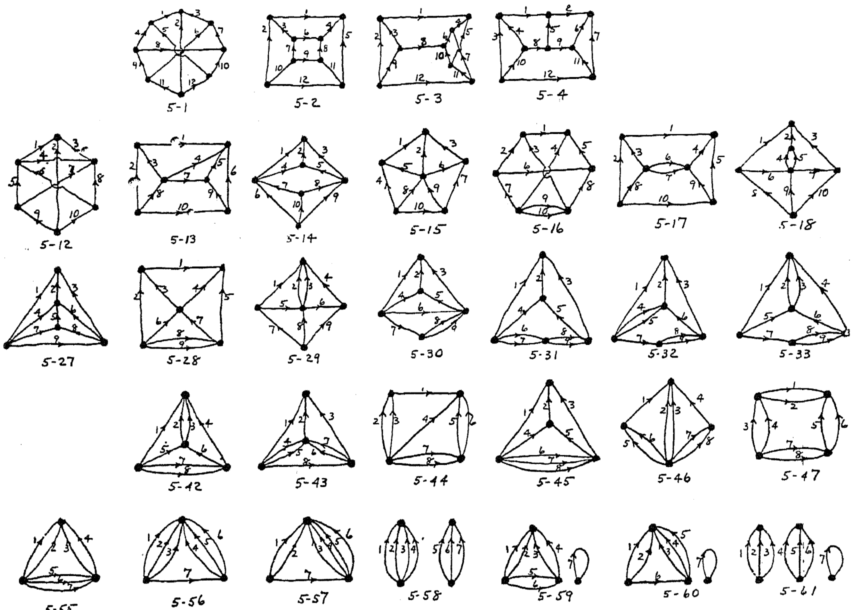

FIG. 3—GEOMETRICAL

CLASSIFICATION BY NULLITY

Of the possible methods of classifying geometrical circuits, one of the most useful for our purpose is by the nullity.[4] If the number of branches, of vertexes, and of separate parts of the circuit are designated by B, V, and S, respectively, then the nullity N is given by the relation:

$$N = B - V + S.$$

Thus in Fig. 1, the upper circuits are of nullity four, the middle circuits four, and the lower circuits five. The nullity N is sometimes called either the cyclomatic number or the number of degrees of freedom. It is important to note that a closed loop, consisting of a single branch with its two terminals coinciding, has one vertex. In this case $B = V = S = 1$, and so $N = 1$, as it should. Equivalent circuits have the same nullity. This classification by nullity seems a natural method for our purpose since the electrical properties of networks are very often specified in terms of independent meshes, the number of degrees of freedom of the electrical network corresponding to the nullity of the geometrical circuit.

It is also important to note that the nullity of a geometrical circuit is not changed by adding or by taking away an isolated, open branch, or by combining two branches in series into a single branch, or by separating a single branch into two branches in series. Accordingly, in enumerating circuits by their nullity, we shall assume that there are no isolated, open branches, and that all branches in series are counted as a single branch. In going over to the physical electrical circuit, a single branch may thus be replaced either by a single electrical element or by two or more such elements in series. It is not likely that electrical elements would be placed in isolated, open branches; but that possibility must be kept in mind.

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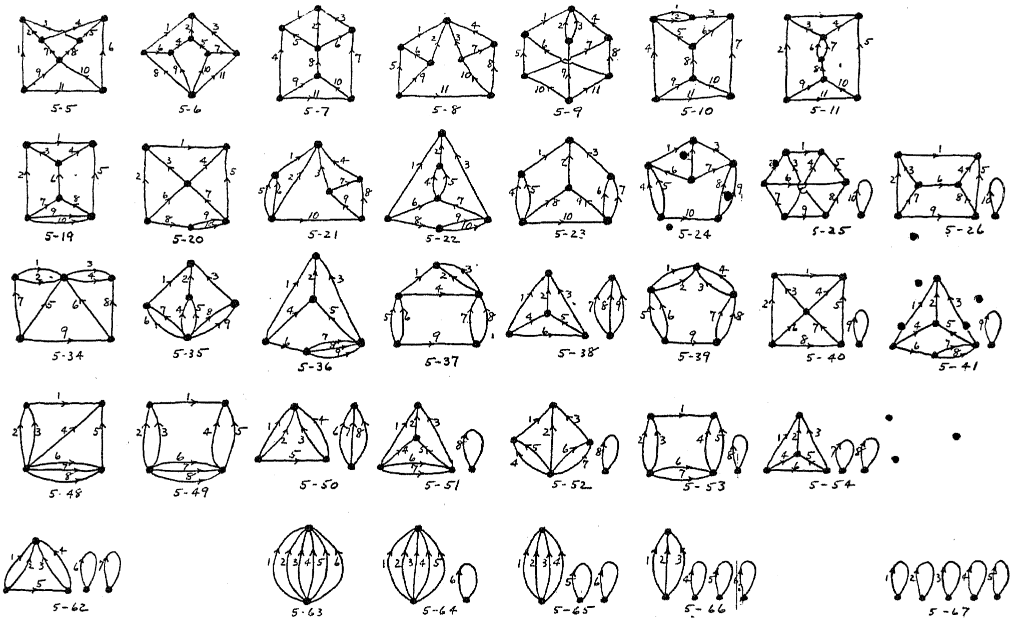
[4] This term is apparently due to Hassler Whitney; see "Non-Separable and Planar Graphs," *Proc. Nat. Academy Sciences*, Vol. 17, Feb. 1931, pp. 125-127.

Geometrical Circuits of Nullity Not Greater Than Five

All distinct geometrical circuits of nullity one, two, three, and four are shown in Fig. 2, and those of nullity five in Fig. 3. This list is complete, with the restrictions mentioned in the preceding paragraph, that is, in any one of the circuits shown, any number of isolated, open branches may be added to the circuit, and any single branch of the circuit may be counted as two or more branches in series. For use in applications, all branches are numbered and an arbitrary positive direction marked on each. These drawings may conveniently serve as a reference framework for all electrical networks with not more than five independent meshes, by designating the appropriate circuit number and specifying the particular electrical elements to be inserted in each of the numbered branches.



Circuits of Nullity Five

Derivation of Circuits

All circuits of nullity N are derived from a smaller group of circuits, called the basic circuits, of the same nullity. In these basic circuits, three branches terminate at each of the vertexes; the circuits are non-separable, that is, each one is connected in a single part which cannot be separated into two parts which have only a single vertex in common; and there are no pairs of branches connected in series. Each of the basic circuits of nullity N thus has $2\ (N - 1)$ vertices and $3\ (N - 1)$ branches. The number of basic circuits is shown by the following table:

Nullity N	1	2	3	4	5	6
Number of basic circuits	1	1	1	2	4	14
Total number of circuits	1	2	5	16	67	435

In Figs. 2 and 3, the basic circuits are placed first, for each value of N. The 14 basic circuits for $N = 6$ are shown in Fig. 4. In view of the large number involved, the complete set of circuits of nullity six is not shown in this paper.

The basic circuits of nullity N are found from the basic circuits of nullity $(N - 1)$ by adding a branch joining the midpoints of a pair of branches, in all possible ways, and then selecting the distinct circuits from the entire set of circuits obtained by this procedure. The fact that this method will yield the entire set of basic circuits of any given nullity follows from theorems proved by H. Whitney.[5]

Then starting with the basic circuits, with $3\ (N - 1)$ branches, all circuits with fewer branches are found by allowing first one and then another branch of the basic circuits to shorten and vanish, thus obtaining first a set of circuits each with $(3\,N - 4)$ branches, then from these the circuits with $(3\,N - 5)$ branches, and so on, until we eventually end with the single circuit with N branches, consisting of N separate closed loops.

From the electrical standpoint this process seems quite natural since allowing a branch to shorten and vanish corresponds to the insertion of an electrical element of zero impedance in the branch, that is, a short circuit. The basic networks for any specified value of N taken alone thus give a complete picture of the entire set of circuits of nullity N, and for some purposes are sufficient, since any electrical network with N independent meshes can then be specified in terms of one or more basic circuits of nullity N by assigning the elements, including possible short circuits, to be distributed among the branches in a specified manner.[6]

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[5] "Basic Graphs," Hassler Whitney,—a paper presented to the American Mathematical Society, New Orleans, Dec. 28, 1931.

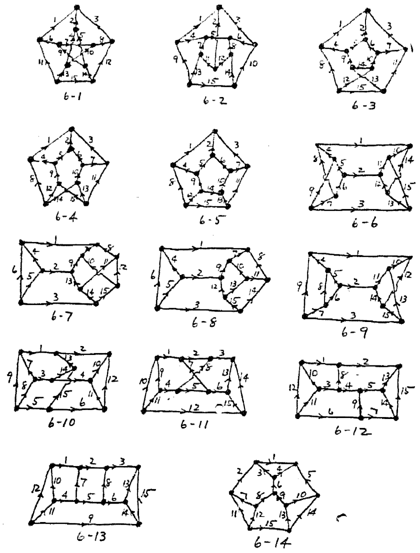

FIG. 4—BASIC GEOMETRICAL CIRCUITS OF NULLITY SIX

CLASSIFICATION BY RANK

Another method of classification of geometrical circuits is by the rank.[7] The rank R of a circuit is defined by the relation:

$$R = V - S.$$

Thus in Fig. 1, the upper circuits are of rank five, the middle circuits two, and the lower circuits four. Equivalent circuits have the same rank.

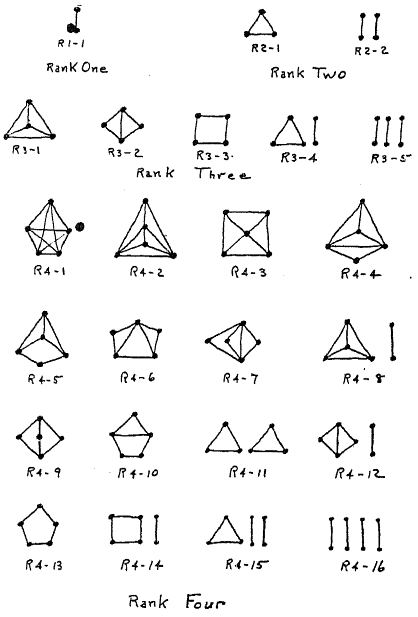

FIG. 5—GEOMETRICAL CIRCUITS OF RANK ONE, TWO, THREE, AND FOUR

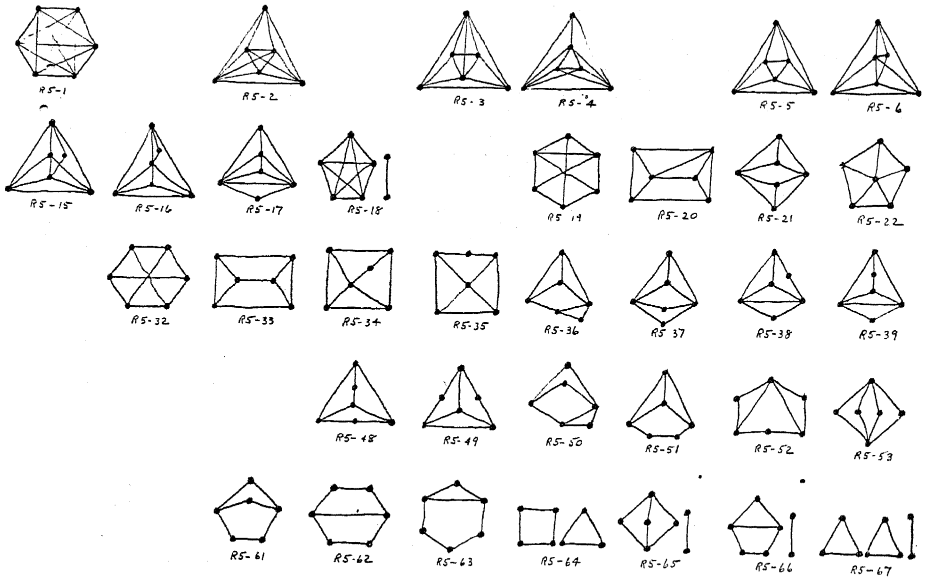

FIG. 6—GEOMETRICAL

It is important to note that the rank of a geometrical circuit is not changed by adding or by taking away an isolated closed loop consisting of a single branch with its two terminals coinciding, or by combining two branches in parallel into a single branch, or by replacing a single branch by two branches in parallel. Accordingly, in enumerating circuits by their rank, it is assumed that there are no isolated closed loops consisting of a single branch, and that all branches in parallel are counted as a single branch. In going over to the physical electrical circuit, a single branch may thus be replaced either by a single electrical element or by two or more such elements in parallel; also, electrical elements may be placed in isolated closed loops, a possibility that may actually arise, such as a short-circuited winding of a transformer.

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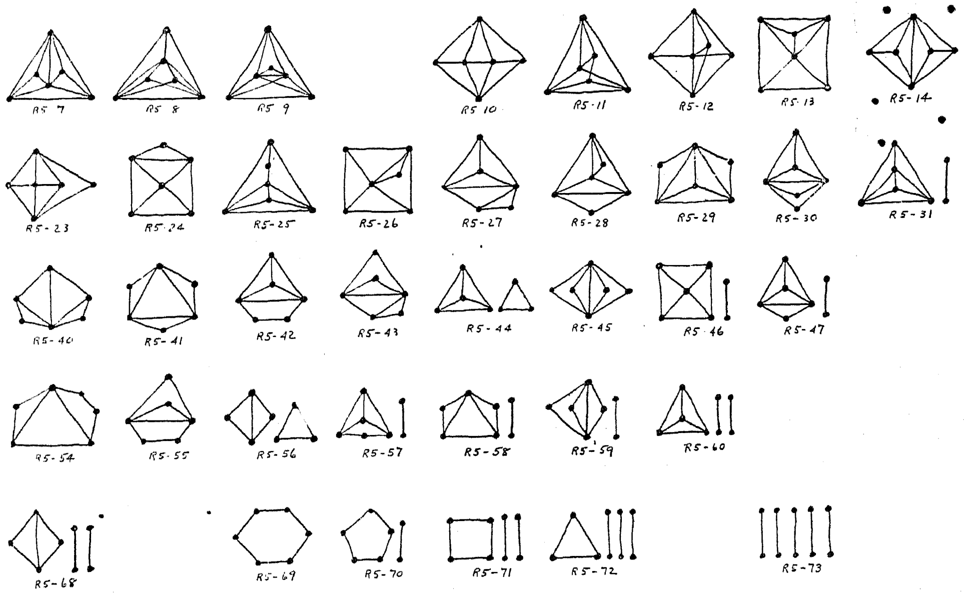
[6] Figs. 2-4 also illustrate the wide variety of forms which, under certain circumstances, equivalent electrical networks may assume.

[7] This term also is apparently due to Hassler Whitney; see "Non-Separable and Planar Graphs," *loc cit.*

GEOMETRICAL CIRCUITS OF RANK NOT GREATER THAN FIVE

All distinct geometrical circuits of rank one, two, three, and four are shown in Fig. 5, and those of rank five in Fig. 6. This enumeration is complete, with the restrictions mentioned in the preceding paragraph, that is, to each of the circuits shown, any number of isolated, closed loops, each consisting of a single branch, may be added, and any single branch of any circuit may be replaced by two or more branches in parallel.

All circuits of rank R are derived from the single circuit consisting of $(R+1)$ vertexes with every pair of vertexes connected by a single branch. Starting with this circuit, we allow first one and then another of the branches to be cut and removed entirely from the circuit. Eventually we end with the single circuit consisting of N isolated open branches. From the electrical standpoint this process corresponds to the insertion of an electrical element of infinite impedance in the branch which is to be cut, that is, the branch is open-circuited.



CIRCUITS OF RANK FIVE

The number of circuits, classified by the rank, is shown by the following table:

Rank R	1	2	3	4	5	6
Total number of circuits	1	2	5	16	73	538

In view of the large number, the circuits of rank six are not shown in this paper.

There is an inverse or dual relationship between the rank R and the nullity N. The two parameters change places when all series connections are changed to parallel and vice versa, with corresponding transformations in the case of bridge circuits; to every set of branches forming a closed path in one circuit there corresponds a set of branches in the other circuit separating the vertexes into two groups which are not connected by any other branch. H. Whitney[8] has shown that if the geometrical circuit is applicable to the surface of a sphere, there exists a dual or inverse circuit with $N' = R$ and $R' = N$, and that this does not hold for circuits which are not applicable to a sphere, that is, for circuits which cannot be drawn on a sphere or plane with no branches crossing one another. As a result, the enumeration by given values of R is not completely symmetrical with the enumeration by given values of N. There is complete correspondence through $N = R = 3$. For $N = R = 4$, however, although the total numbers are the same, circuits N 4-1 and R 4-1 are not duals. For values greater than four the classification by rank leads to larger total numbers than does that by nullity. It should be pointed out, however, that the enumeration by R is a more straightforward process, since we start in each case with a single circuit, each point connected with every other point, instead of starting with a set of basic circuits which must first be derived.

OTHER METHODS OF CLASSIFICATION

For some purposes neither of the above methods of enumeration may prove quite satisfactory in view of the special restrictions involved. It may be desirable to have a complete enumeration of circuits with B branches, whether or not some of these branches may be arranged in series or parallel or left isolated. Fig. 7 shows a beginning of such an enumeration, the circuits being classified according to values of both N and R. It is to be noted that $N + R = B$. This enumeration may be derived from the classification by nullity, upon inserting branches in series and adding isolated open branches; or it may be derived from the classification by rank, upon inserting branches in parallel and adding isolated closed branches. In either case, given the fundamental enumerations, by nullity or by rank, the problem can then be handled by the regular methods of the theory of combinations.

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[8] "Non-Separable and Planar Graphs," Hassler Whitney, *loc. cit.*,—Theorem 15.

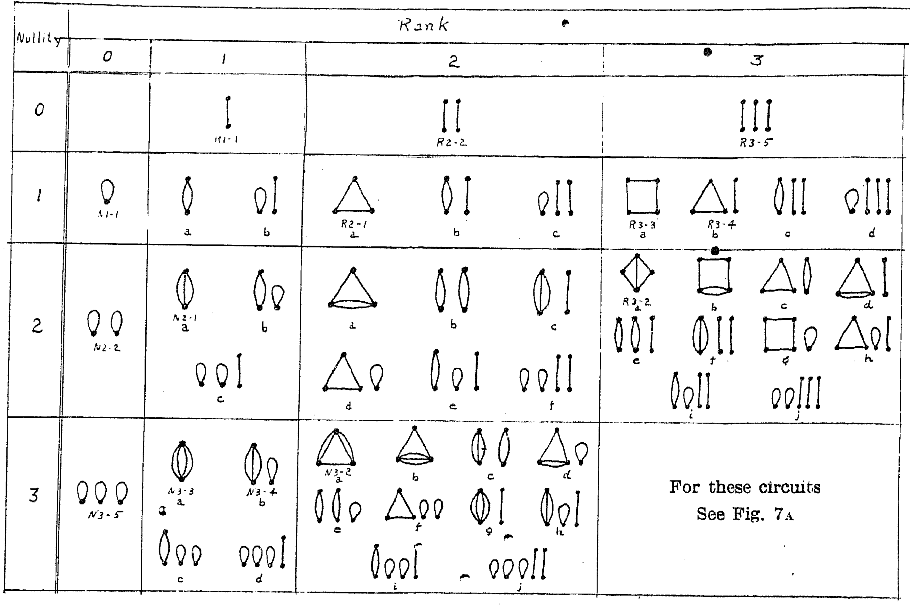

FIG. 7—GEOMETRICAL CIRCUITS CLASSIFIED BY NULLITY AND RANK

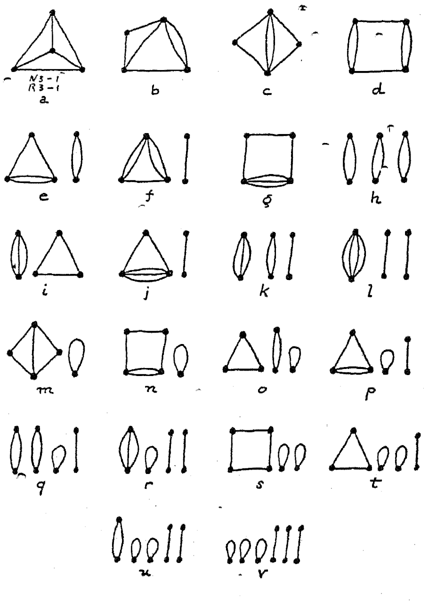

FIG. 7A—GEOMETRICAL CIRCUITS OF NULLITY THREE AND RANK THREE

Such large numbers of circuits are involved in this scheme, however, that it does not seem worth while in this paper to extend it to higher values. A more useful plan, perhaps, is to exclude all series and parallel combinations, as well as all isolated single elements, whether open or closed. Such a classification is shown by Fig. 8, for values of N and R not greater than six. This scheme can be obtained either from the classification by nullity or from the classification by rank, upon excluding the circuits not desired. These excluded circuits can always be restored by simple means.

SYMMETRICAL GEOMETRICAL CIRCUITS

In some of the circuits shown by Figs. 2-6 all the branches enter symmetrically, and also all the vertexes. One example is the familiar wheatstone bridge circuit N 3-1 of Fig. 2. In the following discussion attention is confined to those symmetrical circuits which are composed of a single non-separable part, and in which at least three branches terminate at each vertex.

Nine examples of symmetrical circuits with three branches terminating at each vertex are shown in Fig. 9. The circuits formed by the edges of three of the regular polyhedra appear on this list, the tetrahedron III-3, the cube III-5, and the dodecahedron III-11-A. Apart from the trivial case III-2, the other five are of considerable interest as generalizations, in a certain sense, of the regular polyhedra, or more exactly, generalizations of the geometrical circuits formed by the edges of the regular polyhedra. The circuit III-4 has been used in several places[9] as an illustration of a rigid structure, in the theory of statics, though its symmetrical character has apparently not been pointed out in that connection. The circuit III-6 was used by J.

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[9] See, for example, *Encyklopädie der mathematischen Wissenschaften*, Vol. 4, part 1, p. 403. The symmetrical character of this circuit was pointed out in the 1920 paper by G. A. Campbell and R. M. Foster previously mentioned.

Petersen[10] in showing that Tait's theorem on the coloring of graphs could not be extended to graphs which were not applicable to a sphere, but here again the symmetrical character of the circuit was not mentioned. The three others, III-9, III-11-B, and III-13, are apparently new. It is probable that there exist other symmetrical circuits of nullity greater than six and with three branches at each vertex, but a preliminary investigation has failed to disclose them. On the other hand, it is extremely interesting that there are at least two such circuits of nullity eleven, namely, III-11-A and III-11-B.

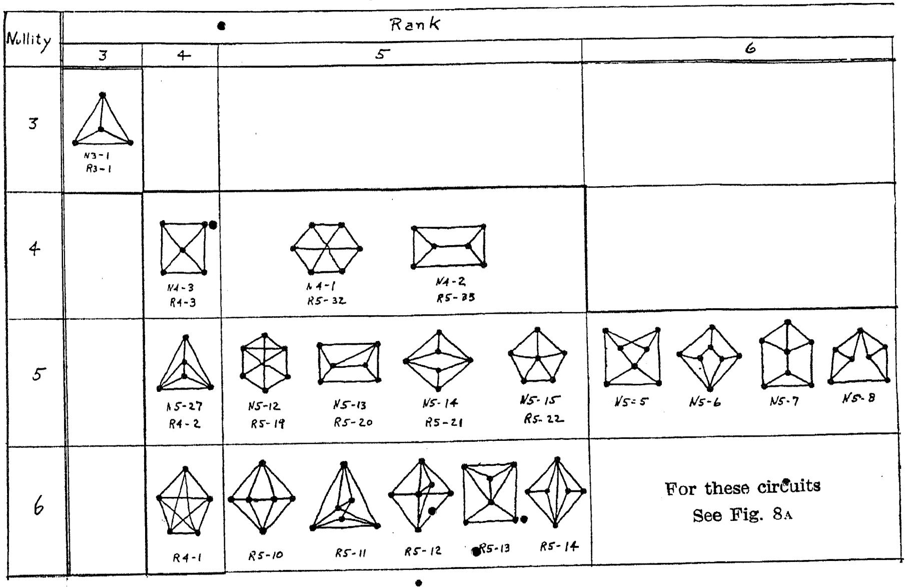

FIG. 8—GEOMETRICAL CIRCUITS CLASSIFIED BY NULLITY AND RANK—OMITTING CIRCUITS WITH BRANCHES IN SERIES OR IN PARALLEL OR ISOLATED

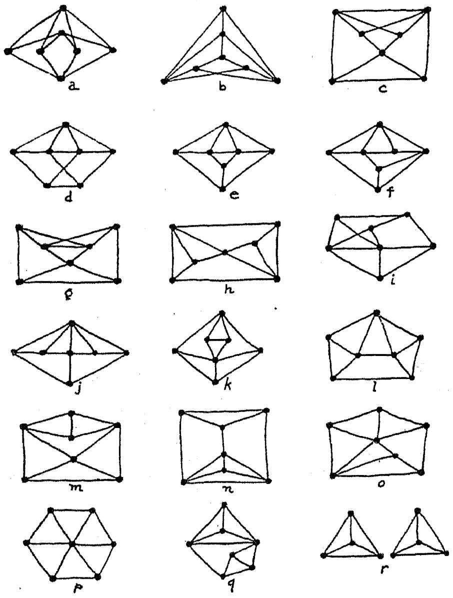

FIG. 8A—GEOMETRICAL CIRCUITS OF NULLITY SIX AND RANK SIX—OMITTING CIRCUITS WITH BRANCHES IN SERIES OR IN PARALLEL OR ISOLATED

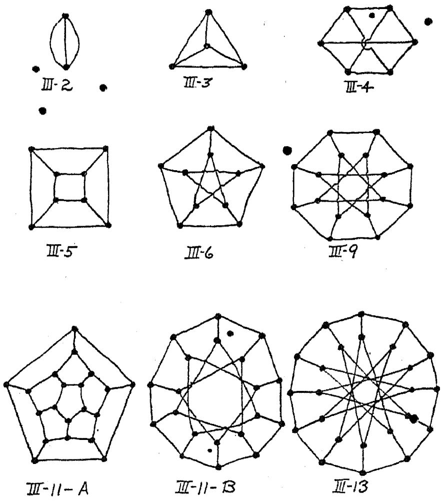

FIG. 9—SYMMETRICAL GEOMETRICAL CIRCUITS WITH THREE BRANCHES AT EACH VERTEX

No attempt has yet been made to enumerate completely the symmetrical circuits with four or more branches terminating at each vertex, although a few examples are shown in Fig. 10, including the circuits of the octahedron IV-7 and the icosahedron V-19. Here again it is interesting to note that there are at least two such circuits of nullity eleven, namely, IV-11-A and IV-11-B. A complete list would include many trivial cases, such as, for example, the symmetrical circuits with four branches at each vertex obtained by putting a branch in parallel with each edge of a regular polygon, as is illustrated by circuits N 4-9 and N 5-47 of Figs. 2 and 3. A complete list of symmetrical circuits would also include the circuits used as starting points in the classification by rank, that is, the circuits with every pair of vertexes connected by a single branch, for example, III-3, IV-6, and V-10.

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[10] Julius Petersen, *Intermédiaire des Mathématiciens*, Vol. 5, Oct. 1898, pp. 225-227.

GEOMETRICAL FORMS OF TRANSDUCERS

As an illustration of one application of the method described in this paper, Fig. 11 shows the basic geometrical forms of transducers with two, three, and four meshes, obtained from the geometrical circuits of Fig. 2. The transducer has two pairs of terminals; so the problem of enumeration reduces to the problem in simple combinations of picking out a pair of branches from the basic circuits N 2-1, N 3-1, N 4-1, and N 4-2. This can be done in 1, 2, 2, and 6 different ways, respectively, giving a total of 11 different forms. The single basic two-mesh transducer is the familiar T network. The first of the basic three-mesh transducers is the series extended lattice, and the second is the series extended bridged-T. No one of the eight four-mesh transducers appears to have been investigated to any extent. It should be noted that the assigned positive directions on Fig. 2 have been ignored in this enumeration; the number of forms given here could be approximately doubled by twisting one pair of terminals; and the number could be further increased by interchanging the two pairs of terminals in the dissymmetrical cases.

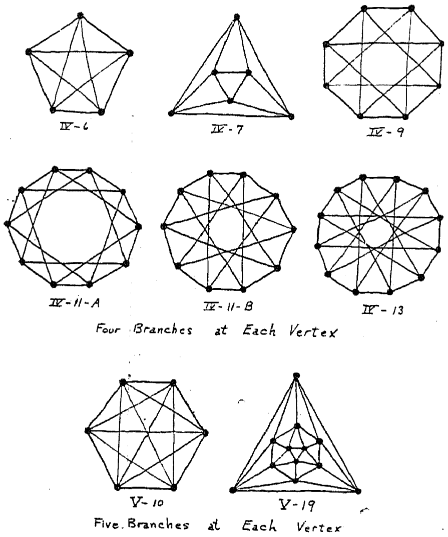

FIG. 10—SYMMETRICAL GEOMETRICAL CIRCUITS WITH FOUR OR MORE BRANCHES AT EACH VERTEX

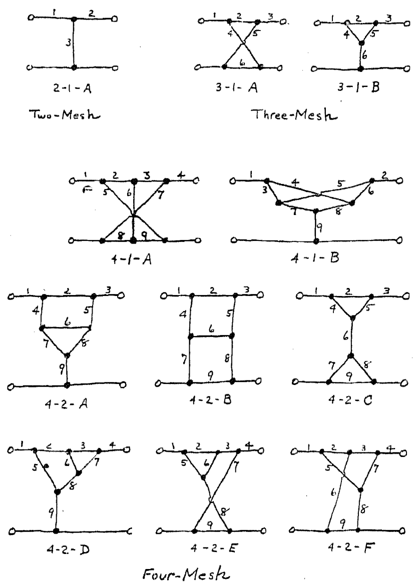

FIG. 11—BASIC GEOMETRICAL FORMS OF TWO-, THREE-, AND FOUR-MESH TRANSDUCERS

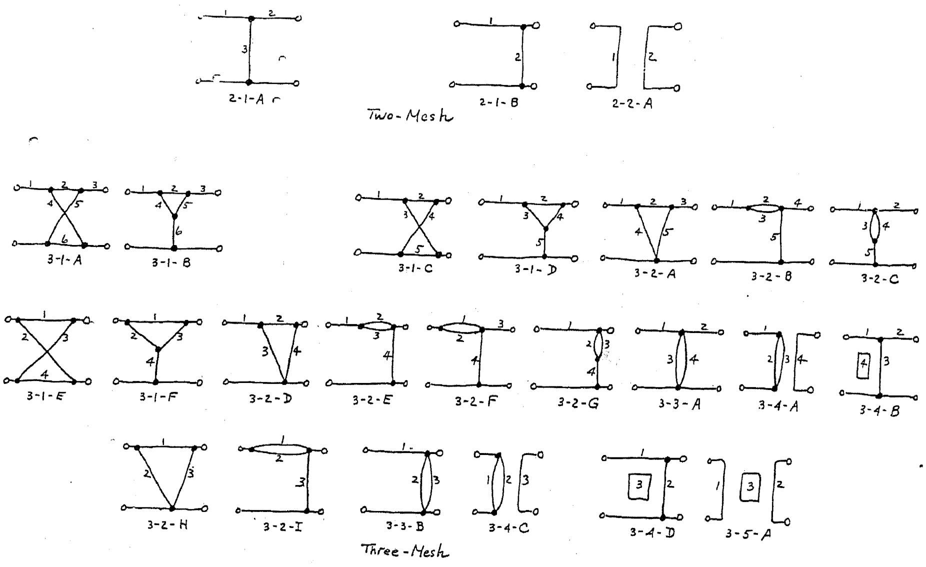

FIG. 12—GEOMETRICAL FORMS OF TWO- AND THREE- MESH TRANSDUCERS

Fig. 11 shows only the basic forms. Thus the familiar Π type of network is not illustrated directly in this figure, although it may be obtained from either 3-1-A or 3-1-B by allowing branches 1, 3, and 6 to shorten and vanish. Since such cases may be of practical interest the complete set of two- and three-mesh transducers is shown in Fig. 12. As before, however, no attempt has been made to show the alternative forms obtained by twisting one pair of terminals or by interchanging the two pairs; nor are the limiting cases shown where the transducer reduces to a mere series or parallel connection. In certain cases, such as 3-2-B, the replacement of a group of branches connecting two terminals by a single branch would reduce the number of meshes from three to two; but this cannot be done, in general, if there is mutual inductance between any branch of the group and a branch outside the group. Accordingly, these cases are included in the enumeration.

Discussion

For discussion of this paper see page 321.

Vector Theory of Circuits Involving Synchronous Machines

BY I. H. SUMMERS*

Synopsis.†—*It has been shown in previous publications that calculations of the performance of a synchronous machine may be simplified by the use of two scalar impedance operators which apply separately to the component of armature current which magnetizes in the axis of the poles, and to the component which magnetizes in the axis of the interpolar space.*

The present paper describes a method of combining these two scalar operators into a single vector operator which applies directly to the total current. With the new operator it becomes possible to write out the complete equations which apply to various numbers of interconnected machines, under various operating conditions, as readily as would be possible if each machine was replaced by a fixed source of voltage operating through a constant value of impedance. The effect is to simplify and thus to extend the scope of synchronous machine analysis. It is believed that the theory developed will be of value to engineers who are working on problems such as stability, hunting, and other problems of this general character which involve detailed analysis of the operation of two or more interconnected synchronous machines under transient conditions.

* * * * *

INTRODUCTION†

THE present paper deals with circuits involving synchronous machines having three armature circuits, salient poles, an arbitrary number of closed circuits on the rotor, and two rotor field circuits, one magnetizing in the axis of the poles, and one magnetizing in the axis of the interpolar space. The performance of such a machine is determined by the history of the three armature terminal voltages, the two field voltages, and the angular position of the rotor with respect to the armature. Let the instantaneous values of these quantities by e_a, e_b, e_c, e_{fd}, e_{fq}, θ, and let the instantaneous values of armature current be i_a, i_b, i_c.

Then in problems which do not involve the flow of neutral current, it can be shown[1] that the equations determining current may be written in the form

$$e_d = p\psi_d - r i_d - \psi_q p\,\theta$$

$$e_q = p\psi_q - r i_q + \psi_d p\,\theta$$

where $$p = \frac{d}{dt}$$

$$\psi_d = g_d(p)\,e_{fd} - x_d(p)\,i_d$$

$$\psi_q = g_q(p)\,e_{fq} - x_q(p)\,i_q$$

r = armature resistance

$g_d(p)$, $g_q(p)$, $x_d(p)$, $x_q(p)$ are operators and e_d, e_q, i_d, i_q are particular functions of e_a, e_b, e_c, θ, and i_a, i_b, i_c, θ, and are referred to as the direct and quadrature components of voltage and current respectively.

The operators $g_d(p)$, $g_q(p)$, etc., may be computed from design data, or determined by tests, and may be visualized if desired in terms of appropriate equivalent circuits.

It will be seen that the solution of the equations is complicated by the fact that there is a mutual relationship between the direct and quadrature quantities, that is by the fact that direct component current gives rise to both direct and quadrature components of voltage, and conversely. Nevertheless, when the values of terminal voltage are definitely known, either as a function of time, or of angular displacement, as when only a single machine is being considered, the analysis is comparatively simple.

However, difficulties are encountered when it is desired to study several interconnected machines. These difficulties may be appreciated by considering the case of two machines connected directly together. Let subscripts 1 and 2 be used to designate these machines. Then it is known that $e_{a1} = e_{a2}$, $i_{a1} = -\,i_{a2}$, and similarly for the other phases. It does not follow, however, that $e_{d1} = e_{d2}$, $i_{d1} = -\,i_{d2}$, $e_{q1} = e_{q2}$, $i_{q1} = -\,i_{q2}$ except in the particular case $\theta_1 = \theta_2$. It can be shown, however, that

$$\mathbf{e}_1 = \mathbf{e}_2$$

$$\mathbf{i}_1 = -\,\mathbf{i}_2$$

where

$$\mathbf{e}_1 = (e_{d1} + j\,e_{q1})\,\underline{|\theta_1}$$

$$\mathbf{e}_2 = (e_{d2} + j\,e_{q2})\,\underline{|\theta_2}$$

$$\mathbf{i}_1 = (i_{d1} + j\,i_{q1})\,\underline{|\theta_1}$$

$$\mathbf{i}_2 = (i_{d2} + j\,i_{q2})\,\underline{|\theta_2}$$

This suggests that the solution for current could be simply effected if suitable symbolic relations could be established between $\mathbf{e}_1$ and $\mathbf{i}_1$, and $\mathbf{e}_2$ and $\mathbf{i}_2$.

Were it not for the salient pole construction it would be possible to write a relation between the current and voltage of any machine in the form

$$\mathbf{e} = g(p)\,\mathbf{e}_f - z(p)\,\mathbf{i}$$

where $$\mathbf{e}_f = (e_{fd} + j\,e_{fq})\,\underline{|\theta}$$

and $g(p)$ and $z(p)$ are scalar operators which contain speed as a parameter. However, as previously noted, it is a consequence of salient pole construction that the component of $\mathbf{e}$ due to $\mathbf{e}_f$ need not be in phase with $\mathbf{e}_f$ and the component $\mathbf{e}$ due to $\mathbf{i}$ need not be exactly 180 deg. out of phase with $\mathbf{i}$.

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*Deceased. Formerly with the General Electric Co., Schenectady, N. Y.

†Prepared by R. H. Park, Stone & Webster Engg. Corp., Boston, Mass.

[1] *Two Reaction Theory of Synchronous Machines*, R. H. Park, A.I.E.E. TRANS., Vol. 48, July 1929, p. 716-727.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

Mr. Summers shows in this paper, however, that a relation can be written between $\mathbf{e}$ and $\mathbf{i}$, in the form

$$\mathbf{e} = \hat{g}\,(p)\,\mathbf{e}_f - \hat{x}\,(p)\,\mathbf{i}$$

where $g\,(p)$ and $x\,(p)$ are linear vector operators which involve speed, armature resistance and the scalar operators $g_d\,(p)$, $g_q\,(p)$, $x_d\,(p)$, and $x_q\,(p)$. These vector operators effect a very great simplification of the differential equations which determine the operation of the machines. In the special case in which the machine speeds are constant, although not necessarily equal, they permit direct solution of the problem in operational form, since the operators are subject to the ordinary rules of the algebra of linear vector operators when they do not contain parameters which are functions of the independent variable. Also, it becomes as simple to write out, in symbolic form, the solution of problems involving the performance of a number of interconnected machines operating at constant or nearly constant speed, under transient or steady load conditions, as it is to write out the solution of the equations which determine the flow of current in a d-c. network. The final explicit form of the result may then be obtained by evaluating the vector operational forms in accordance with established mathematical procedure.

The evaluation of the linear vector operations may be accomplished either by means of dyadics, or by matrix algebra. To quote from Summers' manuscript notes:

"Matrix algebra is of value in linear vector operations for the same reason that ordinary algebra is used in differential operations. Special rules are used in either case when the operations are actually performed on the operand, but transformations are facilitated in the meantime. In the case of the differential operators the expansion theorem is used eventually, and in the case of linear vector operators dyads are supplied and scalar products are formed. Algebra and matrix algebra are used previous to the final operations in the respective cases."

The resultant effect is to simplify and thus to extend the scope of synchronous machine analysis, particularly in cases in which more than one machine is involved.

This work has been taken directly from the author's manuscript notes, except for such minor changes as were considered obviously necessary. The body of the paper will be seen to be incomplete, but it is believed that the development of the thought is adequate to permit any one familiar with the general subject to complete the unfinished portion.

ANALYSIS

It has been shown* that synchronous machine electrical quantities are expressible in terms of linear equations involving direct axis, quadrature axis, and zero phase sequence quantities and that the coefficients of these linear equations are differential operators. Thus the following sets of relations are fundamental:

$$\left.\begin{aligned} \psi_d &= g_d\,(p)\,e_{fd} - x_d\,(p)\,i_d \\ \psi_q &= g_q\,(p)\,e_{fq} - x_q\,(p)\,i_q \\ \psi_o &= -\,x_o\,i_o \end{aligned}\right\} \qquad (1)$$

$$\left.\begin{aligned} e_d &= p\,\psi_d - ri_d - \psi_q\,p\,\theta \\ e_q &= p\,\psi_q - ri_q + \psi_d\,p\,\theta \\ e_o &= p\,\psi_o - ri_o \end{aligned}\right\} \qquad (2)$$

$$\left.\begin{aligned} i_{fd} &= y_{fd}\,(p)\,e_{fd} + k_{fd}\,(p)\,i_d \\ i_{fq} &= y_{fq}\,(p)\,e_{fq} + k_{fq}\,(p)\,i_q \end{aligned}\right\} \qquad (3)$$

The differential operators in these equations are deducible by means of auxiliary relations, and other auxiliary relations enable phase quantities to be determined from the direct, quadrature and zero quantities.

For the present purpose we will assume that the differential operators are known, and that the problem is solved when the direct, quadrature and zero quantities are known.

It happens that the presence of zero phase sequence quantities is a fairly rare occurrence in a synchronous machine connected to a large power system so that a large class of problems occurs where the direct and quadrature quantities completely specify the performance. It is with this class of problems that the present note is concerned.

We may define $\mathbf{d}$ and $\mathbf{q}$ as unit vectors in the direct and quadrature axes of the machine respectively. Further define:

$$\left.\begin{aligned} \psi &= \psi_d\,\mathbf{d} + \psi_q\,\mathbf{q} \\ \mathbf{e} &= e_d\,\mathbf{e} + e_q\,\mathbf{q} \\ \mathbf{i} &= i_d\,\mathbf{e} + i_q\,\mathbf{q} \\ \mathbf{e}_f &= e_{fd}\,\mathbf{e} + e_{fq}\,\mathbf{q} \\ \mathbf{i}_f &= i_{fd}\,\mathbf{e} + i_{fq}\,\mathbf{q} \end{aligned}\right\} \qquad (4)$$

Then neglecting zero phase sequence quantities and replacing $p\,\theta$ by $1 - s$, where s = slip with respect to normal speed, there is obtained:

$$\psi = \hat{g}\,(p)\,\mathbf{e}_f - \hat{x}\,(p)\,\mathbf{i} \qquad (5)$$

$$\mathbf{e} = (p + (1 - s)\,j)\,\psi - r\mathbf{i} \qquad (6)$$

$$\mathbf{i}_f = y_f\,(p)\,e_f + k_f\,(p)\,\mathbf{i} \qquad (7)$$

In these expressions a caret over a quantity means that it is a linear vector operator, transforming a vector into another by extensions and rotations. It is equivalent to a dyadic[2] followed by a dot to indicate the dot (scalar) product. Inasmuch as a dyadic is a matrix[3] and operations are wanted between scalars and linear vector operators it will be convenient to designate the operator by its matrix, the vector coordinate and the dot

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*R. H. Park, *loc. cit.*

[2] "Vector Analysis," Gibbs-Wilson, Yale University Press.

[3] "Introduction to Higher Algebra," Bocher, M., McMillan.

being understood. The coordinates may be stated as subscripts when desired. Thus:

$\hat{a}\ r$ is understood to be

$$\begin{pmatrix} a_{11}\,\mathbf{dd} + a_{12}\,\mathbf{dq} \\ + a_{21}\,\mathbf{qd} + a_{22}\,\mathbf{qq} \end{pmatrix} . \mathbf{r}$$

and will be characterized by the convention.

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \mathbf{r}$$

In this way the symbolism is freed from superfluous unit vectors and product symbols and matrix algebra is available to carry out operations between scalars and matrices. When vector operations are to be performed the vector dyads and the dot may be written in at will,[4] or may be supplied mentally.

Thus in (5), (6) and (7):

$$\left.\begin{aligned} \hat{g}\ (p) &= \begin{pmatrix} g_d\,(p) & o \\ o & g_q\,(p) \end{pmatrix} \\ \hat{x}\ (p) &= \begin{pmatrix} x_d\,(p) & o \\ o & x_q\,(p) \end{pmatrix} \\ \hat{x} &= \begin{pmatrix} x_d & o \\ o & x_q \end{pmatrix} \\ j &= \begin{pmatrix} o & -1 \\ 1 & o \end{pmatrix} \\ \hat{y}_f\,(p) &= \begin{pmatrix} y_{fd}\,(p) & o \\ o & y_{fq}\,(p) \end{pmatrix} \\ \hat{k}_f\,(p) &= \begin{pmatrix} k_{fd}\,(p) & o \\ o & k_{fq}\,(p) \end{pmatrix} \end{aligned}\right\} \qquad (8)$$

Where any terms in a matrix equation do not contain a matric explicitly as written, unit matrix, $\begin{pmatrix} l & o \\ o & l \end{pmatrix}$, is understood.

Equations (5) and (6) may be combined to give:

$$\mathbf{e} = \hat{h}\ (p)\,\mathbf{e}_f - \hat{z}\ (p)\,\mathbf{i} \qquad (9)$$

The method of working may be illustrated by evaluating $\hat{h}\ (p)$ and $\hat{z}\ (p)$.

Thus, directly:

$$\hat{z}\ (p) = [p + (1 - s)\,j]\,\hat{x}\ (p) + r$$

This gives:

$$\begin{pmatrix} p & s-1 \\ 1-s & p \end{pmatrix} \begin{pmatrix} x_d\,(p) & o \\ o & x_q\,(p) \end{pmatrix} + r$$

Carrying out the multiplication and adding r there is obtained

$$\left.\begin{aligned} \hat{z}\ (p) &= \begin{pmatrix} z_d\,(p) & (s-1)\,x_q\,(p) \\ (1-s)\,x_d\,(p) & z_q\,(p) \end{pmatrix} \\ &\text{where} \\ z_d\,(p) &= p\,x_d\,(p) + r \\ z_q\,(p) &= p\,x_q\,(p) + r \end{aligned}\right\} \qquad (10)$$

Similarly originally

$$\hat{h}\ (p) = (p + (1 - s)\,j)\,\hat{g}\ (p)$$

which gives

$$\hat{h}\ (p) = \begin{pmatrix} p g_d\,(p) & (s-1)\,g_q\,(p) \\ (1-s)\,g_d\,(p) & p g_q\,(p) \end{pmatrix} \qquad (11)$$

As an illustration of the working out of the equations, consider two machines connected together,

$$\left.\begin{aligned} \mathbf{e} &= \hat{h}_1\,(p)\,\mathbf{e}_{f1} - \hat{z}_1\,(p)\,\mathbf{i} \\ &= \hat{h}_2\,(p)\,\mathbf{e}_{f2} + \hat{z}_2\,(p)\,\mathbf{i} \end{aligned}\right\} \qquad (12)$$

Subtracting

$$(\hat{z}_1\,(p) + \hat{z}_2\,(p)\,) = \mathbf{i} = \hat{h}_1\,(p)\,\mathbf{e}_{f1} - \hat{h}_2\,(p)\,\mathbf{e}_{f2}$$

Multiplying through by the inverse operator[5]

$$\mathbf{i} = [\hat{z}_1\,(p) + \hat{z}_2\,(p)\,]^{-1}\,[\hat{h}_1\,(p)\,\mathbf{e}_{f1} - \hat{h}_2\,(p)\,\mathbf{e}_{f2}\,] \qquad (13)$$

Now in writing equations (12) it was tacitly assumed that the matrix operators were expressible as vector dyads on the same set of axes.

In order to carry out this requirement consider:

$$a_{ij} = \begin{pmatrix} a_{11}\,\mathbf{ii} & a_{12}\,\mathbf{ij} \\ a_{21}\,\mathbf{ji} & a_{22}\,\mathbf{jj} \end{pmatrix} \qquad (14)$$

Now if

$$\left.\begin{aligned} \mathbf{i} &= l_1\,\mathbf{f} + m_1\,\mathbf{g} \\ \mathbf{j} &= l_2\,\mathbf{f} + m_2\,\mathbf{g} \end{aligned}\right\} \qquad (15)$$

Substitution of (15) in (14) and collecting terms gives:

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[4] The question of having special symbolism to indicate that a quantity is a scalar, a vector, a differential operator or a linear vector operator, should be given careful thought. It would facilitate writing, typing, printing, and reading if no special symbolism were used, the quantities being completely flexible and capable of taking on the proper significance at will. On the other hand, interpretation is facilitated by having the precise meanings of all symbols indicated at all times. Perhaps a compromise would be best in which the actual equations are written without special symbolism but the interpretations are carefully noted in the text. Special operators, such as synchronous reactance could be indicated, for example, as $x(\infty)$ or x_s and pure vectors could be written as a scalar with the proper unit vector, special letters being reserved for the unit vector to make them readily distinguishable.

[5] This implies that s_1 and s_2 are constant. For uniplanar matrixes the inverse operator $- x^{-1}$ is

$$\frac{\begin{pmatrix} x_{22} & -x_{12} \\ -x_{21} & x_{11} \end{pmatrix}}{\begin{vmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{vmatrix}}$$

$$a_{fg} = \begin{pmatrix} b_{11}\,\mathbf{ff} & b_{12}\,\mathbf{fg} \\ b_{21}\,\mathbf{gf} & b_{22}\,\mathbf{gg} \end{pmatrix}$$

where

$$\left.\begin{aligned} b_{11} &= l_1^2\,a_{11} + (a_{12} + a_{21})\,l_1\,l_2 + l_2^2\,a_{22} \\ b_{12} &= l_1\,m_1\,a_{11} + l_1\,m_2\,a_{12} + l_2\,m_1\,a_{21} + l_2\,m_2\,a_{22} \\ b_{21} &= l_1\,m_1\,a_{11} + l_2\,m_1\,a_{12} = l_1\,m_2\,a_{21} + l_2\,m_2\,a_{22} \\ b_{22} &= m_1^2\,a_{11} + (a_{12} + a_{21})\,m_1\,m_2 + m_2^2\,a_{22} \end{aligned}\right\} \quad (16)$$

For a set of rectangular axes rotated an angle δ from the reference there is

$$\left.\begin{aligned} l_1 &= \cos\delta & m_1 &= \sin\delta \\ l_2 &= -\sin\delta & m_2 &= \cos\delta \end{aligned}\right\} \quad (17)$$

and

$$b_{11} = \frac{a_{11} + a_{22}}{2} + \frac{a_{11} - a_{12}}{2}\,\text{Cos}\,2\,\delta - \frac{a_{12} + a_{21}}{2}\,\text{Sin}\,2\,\delta$$

$$b_{12} = \frac{a_{11} - a_{22}}{2}\,\text{Sin}\,2\,\delta + \frac{a_{12} - a_{21}}{2} + \frac{a_{12} + a_{21}}{2}\,\text{Cos}\,2\,\delta$$

$$b_{21} = \frac{a_{11} - a_{22}}{2}\,\text{Sin}\,2\,\delta + \frac{a_{21} - a_{12}}{2} + \frac{a_{12} + a_{21}}{2}\,\text{Cos}\,2\,\delta$$

$$b_{22} = \frac{a_{11} + a_{22}}{2} - \frac{a_{11} - a_{22}}{2}\,\text{Cos}\,2\,\delta + \frac{a_{12} + a_{21}}{2}\,\text{Sin}\,2\,\delta$$

Discussion

EQUIVALENT CIRCUITS—I

(Starr)

TRANSIENT OSCILLATIONS OF MUTUALLY COUPLED CIRCUITS

(Bewley)

GEOMETRICAL CIRCUITS OF ELECTRICAL NETWORKS

(Foster)

VECTOR THEORY OF CIRCUITS INVOLVING SYNCHRONOUS MACHINES

(Summers)

J. E. Clem: Mr. Starr's paper presents a thorough and comprehensive treatment of the derivation and application of equivalent circuits. Considerable emphasis is given to the greater utilization of the calculating board, by applying equivalent circuits, to the solution of certain problems too difficult to solve analytically. For example, in Part III of the paper, a number of simplified equivalent circuits to represent groups of parallel transmission lines are derived. It is shown in the paper how these circuits may be used to represent the zero-phase diagram of a transmission system on a calculating board in a very simple manner.

It is interesting to note that the group of equivalents shown in Figs. 8 to 15 involve no negative resistance and in most cases involve no negative links of any kind. It is evident that these properties make these circuits of particular value in calculating board work. Anyone interested in system studies should find these circuits, as well as the generalized equivalents developed in Parts I and II of the paper of considerable value, whether or not it is intended to use the calculating board.

More important than the particular applications treated are, perhaps, the general methods and rules which are given to apply equivalent circuits to any type of problem. Thus, the methods developed here might be applied directly to obtain equivalent circuits to represent various types of rotating machinery, and particularly certain multi-circuit motors for which adequate equivalents have not as yet been developed. Representations of the proposed three and four winding generators for use in a single-line diagram also suggest the use of circuits similar to those shown in Figs. 9 and 10.

Many other circuit problems lend themselves to the use of an equivalent circuit as an aid to the solution. For example, they are useful in analyzing any multi-circuit transformer problem where each winding is loaded with a particular type of load and especially if the loads are interconnected electrically. Another suggested application is the use in analyzing the operation of certain types of relays involving combinations of coupled circuits.

Although the equivalent circuit is shown to be quite useful in the analysis of coupled circuits, this is not the only general application. In Part II of the paper there is developed a general equivalent which may be used to replace any part or all of a general network composed of any number of erratically placed links and having any number of points of entry. The value of such an equivalent is evident. The number of links required to represent the network is greatly reduced by using the equivalent.

In Mr. Summers' paper there is presented a new variant of old methods of mathematical manipulation. We have had matrix algebra, vector analysis, and operational mathematic, for a long time. Here we find suggested a method of adapting and combining matrix algebra and vector analysis so that the work may be carried on in the operational form. With the aid of a few rules it is possible to work out the solution symbolically as easily as for an ordinary algebraic problem. The work to obtain this solution is reduced through the simplification and decrease in the number of terms which must be carried along and it is much easier to retain throughout the work a clear conception of the physicial meaning of the equations. This latter concept is of great importance.

The method proposed has been illustrated by an application to an a-c. generator and this immediately suggests that it can be advantageously applied to problems of system operation such as stability, hunting, and synchronizing power. Design of synchronous apparatus can thus be studied in relation to system operation to the end that machines giving better system performance may be built.

This method of analysis should find a broad field of application in all problems involving the use of vector operators. Whenever the reaction coefficient along one axis is different than that along another, as in non-isotropic substances, vector operators can be brought into use. Such problems may occur not only in electrical work, but also in the field of acoustics, mechanics, thermics, and hydraulics.

The paper by Mr. Bewley makes public a mathematical solution of a problem which has been recognized for some time, so that there is now generally available a method by which the internal behavior of a transformer resulting from impact of an impulse may be analyzed. Although the analysis has been worked out on the basis of an applied rectangular wave, the solution for any applied wave shape is immediately available by the use of the well-known principle of super-position.

This paper will be of inestimable value in analyzing the insulation design of transformers. It makes it possible for anyone to evaluate the stress at various points of the windings and correlate the amount of insulation with the requirements. Further, it now becomes possible to insulate the secondary windings of a transformer not only from consideration of the overvoltages coming from the secondary side, but also of the overvoltages coming through the transformer from the primary side. In short, the correlation of the insulation of transformers with experience can now be placed upon a combined rational and empirical basis.

Although written primarily in consideration of transformer needs, this paper will be of distinct value in other fields, particularly for rotating apparatus. Proper insulation of rotating apparatus requires a knowledge of the voltage distribution throughout the windings due to impact of a traveling wave. The method of attack set forth by Mr. Bewley can be applied to this problem. With the modifications requisite for the different types of apparatus, distribution of the internal voltages can be analyzed and the insulation and protective requirements determined.

In Mr. Foster's paper there is given a method of enumerating all possible circuits having certain specified characteristics, defined as "nullity" and "rank." This information is of particular use to communication engineers in the study and design of maximum output networks for substation and repeater circuits.

R. D. Evans: One of the early problems in power system networks was in connection with the supply systems for a-c. railways. Here the layout was determined largely because of transformer loading and voltage regulation which depended upon unusual concentrations that would occur under practical operation. A particular feature of this network problem is the fact that the loads are taken off at varying points along a network branch. Quite elaborate methods were developed to take care of simultaneous loading at different points and to take into account changes in circuit condition arising from the disconnection of a line or of a transformer.

Following this came a very important step in network theory, namely, the introduction of the method of symmetrical components by Dr. Fortescue to make possible the solution of unbalanced power system networks. In the last ten years numerous contributions to the theory have been made which have facilitated its practical application to a large variety of current problems and it is now considered basic for relaying and other applications which are dependent upon unbalanced faults.

In connection with the stability problem it was necessary to develop new forms of circuit constants in order to facilitate the solution of general networks taking into account not only distributed constants but also their combination with lump constants. These constants were also used in extensions of the power circle diagram to take care of the general network. In addition methods for solving networks with synchronous machines in various branches were devised.

The most striking development in the last few years has been the introduction of the a-c. calculating board which has made it possible to solve many problems in a more complete manner and in less time than was possible heretofore. The calculating board has been used extensively in connection with problems of power system stability and relay operation. It also has been used for the purpose of simplifying complicated networks of hundreds of branches to the simplest possible equivalent general network for the smallest number of terminals permissible for the particular case under consideration.

J. W. Butler: The method which Mr. Starr has developed in Part III of his paper for setting up his several simplified equivalent circuits is an interesting and useful one. For example, the circuit shown in Fig. 8 of the paper may be recognized as the equivalent linkage to represent the single-line diagram of the three-winding transformer. For this application, the links a, b and c should be expressed in terms of the Z_{j-k} coefficients rather than the M_{jk} coefficients since the through impedances, that is, the impedances between pairs of windings, are those generally known. We may of course always change any function of M coefficients to one of the Z coefficients by using the general equations (**13b**), (**14b**) and (**15b**) of the paper.

Similarly, Fig. 9 may be interpreted to represent the four-winding transformer, Fig. 10 may be interpreted to represent the five-winding transformer, etc.

As pointed out in the paper, the equivalent circuit used for any particular application may be set up to fit the application in question in the simplest manner.

For example, let it be required to set up an equivalent single-line circuit to represent a four-winding transformer whose six impedances are:

$$Z_{1-3} = Z_{1-2} = 12.0 \text{ per cent}$$
$$Z_{1-4} = 12.53 \text{ per cent}$$
$$Z_{2-3} = 24.0 \text{ per cent}$$
$$Z_{2-3} = Z_{2-4} = 11.9 \text{ per cent}$$

Since there are four independent values of impedance, the equivalent circuit need have only four independent links. It must, of course, have four points of entry and must satisfy the above conditions. The circuit shown below is a simple one which satisfies the particular conditions of this case. For example, one may observe at once that $Z_{1-3} = Z_{1-2}$ and $Z_{3-4} = Z_{2-4}$.

The through impedances are from Fig. 1

$$Z_{1-2} = a + b + \tfrac{3}{4}d$$
$$Z_{1-4} = a + c + d$$
$$Z_{2-3} = 2b + d$$
$$Z_{2-4} = b + c + \tfrac{3}{4}d$$

Solving for a, b, c and d.

$$a = \tfrac{1}{2}(Z_{2-4} - Z_{1-4}) - X_{2-3} + 3/2Z_{1-2}$$
$$b = Z_{1-2} + Z_{2-4} - Z_{1-4} - \tfrac{1}{2}Z_{2-3}$$
$$c = \tfrac{1}{2}(Z_{1-2} - Z_{1-4}) - Z_{2-3} + 3/2Z_{2-4}$$
$$d = 2(Z_{1-4} + Z_{2-3} - Z_{1-2} - Z_{2-4})$$

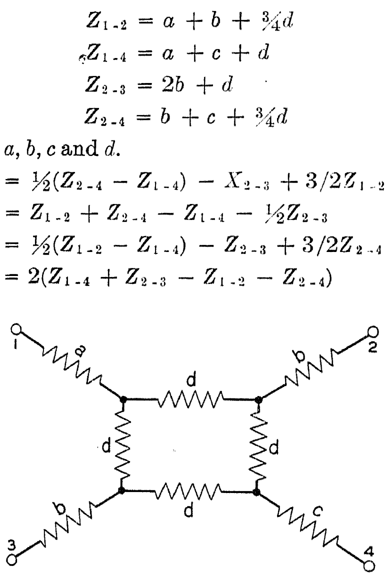

FIG. 1

Substituting the impedance values in the above equations

$$a = -\ 6.32 \text{ per cent}$$
$$b = -\ 0.63 \text{ per cent}$$
$$c = -\ 4.42 \text{ per cent}$$
$$d = 25.26 \text{ per cent}$$

Note that the negative links appear in the terminals and that these may be balanced out by the positive impedance of connected apparatus.

Edith Clarke: Equations corresponding to those given under (**1b**) of Appendix B of Mr. Starr's paper, have been used to advantage in the solutions of the positive sequence system during simultaneous faults when the faults were on different phases and on lines which were not symmetrical with respect to the rest of the system, such as parallel lines not bussed at either end or bussed at one end only. Mr. Starr has expressed the C coefficients of these equations in terms of the "through impedances" between the terminal points taken two at a time, the through impedance being the impedance between the two points under consideration with all other points ungrounded. Since in transmission systems it is usually a simple matter to calculate the through impedance between any two points of the system, this method of determining the numerical value of the C coefficients is a simple and direct one.

Another simplification which will be found of advantage where an a-c. calculating table is not available or when greater accuracy is required than can be secured by its use, is that of placing the self-impedance at the terminals and not in the mesh of the equivalent circuits. The labor of numerical calculations is materially reduced by this procedure since the mesh will be composed of ewer branches all of which will be reactive. For example, two arallel lines in the zero sequence diagram, not bussed at either nd, may be represented by equivalent circuits $I(d)$ or $I(e)$. Fig. e, which has purely reactive branches in the mesh with the self-mpedances at the terminals, is a much more wieldy circuit.

R. G. Lorraine: Mr. Starr has developed equivalent circuits or two, three, and four parallel transmission lines. Mutual ffects between lines may be represented by mutual reactors transformers) instead of by equivalent circuits as pointed out by Mr. Lewis. However, the number of mutuals frequently exceeds he number of mutual reactance units available on the a-c. table ınd the equivalent circuit must be used. In these circuits all ranches representing mutual coupling are pure reactances. For urposes of calculation this is, of course, desirable. However, ure inductive reactances cannot be represented directly on the ı-c. calculating table.

The reactance units on a calculating table, to be of reasonable ize, must necessarily have an appreciable amount of resistance ssociated with the reactance. For the accuracy usually required, his resistance cannot be neglected when it occurs in a unit used o represent a pure reactance. To specify arbitrarily this re-sistance in the mutual branches of the equivalent circuit, is one way of securing proper representation, but at the expense of ncreasing the number of branches of the equivalent circuit. A etter way is to change arbitrarily the impedance reference vec-tor, so that all the mutual impedance branches have a smaller mpedance angle than that of the reactance units. This rotation n addition to introducing the required resistance in the mutual ranches of the equivalent circuit, produces a different phase ıngle relation between the currents and voltages throughout the network. Therefore, the resultant currents must be rotated back to obtain the solution. The polar form of the expression for voltage drop through an impedance may be used to find the rela-tions between the impedance and current angles.

The rotation of impedances may produce a negative resistance in any capacity reactance branches which are present. Usually the effect of this resistance may be neglected as the capacity effects are generally small. If they are not the negative resistance may be represented by a potentiometer.

Thus, even though Mr. Starr's equivalent circuits contain pure reactance branches, means are available to represent them adequately on the a-c. calculating table.

W. A. Lewis: Several of the equivalent circuits presented by Mr. Starr are both new and interesting, and should prove a valuable addition to the work on the subject. In particular the equivalent circuits for the transformer in Fig. 1e and for 0 or more coupled lines in Figs. 9 to 14 should prove useful. The circuit of Fig. 8 for the common case of two lines is rather well known and has been used by others.*

It seems desirable to point out a slight error in the statements that the mutual impedance between lines is purely reactive. The mutual impedance is due not only to inductive coupling but also to the common use by all circuits of the earth and ground wires, if present. This resistance may often be as large a proportion of the mutual impedance as the combined resistances of earth circuit and conductor are of the zero-phase sequence self impedance, and so can seldom be neglected unless the accuracy required permits neglecting all resistances.

In the footnote on page 292 it is suggested that actual transformers might be used to represent mutual impedance. This has been done with considerable success with one design of a-c. calculating board, where the number of mutual impedances with any one circuit is small. The method, suggested by Mr. C. F. Wagner, is shown in Fig. 2. The transformer has a 1 to 1 ratio and is wound with a bifilar conductor so that the leakage reactance is negligible. It is seen from Fig. 2 that the mutual impedance, M, causes the same drop in both circuits, in one case directly and in the other case through the transformer. To provide the proper value of self-impedance, the mutual impedance must be subtracted from the self-impedance to obtain the values of the external branches, resulting occasionally in negative branches when several mutuals with the same circuit must be represented. The most generally satisfactory method appears to be a judicious combination of equivalent circuits and transformers.

One practical point mitigates against the general use of the rigorous transformer equivalent circuit. Reactance as set on the calculating board inevitably includes with it a certain small proportion of resistance which is usually negligible. When, however, the reactance is neutralized to a small value by means of capacitance, the resistance may no longer be a negligible proportion of the resultant effective reactance.

George W. Hampe: It will be found that the complete or lattice mesh of which Fig. 1d and Fig. 3b of Mr. Starr's paper are examples can be handled more readily if admittances instead of impedances are used to express its links. A simple process for finding the branches of the complete mesh equivalent to the general concealed network was given by Mr. Campbell.* It can be derived without the use of determinants as follows:

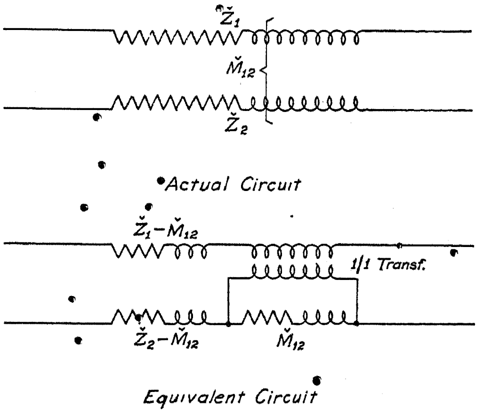

FIG. 2

Let Y_j be the admittance from a free entry point j to ground, with all other points of entry grounded. Let each entry point function in turn as the free point.

Let $Y_{j,k}$ be the admittance from the entry points j and k, which are joined but otherwise free, to ground, with all other points of entry grounded. Let each pair of points function in the same manner.

Consider both the concealed network and its equivalent complete mesh to be measured in the above ways. On the equivalent, of which Fig. 3b is an example, some links are thus placed in parallel and the resultant admittance can be equated to a measured quantity from the concealed network.

Thus if $Y_{\overline{jk}}$ is the admittance corresponding to $Z_{\overline{jk}}$ (Appendix E), a set of equations corresponding to each type of measurement can be written. For the first set, the following are instances:

$$Y_{\overline{ja}} + Y_{\overline{jb}} + \ldots + Y_{\overline{ji}} + Y_{\overline{jk}} + \ldots + Y_{\overline{jn}} = Y_j \qquad (1)$$

$$Y_{\overline{ka}} + Y_{\overline{kb}} + \ldots + Y_{\overline{kj}} + Y_{\overline{kl}} + \ldots + Y_{\overline{kn}} = Y_k \qquad (2)$$

From the second set:

$$Y_{\overline{ja}} + Y_{\overline{jb}} + \ldots Y_{\overline{ji}} + Y_{\overline{jl}} + \ldots Y_{\overline{jn}} + Y_{\overline{ka}} + Y_{\overline{kb}} + \ldots Y_{\overline{ki}} + Y_{\overline{kl}} + \ldots Y_{\overline{kn}} = Y_{j,k} \qquad (3)$$

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*Theory and Application of Relay Systems," by I. T. Monseth and P. H. Robinson, *Elec. Jl.*, July 1930, p. 411.

Fundamental Basis for Distance Relaying, by W. A. Lewis and L. S. Tippett, A.I.E.E. paper **No. 31-61.**

Cisoidal Oscillations, by G. A. Campbell, A.I.E.E. TRANS., Vol. 30, 1911, p. 873.

Substituting from (1) and (2) into (3) yields:

$$Y_j + Y_k - 2Y_{\overline{jk}} \text{ (or } 2Y_{\overline{kj}}) = Y_{j,k} \qquad (4)$$

or

$$Y_{\overline{jk}} = (Y_j + Y_k - Y_{j,k}) \div 2 \qquad (5)$$

For determination of the complete mesh by direct measurement (for instance on the calculating table) this solution is sufficient and leads to easy manipulation. Many other forms of equivalents from which mutual couplings are excluded can be found from the complete mesh by delta-star and star-mesh conversions. For the four-point case there are six in the basic group of which Figs. 1d and e are members. Star-mesh conversions on a general network can be used to show that the complete mesh equivalent of concealed positive resistance will never contain a negative link.

For determination of equivalents analytically, determinants may be useful for the expression of driving point and transfer admittances. The type form equation (5) can be used as the key to the situation. Thus for the n winding transformer it is merely necessary to ascertain which driving point and transfer admittances apply for the measurement conditions specified. The general equations expressing the links of the complete mesh then follow. Parallel lines are coupled in the same manner as the windings of a transformer.

The writer would also like to ask several questions concerning the paper. Why is the term "characteristic impedance" used in defining some of the symbols in Appendix C, but not others? In the introduction it was said that, in general, equivalence held only for a single frequency.

In the transformer equivalent the value of some links is the negative of others. Mr. W. C. Hahn (*General Elec. Rev.*, V. 34, Aug., 1931, pp. 483-489) explained a way of setting up negative resistance on a d-c. calculating board. I wonder if it has been found practical in this connection. Not all coupled lines can be traced to a common bus at one end.

The foregoing points are mentioned to supplement, not to criticize, Mr. Starr's work. Mr. Starr and I, working independently on similar problems, found similar solutions.

Mr. Campbell's remarkable paper had remained practically unused, at least in the power engineering field, for over twenty years. It is to be hoped that since it has been rediscovered the application of other parts of it will lead to the solution of modern problems.

H. L. Rorden: Specific calculations based on Mr. Bewley's derivations illustrate several points of interest. Table I lists the essential terms for three space harmonics as calculated for the given constants.

These calculations show:

1. That of the two time harmonics for each space harmonic, only one of them is important to the primary.
2. The two-winding theory gives practically the same amplitudes of oscillation as the single-winding theory.
3. For practical purposes $A_s' = X_s$.
4. The harmonic frequencies are substantially the same for the single and for the two-winding theory.
5. Both time harmonics are of practically equal importance in the secondary. At the initial instant these time harmonics are of opposite sign for grounded neutral, but due to the great difference in their frequencies they are quite likely to become additive numerically.
6. The first three harmonics are practically of the same importance in the secondary.

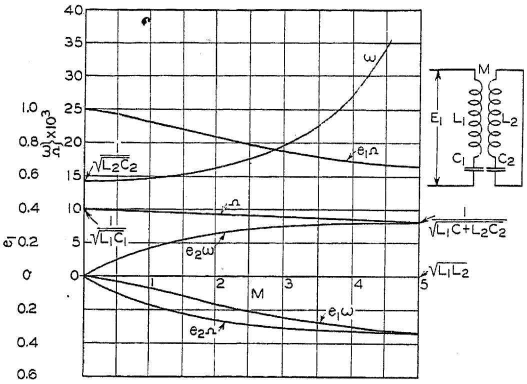


FIG. 3

7. The axes of oscillation in the secondary are quite different, depending upon the terminal connections. For grounded primary neutral these oscillations are about the axes,

$$e_2 = \left[\frac{x}{2} - \frac{1}{12}\right] E$$

for isolated open-circuited secondary and

$$e_2 = 0$$

for short-circuited, grounded secondary.

8. Calculations made with the two-winding theory are extremely sensitive to error, and the computations must be carried out with considerable care to get accurate results. A small error in the calculation of frequencies may cause an enormous error in the amplitudes.

TABLE I—CALCULATED TRANSIENTS

Basic Constants

$K_1 = 1 \times 10^{-11}$ $C_1 = 1 \times 10^{-9}$ $L_1 = 10$

$K_2 = 1 \times 10^{-11}$ $C_2 = 2 \times 10^{-9}$ $L_2 = 2.5$

$C_3 = 1 \times 10^{-9}$ $M = 2.5$

	Short-circuit and grounded secondary			Open-circuited and isolated secondary		
s	1	2	3	1	2	3
ω_s	132,000	500,000	1,140,000	132,000	500,000	1,140,000
Ω_s	74,500	276,000	553,000	74,500	276,000	553,000
r_s	5.65	10.08	81.3	5.65	10.08	81.3
r_s'	0.216	0.308	0.88	0.216	0.308	0.88
X_s	−0.592	0.252	−0.1375	−0.563	0.234	−0.1126
U_s	0.0114	−0.0175	0.0181	−0.2032	0.123	−0.0995
A_s	0.0254	−0.00974	0.00173	−0.015	0.0052	
A_s'	−0.617	0.259	−0.1394	−0.547	0.229	−0.1125
A_{2s}	0.144	−0.0974	0.141	−0.0845	0.0524	
A_{2s}'	−0.133	0.0798	−0.123	−0.118	0.0706	−0.099
$^*A_s$	−0.601	0.257	−0.138	−0.601	0.257	−0.138
$^*2\pi f$	74,700	278,000	563,000	74,700	278,000	563,000

*Based on single winding theory.

It is interesting, as an analogy, to compare the general distributed circuit with the lumped constant circuit shown in the figure, which exhibits similar characteristics. This consists of two *L-C* circuits coupled by a mutual inductance M. The equations for the voltages across the capacitors are:

$$e_1 = \frac{E_1}{(L_1L_2 - M^2)C_1C_2}\left[\frac{1}{\omega\,\Omega^2} + \left[\frac{1 - \omega^2 L_2C_2}{\omega^2(\omega^2 - \Omega^2)}\cos\omega t - \frac{1 - \Omega^2 L_2C_2}{\Omega^2(\omega^2 - \Omega^2)}\cos\Omega t\right]\right]$$

$$e_2 = \frac{E_1M}{(L_1L_2 - M^2)\,C_2\,(\omega^2 - \Omega^2)}[\cos\omega t - \cos\Omega t]$$

$$\text{where}\quad \left.\begin{matrix}\omega^2\\ \Omega^2\end{matrix}\right\} = \frac{L_1C_1 + L_2C_2 \pm \sqrt{(L_1C_1 - L_2C_2)^2 + 4\,M^2C_1C_2}}{2(L_1L_2 - M^2)\,C_1C_2}$$

The comparison illustrates several points of similarity:

A. The constant term in e_1 of the lumped circuit corresponds to the axis of oscillation of the distributed circuit.

B. The lumped circuit has two distinct frequencies of oscillation, corresponding to two distinct fundamental frequencies in the distributed circuit.

C. The two frequencies are practically of equal importance in the secondary of either the lumped or the distributed circuit.

D. One of the frequencies (that nearest to the natural frequency of the primary alone) is the predominating component in the primary oscillations.

E. As the mutual inductance is decreased in either circuit, the two frequencies approach the natural frequencies of the primary and secondary alone. As the mutual inductance is increased, one of the frequencies increases towards infinity and the other frequency approaches the compromise

$$1/2\pi\sqrt{L_1C_1 + L_2C_2}.$$

These conclusions are illustrated in Fig. 3.

Gabriel Kron: For years the customary procedure has been to analyze the transient and steady-state behavior of rotating machinery in terms of currents and voltages appearing at the terminals. Lately a new point of view is applied to the analysis of salient-pole synchronous machines; it is tacitly assumed that at any instant the current-density wave and the actual flux-density wave are sinusoidally distributed in *space* in both field and armature windings, hence they are replaced by two-dimensional space vectors and the behavior of the machine is expressed in terms of these space vectors instead of the terminal currents and voltages. Park in his classical paper "Two-Reaction Theory of Synchronous Machines" assumes two mutually perpendicular, stationary coordinates along the field poles and along the interpolar space and analyzes the *projections* of the space vectors along these coordinates. As a result, for each layer of winding (the field winding being one layer, the armature winding another layer) he sets up two *scalar* equations, altogether four scalar equations, as given in the first column of this paper. Since practically all types of asymmetrical rotating machines, having brushes, or sliprings in any combination with any number of layers of windings on stator and rotor, can be assumed to have sinusoidal current-density and flux-density waves in *space* under any transient or steady-state, balanced or unbalanced conditions, it follows that any rotating machine requires for its analysis *twice* as many *scalar* equations as there are layers of windings on it (neglecting, for the time being, space-harmonics).

The most logical step for simplification lies in the introduction of *two-dimensional vectors* for currents, fluxes, voltages and of *dyadic operators* for impedances and admittances, each operator consisting of the sum of 2 × 2 = 4 dyads, as has been done by Summers for synchronous machines. With the use of these vectors and dyadics (or tensors of rank two) the number of equations is halved and one has to manipulate only as many *tensor* equations as there are layers of windings. In the example used by Summers, the salient-pole alternator, the number of the necessary tensor equations is two [equations (5) and (6)]; in a repulsion motor with an additional squirrel-cage the number of tensor equations is three, etc.

My studies of the vector currents and vector fluxes existing in different types of rotating machines led me first to a graphical treatment of these vectors. When a limit of their practical use was reached the geometrical relations existing between the different vectors were put in the form of scalar equations. Then quite ignorant of the studies of Summers I also began to use two-dimensional vectors and dyadics with four dyads as being the most obvious tools for a simplified treatment. Soon however, I was led to a more simplified procedure which seemed to be a necessary and *inevitable* step forward.

I assumed that the two stationary unit coordinates of the armature (rotor) $\mathbf{d}_r$ and $\mathbf{q}_r$ are mutually perpendicular to the two unit coordinates of the field (stator) $\mathbf{d}_s$ and $\mathbf{q}_s$ and instead of having a two-dimensional current vector in the field winding and another two-dimensional current vector in the armature winding, I assumed that there is only *one four-dimensional* current in the machine with different components along the four normal coordinates $\mathbf{d}_s$, $\mathbf{q}_s$, $\mathbf{d}_r$ and $\mathbf{q}_r$. Similarly I assumed that the terminal voltage consists of *one* four-dimensional voltage vector

$$\mathbf{e} = e_{ds}\,\mathbf{d} + e_{qs}\,\mathbf{q} + e_{dr}\,\mathbf{d} + e_{qr}\,\mathbf{q}\ .\ .\ .$$

where each scalar coefficient can be any function of time, d-c., a-c., Heaviside unit function, etc., (also zero) or any combination of these. Corresponding to these assumptions I introduced the following *impedance* operator as a dyadic consisting of 4 × 4 = 16 dyads which transforms a four-dimensional current vector into a four-dimensional voltage vector in any machine with two layers of windings

$$\begin{aligned}&(r_{ds} + L_{ds}p)\,\mathbf{q}_s\,\mathbf{q}_s + M_d\,\mathbf{q}_s\,\mathbf{q}_r - M_dp\theta\,\mathbf{q}_s\,\mathbf{d}_r + O\,\mathbf{q}_s\,\mathbf{d}_s\\ &+ M_dp\,\mathbf{q}_r\,\mathbf{q}_s + (r_{dr} + L_{dr}p)\,\mathbf{q}_r\,\mathbf{q}_r - L_{dr}p\theta\,\mathbf{q}_r\,\mathbf{d}_r + O\,\mathbf{q}_r\,\mathbf{d}_s\\ &+ O\,\mathbf{d}_r\,\mathbf{q}_s + L_{qr}p\theta\,\mathbf{d}_r\,\mathbf{q}_r + (r_{qr} + L_{qr}p)\,\mathbf{d}_r\,\mathbf{d}_r + M_qp\,\mathbf{d}_r\,\mathbf{d}_s\\ &+ O\,\mathbf{d}_s\,\mathbf{q}_s + M_qp\theta\,\mathbf{d}_s\,\mathbf{q}_r + M_qp\,\mathbf{d}_s\,\mathbf{d}_r + (r_{qs} + L_{qs}p)\,\mathbf{d}_s\,\mathbf{d}_s\end{aligned}$$

In setting up this impedance operator the "per-unit" concepts have been abandoned, self and mutual inductances were used and the *stator* was assumed to have salient poles, in order to make the tensor available for the transient analysis of asymmetrical commutator machines also. Rotating machines with *three* layers of windings have *six-dimensional* currents and terminal voltages and their impedance operator contains 6 × 6 = 36 dyads, of which the above 16 dyads form the inner square. With greater number of layers of windings proportional relations hold true. This operator, the "magnetic-field tensor" contains all the design constants, is an *invariant*, its value is independent of the presence of brushes, or sliprings and is independent of the terminal conditions.

The presence of brushes, sliprings, taps, and the various possible connections between them is represented by a *transformation of coordinates*. The transformation is made with the aid of another tensor, the "connection-diagram tensor" showing the location of brushes, sliprings, the various connections, the ratio of turns, etc., this tensor being a mathematical representation of the usual connection diagrams of different types of machines. While all types of rotating machines have the same "magnetic-field tensor" containing all the design constants, each has a different "connection-diagram tensor."

With these new assumptions the transient and steady state behavior of all types of rotating machines with any number of layers of windings, under balanced or unbalanced conditions can be expressed (assuming speed as a parameter) in terms of *one* first degree tensor equation

$$\mathbf{i}\ .\ \mathbf{Z} = \mathbf{e} \tag{2}$$

from which the unknown current $\mathbf{i}$ for a given terminal voltage $\mathbf{e}$ is found

$$\mathbf{i} = \mathbf{e}\ .\ \mathbf{Z}^{-1} \tag{3}$$

giving the instantaneous values of currents along each coordinate axis by the routine procedure of calculating the *inverse* of the impedance tensor.

The "magnetic-field tensor" with any number of dyads can be expressed as the sum of three dyadics

$$\mathbf{Z} = \mathbf{R} + \mathbf{L}p + \mathbf{R}_v p\theta \qquad (4)$$

where **L** represents all dyads containing p and $\mathbf{R}_v$ those containing $p\theta$ showing that any rotating machine consists of a constant (tensor-) resistance and a constant (tensor-) inductance in series with a variable (tensor-) resistance. Also:

$$\mathbf{e} \,.\, \mathbf{i} = \text{power input} \qquad (5)$$

$$\mathbf{i} \,.\, \mathbf{R} \,.\, \mathbf{i} = \text{copper losses inside the machine} \qquad (6)$$

$$\mathbf{i} \,.\, \mathbf{L} \,.\, \mathbf{p} \,.\, \mathbf{i} = \text{rate of increase of stored magnetic energy} \qquad (7)$$

$$\mathbf{i}\, \mathbf{R}_v\, \mathbf{p}\theta \,.\, \mathbf{i} = \text{mechanical output} \qquad (8)$$

Hence

$$\text{Torque} = \mathbf{i} \,.\, \mathbf{R}_v \,.\, \mathbf{i} \qquad (9)$$

Dyadic operators have been set up not only for rotating machines, but also for polyphase networks, vibrating systems, vacuum tubes, etc. One of the most valuable properties of this analysis is that the resultant impedance of any number of interconnected rotating machines and stationary networks (d-c., induction-, synchronous-, commutator-machines, polyphase-networks, vacuum tubes, etc.) is the *sum* of the impedance of each machine of the interconnected system, thereby giving an easier procedure for the transient (and steady state) analysis of systems for speed-control, power-factor control, welding, etc.

In order to have the instantaneous speed of the rotor also as a variable, the concept of current and voltage vectors had to be further extended. I assumed that **e** represents not only the multi-dimensional impressed voltage but also the instantaneous *torque* acting along an additional dimension **t** in the direction of the axis of the rotor; similarly **i** represents not only the multi-dimensional current vector, but in addition the instantaneous angular velocity acting along **t** also. If now **R** includes in addition the frictional resistance dyad **ptt**, **L** includes the moment of inertia of the rotor T**tt** and $\mathbf{L}_v = -\mathbf{R}_v \times \mathbf{t}$, the transient behavior of any machine or combination of machines is described *during acceleration* by the following single, second degree tensor equation

$$\mathbf{i} \,.\, \mathbf{L}_v \times \mathbf{i} + \mathbf{i} \,.\, \mathbf{L}\, p + \mathbf{i} \,.\, \mathbf{R} = \mathbf{e} \qquad (10)$$

By taking infinitesimal changes of vectors in this equation, all problems of hunting and vibration of different types of machines, system stability, starting, etc., can be studied systematically.

It can be proved that this last equation also is a tensor expression of the Maxwell-Lagrangian Equation of Motion for a system containing kinetic (mechanical) and electrokinetic (magnetic) energy.

F. M. Starr: As pointed out by Mr. Lewis, it is stated in the paper that the mutual impedance between parallel lines is purely reactive. I do not agree with Mr. Lewis that there is the slightest inaccuracy in this statement. I do not see any reason why the connotation of the phrase "mutual impedance between the lines" should be construed as including "mutual resistance between ground return circuits."

However, Mr. Lewis has brought up an interesting question, and that is: how may mutual resistances between ground return circuits be represented when the resistance is not negligible? One exact method of doing this would be to represent the ground circuit as an equivalent mesh whose links would be evaluated in terms of resistance measurements made between the various grounding points (with all lines disconnected) in accordance with the method outlined in Part II of the paper. The zero-phase diagram of the system shown in Fig. 5, then, would consist of the equivalent layout shown in Figs. 6 or 7 with the various grounding points connected, not to a common ground, but to the respective terminals of an equivalent resistance mesh representing the ground. Of course, in order to represent the ground circuit by such an equivalent, or in any manner at all, it is necessary that the resistance coefficients be known or possible of measurement, and this may be rather difficult or at least impracticable.

Mr. Lewis further pointed out that the small amount of resistance associated with reactor units on the calculating board might be objectionable when applied to pure reactance links of an equivalent circuit. Although such a condition would rarely occur, if it were important in some particular case, it could be completely corrected by rotating all impedances through the proper angle before setting up on the board, as pointed out by Mr. Lorraine.

Mr. Hampe, in his equations (4) and (5) has given an interesting equivalent for the general network which is somewhat different than my own. I do not agree with Mr. Hampe in his statement that the general network equivalent can be handled more easily if the links are expressed as admittances rather than impedances. It really makes very little difference whether driving point impedances or driving point admittances are used since they are reciprocals. (See equation (**6b**) in the paper.) Mr. Hampe implies that the links of this equivalent circuit may be calculated without the use of determinants. I think great care should be taken when speaking of measured or calculated quantities. The Y coefficients of Mr. Hampe's equation (**5**) are entirely similar to my B or D coefficients in equation (**9a**) and (**6b**) respectively. In either case these coefficients may be measured directly, but if measurements are not possible, they must be calculated by means of determinants. In this connection, it should be pointed out that to evaluate a link in Mr. Hampe's equivalent requires three impedance (or admittance) measurements or calculations as the case may be, whereas the circuit derived in Appendix B requires only one measurement or calculation. (See equation (**6b**).)

The answers to Mr. Hampe's questions are given briefly in the order in which they were asked:

1. "Characteristic impedance" or "characteristic admittance" is used in defining symbols used in Appendixes A and B (not Appendix C) in accordance with standard nomenclature for determinantal coefficients.
2. Mr. Hahn's method of representing negative resistance is always applicable on a d-c. board, although this method would be unwieldy and impracticable if very many links required it. If an a-c. board is used, there is no reason why negative resistances need ever appear. Negative resistance can always be avoided by applying the reasoning used in Part I of the paper in deriving the circuit of Fig. 1e. That is, resistance may always be lumped in the terminals of a circuit leaving any negative links purely reactive.

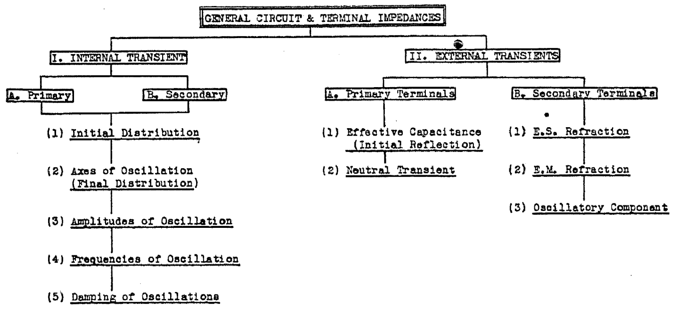
Mr. Butler's discussion shows an interesting application of the method developed in Part III of the paper for setting up simple equivalent circuits. This type of application should prove particularly valuable in mapping out the single-line positive or negative phase diagrams of a system involving 4- or 5-winding transformer banks.

L. V. Bewley: As pointed out by Mr. Rorden, the equations given in the paper are not only complicated, but extremely sensitive to small errors. For these reasons they are not well adapted to numerical calculation. The real practical utility of the analysis is that it suffices to establish rigorously the range of application and limitations of a number of simplified methods pertaining to certain aspects of transformer high frequency transients; such as the primary internal oscillations; terminal and neutral transients; transfer of waves to the secondary external circuit; etc. In this discussion I have compiled and classified the approximate equivalent circuits of the transformer, based upon the solution to the general circuit. It must not be construed, however, that these circuits are new. They have all been employed in the past by various individuals interested in some particular aspect of the problems. In chronological order, the solution to the general circuit was the last one obtained.

The equivalent circuits discussed here are approximate, as contrasted to the mathematically exact equivalent circuits described in Mr. Starr's paper. It often happens in engineering work that approximate equivalent circuits may be substituted which are sufficiently accurate for certain purposes and under specific conditions. Thus a wide variety of circuits may be employed to describe different aspects of a given circuit or piece of apparatus, depending upon the characteristic which it is desired to investigate.

In the accompanying chart a classification of transformer transients due to traveling waves, or high frequency, is given in the form of a family tree; and in Fig. 4 there are tabulated, in the order of their accuracy, various approximate equivalent circuits which suffice, in specific cases and ranges, for engineering calculations. Theoretically, a solution to the general circuit with arbitrary terminal impedances would yield complete information on both the internal and external transients in both windings. Unfortunately, however, the mathematical difficulties are insurmountable, and even could the general solution be obtained it would be too complicated to prove of much utility in practicable engineering calculations. But solutions have been obtained for zero and infinite values of the terminal impedances (short-circuited or open-circuited terminals, either grounded or isolated) and it has been shown that:

Classification of the High Frequency Transients of Transformers


GENERAL CIRCUIT & TERMINAL IMPEDANCES
I. INTERNAL TRANSIENT
A. Primary
B. Secondary
(1) Initial Distribution
(2) Axes of Oscillation (Final Distribution)
(3) Amplitudes of Oscillation
(4) Frequencies of Oscillation
(5) Damping of Oscillations
II. EXTERNAL TRANSIENTS
A. Primary Terminals
(1) Effective Capacitance (Initial Reflection)
(2) Neutral Transient
B. Secondary Terminals
(1) E.S. Refraction
(2) E.M. Refraction
(3) Oscillatory Component

1. The initial distribution may be calculated for arbitrary terminal impedances.

2. The transformer acts as a capacitance network at the instant of impact of steep waves, or when abrupt changes occur in the terminal voltages. The effective capacitance of this network has been derived.

3. Voltages transmitted to the primary neutral and to the secondary terminal impedances through the capacitance network may be calculated. These electrostatic components are exponential impulses of very short duration when the terminals are connected to their surge impedances; but if the terminals are isolated, the electrostatic potentials last as long as the applied voltage.

4. The axes about which the oscillations of the primary and secondary take place are determined by the inductances of the winding (neglecting the losses), and the equations for these axes of oscillation are identical with the simple equations for two mutually coupled windings with arbitrarily terminal impedances. These equations thus suffice for calculating the purely electromagnetic component of voltage and current at the primary neutral and the secondary terminals. This component is very slow and very long compared with the electrostatic component, and is limited in magnitude to a fraction of the primary applied voltage equal to the turn ratio. It is the dominate component of the external transient when the terminals are connected to surge impedances not exceeding a few thousand ohms; but of course it vanishes if the primary neutral is isolated.

5. The internal oscillations consist of an infinite series of space harmonics, and each space harmonic has two components with different frequencies; that is, there are two *time* harmonics for each *space* harmonic. Only one of these two sets of time harmonics is of importance in the primary oscillations, but both sets are of practically equal importance in the secondary oscillations. Moreover, the *amplitudes* of the primary oscillations are almost identical with those given by the single-winding theory of transformer oscillations; and the *ratio* of harmonic frequencies are in the same order, although the *absolute* values of the frequencies may be somewhat different. In any event, if the inductances used in the single-winding theory are empirically changed to correspond to different connections of the secondary winding, the single-winding theory yields very good quantitative as well as qualitative agreement with the double-winding theory and with test. It may also be shown that for the usual values of surge impedances, that the oscillatory components transmitted to the terminal impedances are insignificant compared with the pure electromagnetic component discussed under (4) above. For obviously, the oscillatory components of current can not exceed in value those which flow when the terminals are short-circuited or grounded (values which can be calculated) and therefore the voltages across any terminal impedance can not be as great as that given by the product of that impedance and the current which would flow if that impedance were reduced to zero.

From the above facts, which have been derived from consideration of the general circuit with arbitrary terminal impedances, a table of equivalent circuits for the transformer, under traveling wave conditions and for different purposes, may be compiled as shown in Fig. 4. Starting with the two-winding circuit P, which is necessary for calculating the internal transient of the secondary it follows by (5) above that the single-winding circuit N may be used, if it is only required to calculate the primary internal transient. Now it may be desirable to include the effect of damping (for example, in investigating the possibility of cumulative oscillations caused by applied waves of resonate frequency), in which case circuit M is appropriate. Circuit O, which neglects the mutual inductance between parts of the same winding, has been used, but fails to give a correct description of the phenomena of transformer oscillations, because its frequency distribution is erroneous.

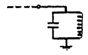
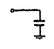
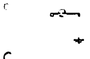
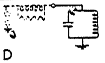
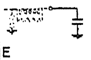
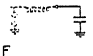
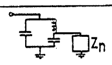
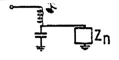
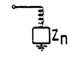
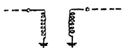
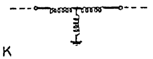
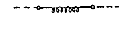
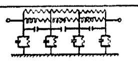
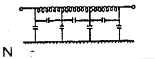
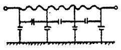
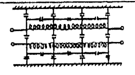
PURPOSE OF CIRCUIT	APPROXIMATIONS IN THE ORDER OF THEIR ACCURACY		
CALCULATING TRANSFORMER TERMINAL VOLTAGE AND REFLECTED WAVES	 A	 B	 C
INVESTIGATING EFFECT OF CHOKE COIL OR EXTERNAL REACTANCE AT TERMINALS	 D	 E	 F
CALCULATION OF THE NEUTRAL TRANSIENT. NEUTRAL IMPEDANCE=Z_n	 G	 H	 I
PASSAGE OF WAVES THROUGH THE TRANSFORMER TO THE SECONDARY CIRCUIT	 J	 K	 L
INTERNAL TRANSIENTS OF A SINGLE WINDING	 M	 N	 O
INTERNAL TRANSIENTS OF MUTUALLY COUPLED WINDINGS	 P	Q	R

Fig. 4—Equivalent Circuits of Transformers to Traveling Waves and High Frequency

For calculating the external transient of the secondary, when connected to normal impedances and when the primary neutral is grounded, it follows by (4) that circuit J may be employed, or its simplifications K and L.

When the purpose of the calculation is to determine the neutral transient, or the transient of a traveling wave at the transformer terminal, the appropriate circuit is G; for at the first instant it acts as a capacitance in conformity with (2); if the neutral is isolated ($Z_n = \infty$) the neutral voltage oscillates at the fundamental natural period of the isolated-neutral transformer; and if the neutral is grounded ($Z_n = O$) the winding becomes an inductance to ground and eventually a short circuit. Now the effect of the terminal capacitance on the neutral current is quite small, so the circuit may be simplified to H, and here again the effect of the fictitious neutral capacitance is dominated by the neutral impedance Z_n, so that circuit I is sufficient for practical calculations.

If, however, the problem is to investigate the effect of an external reactance, the relative importance of the elements of circuit G is completely reversed; for the external reactance (a choke coil, for example) enters into a very high frequency oscillation with the effective capacitance of the transformer, (2), and at this high frequency the inductive winding of the transformer is practically an open circuit. Thus D or E apply.

For calculating the transformer terminal voltage and the reflected waves, circuit G is still the best approximation. If the neutral is grounded, it simplifies to A. For very short waves, or for isolated neutral, circuit B is sufficient. For waves of moderate length, and when slight depression of the wave front by the effective capacitance is not to be considered, the transformer may be regarded as an open-circuit; or as a surge impedance of several thousand ohms if the very small negative reflection is to be included.

R. H. Park: In his discussion of Mr. Summers' paper, Mr. Gabriel Kron suggests a method of analysis which appears to have interesting possibilities. It is to be hoped that Mr. Kron will develop his analysis more completely in the form of a paper for presentation before the A.I.E.E.

Exception must be taken to one point in this discussion. This is the statement that the Summers' vector method of analysis requires "as many tensor equations as there are layers of windings." From this statement, it would be concluded that the form of the equations given by Summer would not be adequate to include synchronous machines equipped with field collars and amortisseurs. However, this is not found to be the case. The only change which is found necessary, in order to consider the effect of field collars or amortisseurs is a modification of the scalar components of the characteristic vector operators.

Auxiliary Drive for Steam Power Stations[1]

A Review of the Characteristics and Economics of Steam and Electric Drive for Steam Power Station Auxiliaries

BY F. H. HOLLISTER[2]
Member, A.I.E.E.

I. *Steam Driven Auxiliaries for Power Plants,* by **W. Poole Dryer**
II. *Electrically Driven Auxiliaries for Steam Power Stations,* by **Lyle W. Smith.**

DURING the past few years there have been important changes in power station and power system design that have reduced the operating costs and now power station designers are giving more attention to reducing the investment costs. A number of large stations has been built recently in which practically all of the essential as well as the non-essential auxiliaries are electrically driven. Other recent stations have used steam-driven auxiliaries quite extensively. The committee felt that it would be of interest to review the characteristics and merits of steam and electric auxiliary drive in light of present conditions and past operating experience, and has therefore sponsored the papers by Messrs. Dryer and Smith.

The central power station industry as a whole enjoys a well deserved reputation for continuity of service. The public has become accustomed to expect continuous service and the large investment charges of the power station companies and their industrial customers demand it. One of the most important factors in continuity of power service is reliability of the auxiliary equipment required at the power stations. Reliability, therefore, deserves the attention that has been given it in these papers.

The value of adjustable-speed auxiliaries makes an interesting study in the application of auxiliary drive. In the past, the station designers have used adjustable-speed drives for fans, pulverizer mills, boiler feed pumps and circulating pumps quite extensively. Recently there have been developments in the manufacture of auxiliary equipment and station design that have made adjustable speeds less necessary. The one exception to this trend is boiler feed pumps for high-pressure boilers. The paper on steam-driven auxiliaries brings out very clearly the advantages of the steam turbine as a simple and efficient means for obtaining adjustable-speed drive.

It has been shown in the papers that the type of drive best suited for the essential auxiliaries such as fans, boiler feed pumps, and circulating pumps is closely related to and dependent upon the general design of the station. The general station design in turn is influenced by the relation of the station to the balance of the power system with respect to interconnections, size, and location. For instance, it is quite apparent that a power station that forms the chief source of power supply for an area and that does not have reliable interconnections with other power systems warrants more reserve equipment and must have a greater number of steam-driven auxiliaries for starting up the station than a station forming part of a system of interconnected stations.

Furthermore, it is apparent that if a new power station forms part of a large interconnected group of power stations, it may be designed and built to operate at a better load factor and thereby reduce the need for adjustable-speed auxiliary drives.

The location of the station will also affect the selection of type of auxiliary drive by reason of the influence of location on the kind of fuel and fuel burning equipment used and on the space limitations. Gas and oil fired, also pulverized fuel fired boilers are somewhat more flexible than stoker fired boilers and can therefore be taken out of service more readily with changes in load. This favors the use of constant speed electric drive. If, on the other hand, the bin system of pulverized fuel supply is used, exhaust steam is usually required to dry out the coal, thus justifying the use of some steam-driven auxiliaries. Furthermore, if ground and building costs are high and adjustable speed is required, the use of steam-driven auxiliaries may result in an appreciable saving.

It is interesting to note that the total auxiliary power used by a modern station is a relatively small part of the gross generated power, being approximately from 4½ to 6 per cent; hence a radical power saving can not be expected in the selection of auxiliary drive. Likewise, the investment cost of the auxiliary power including drive and control equipments is a relatively small part of the total station cost, being in the case cited for electrically driven auxiliaries approximately 7 per cent of the total station cost. It would appear that the station designer is more or less free to use either type of drive without seriously affecting the cost or the efficiency of the power station as a whole. Electric drive

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[1] Foreword prepared under the Auspices of the Committee on Power Generation.

[2] Sargent & Lundy, Inc., Chicago, Illinois.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

for essential auxiliaries is favored from the standpoint of convenience and cleanliness in operation, also because it is more readily adapted to automatic and remote control particularly in the boiler room.

Maintenance costs would be of interest for a comparison between steam and electric drive; however, representative figures are difficult to obtain chiefly because sufficient time has not elapsed since the modern stations were placed in operation. In addition, the advancement in station design has been so rapid in the last few years it is difficult to find stations of sufficient similarity to make a fair comparison. The subject of these papers is inherently one of generalities because of the many factors that may enter in the selection of auxiliary drive. The discussion of the subject matter should, however, be of interest in bringing out points for consideration in determining the type of auxiliary drive best suited for a specific problem in power station design.

Discussion

For discussion of this paper see page 341.

Steam-Driven Auxiliaries for Power Plants

BY W. POOLE DRYER[1]
Associate, A.I.E.E.

REQUIREMENTS

THE degree of reliability required of a generating station is determined by the kind of community or business served by the station and the relation of the particular station to the other sources of supply for the system. The activities of the downtown section of a large metropolis demand service reliable to a degree not expected in a rural district. So also in the industries, some metallurgical and chemical processes need the continuity of their power supply guaranteed to an extent not essential in other industries. Stations in those strategic situations, metropolitan and industrial, must supply service reliable to the highest degree. Once the city station loses its network load, restarting is extremely difficult; once the industrial process stops, the product, and maybe the machinery, is spoiled and to restart the process takes hours. These are the paramount considerations determining the methods to be used for driving essential auxiliaries in power plants. Secondary considerations are simplicity, initial cost, maintenance expense and economy of power.

RELIABILITY OF AUXILIARY SERVICE

Inasmuch as a station's ability to produce an uninterrupted supply of power depends, among other things, on the ability of the essential auxiliaries to function continuously, the arrangements made for guaranteeing the operation of these auxiliaries should be appropriate to the degree of reliability of service expected from the station. These arrangements involve furnishing a reliable supply of energy, as well as turbines, motors, and accessory equipment which are free from physical defects and which possess suitable operating characteristics.

RELIABILITY OF AUXILIARY POWER SUPPLY

The essential auxiliary driving units must be supplied with energy free from disturbances which would interrupt their operation. To the extent that the source of supply is exposed to those disturbances, arrangements must be made to limit the effect of any single disturbance to as few auxiliaries as possible and to provide a duplicate source immediately available when the primary source fails. Minor disturbances of the energy, such as slight reductions in steam pressure or voltage, are to be expected, and should be provided for in the design of the drives. To what extent is the provision of a duplicate source of supply advisable in stations, when the essential auxiliaries are steam driven, and to what extent when motor driven?

RELIABILITY OF AUXILIARY STEAM POWER

Disturbances threatening interruption of the auxiliary service may originate within the station or without. In properly designed and operated stations, with turbine driven auxiliary equipment, no external circumstances could disturb the characteristics of the steam supply to such an extent as to prevent the auxiliaries from always having available sufficient driving power of suitable characteristics. The one circumstance which, if not anticipated and arranged for, could interfere with the ability of the steam power to sustain properly the operation of the auxiliaries, would be the sudden imposition on the station of an abnormally large block of electrical load. By overdrawing on the evaporating ability of the boilers this would lower the pressure in the steam system possibly to the point where some essential auxiliaries would "lie down." The station then would fail to deliver not only the additional load required of it, but also its original load. With this contingency recognized in the modern station, and provided for by proper schedules for holding in reserve banked boilers, and for accepting load at a rate appropriate to the steaming ability of the boilers, the operation of steam driven auxiliary equipment remains immune to events occurring externally to the station.

Troubles originating within the station and affecting the steam supply to the auxiliaries enough to prevent their operation could be caused only by conditions resulting in an excessive lowering of the steam pressure at the throttles of the auxiliary turbines. Then some of the auxiliaries might reach their breakdown torque and stop running. For instance, certain types of pulverized fuel mills which require a substantially constant torque regardless of output would stop. This might result in the boilers, inadequately supplied with fuel, failing to sustain steam pressure. However, conditions resulting in such an excessive decrease of steam pressure are preventable and not encountered in stations properly designed and operated. These conditions are either under-boilering of the station or else serious damage to the steam system equipment—boilers or piping. Under-boilering is avoided in designing modern utility plants as it would deprive the main turbine generators of their proper capacity for service; however, this defect sometimes exists in small industrial plants. Since the failure of auxiliary power, due to damage to the steam system equipment, is a condition as likely to affect the main steam supply as the auxiliary system, it qualifies equally the reliability of all steam stations, whether the driving power for the auxiliaries comes directly through auxiliary turbines or indirectly through turbine generators and motors. The piping required for auxiliary turbines may be as

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[1] Stone & Webster Engineering Corp., Boston, Mass.

Presented at the Winter Convention of the A.I.E.E., New York, N.Y., January 25-29, 1932.

simple and reliable as the main steam piping on which large turbine generators in all steam stations have to depend in producing the station's electrical output.

Reliability of Auxiliary Electric Power

A station with motor-driven auxiliaries supplied from a single source of energy would not be as reliable, in so far as the supply of energy is concerned, as the station with turbine-driven auxiliaries; for once the steam energy destined for auxiliaries is transmuted into electrical energy for use in the motors, it enters an additional realm wherein its stability is both more sensitive to disturbance and more subject to unexpected attack. Hence, to attain the same degree of reliability of the auxiliary energy supply as in the turbine-driven station, the station with motors must have duplicate sources of energy.

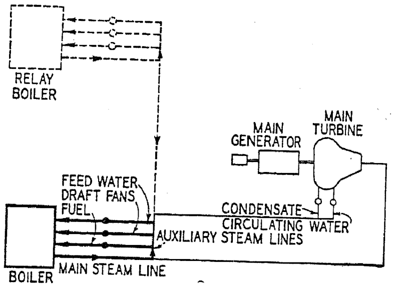


Fig. 1—Station with All Essential Auxiliaries Turbine Driven

—●— Lines and equipment which must function continuously to prevent interruptions

Since the service from the auxiliaries must be of a higher degree of reliability than that of the main power supply, the auxiliaries must be supplied with energy from at least one source not connected to the main generator or to the outgoing lines. This entails furnishing especially for the auxiliaries at least one generator which may be driven either by the main turbine or by a turbine introduced solely for this purpose. In important cases, two special generators are furnished, one driven by each of those turbines. This is frequently done in the case of the downtown stations of metropolitan systems, where so high a degree of reliability of service is expected that it is considered essential to keep the auxiliary system always free from affection by external disturbances.

Not only is it necessary in motor-operated stations to provide special generators and sometimes duplicate distributing systems, but it is customary in the case of such important auxiliaries as boiler feed pumps and exciters, as an ultimate resource, to rely on turbines as the drivers.

In Figs. 1 and 2, one referring to a station with all essential auxiliaries turbine driven, the other to a station with all essential auxiliaries motor driven with power taken from a shaft generator, the solid lines indicate the piping, wiring, and equipment which must always function in order that the supply of auxiliary energy always be maintained. Solid circles indicate auxiliary turbines or motors that similarly must always function; dotted lines and circles indicate duplicate auxiliary energy lines and equipment.

Reliability of Turbines and Motors

The ultimate reliability of the service given by auxiliary equipment to the main units in a station is the compounded reliability of the power supply furnished to the auxiliaries and of the performing abilities inherent in the auxiliary apparatus itself.

Regarding the inherent functional qualities of auxiliary turbines, the main parts in these machines are robust, and, as a concession to low initial cost, simple and inefficient compared with main turbine units. The parts most questioned are the reduction gears, governors, and valve mechanisms. However, the art of gear-cutting has been so developed that gears seldom give trouble without ample warning. Governors and valve mechanisms are less certain in operation unless particular skill is applied to their design.

As a basis for evaluating the degree of functional reliability of turbines, they can be compared with the more usual types of motors; *i. e.*, the squirrel cage, and the adjustable-speed wound rotor types. The simplicity of the construction of squirrel cage motors and their accessories probably place them in approximately the same grade of inherent reliability as turbines. Wound rotor motors and their accessories, however, involve parts and constructions which can be preferred only from considerations of economy of power. It might fairly be said, regarding the qualities of reliability inherent in the drivers and accessories themselves, that steam-driven auxiliaries are as reliable as those electric driven, in the case of constant speed equipment, and are more reliable in the case of adjustable-speed equipment.

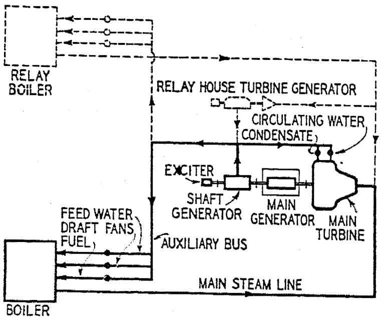


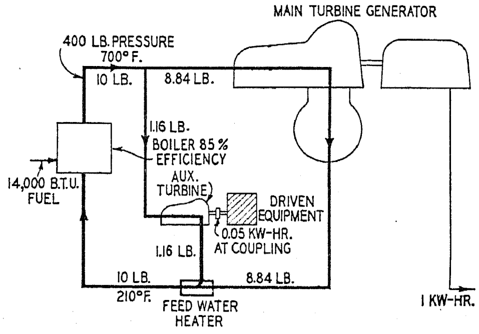
Fig. 2—Station with All Essential Auxiliaries Motor Driven

—●— Lines and equipment which must function continuously to prevent interruptions

A further aspect of the reliability of performance inherent in the driving apparatus itself is the relative ability of turbines and motors to drive adjustable-ed equipment smoothly and without transitional urbances—particularly draft fans. On such duty, turbine with its infinite number of speeds performs nly; the wound rotor motor, with its step action nsition in speeds, is more liable to cause disturbance urner flames, and in the change from the motor of speed to that of another, further operating compli- ons are encountered.

ATIVE EFFICIENCY OF TURBINE AND MOTOR DRIVES

espite their simplicity and reliability, turbine- en auxiliaries were to a large extent displaced, when as discovered that a fuel saving could be made if m for producing auxiliary power was first passed ugh efficient main turbine units to produce electri- energy and then extracted for water heating, ead of being passed through small inefficient ines mechanically driving auxiliaries and then into water heaters. It was found that the additional trical energy thereby generated was more than required for driving the auxiliaries by motors, after deducting the losses in the generators, trans- ners, and motors. The fundamental reason for gain in economy is that the surplus energy thus le available is produced very much more economi- y than the rest of the station output energy due to fact that the steam required for generating the lus energy is used for heating the boiler feed water. result, that steam is utilized at a station thermal iency between 90 per cent and 100 per cent, whereas steam used for generating the rest of the output gy is carried into the condenser, where most of riginal heat energy is lost, and so is used at an iency between 20 and 35 per cent.



3—Station Efficiency Diagram with All Auxiliaries Turbine Driven

Overall thermal efficiency of station 24.4 per cent

ith regard to the economy actually effected by these ges the examples illustrated in Figs. 3 and 4 show a modern station's net output for a given fuel umption is increased about one and three-quarters cent at maximum load by substituting motors for ines on all essential auxiliaries, and by heating the water with extracted steam instead of auxiliary ust steam; the over-all thermal efficiency of the turbine-driven station is 24.4 per cent and of the motor-driven station 24.8 per cent. In this comparison other conditions have been kept similar. It should be observed that in Fig. 3 less steam is needed for water heating than in Fig. 4; this is because the auxiliary turbine in Fig. 3 is inefficient, or a poor converter of heat energy into mechanical energy. As a result, hot steam is exhausted, less of which is required for the water heating effect than is needed of the cooler steam extracted from the efficient main turbine. However, as will be inferred from the reasons for the economy explained above, the greater the percentage of the total steam in the main turbine which can be extracted and used for heating, and the less remaining to go into the condenser, the more will be the net station output for the same fuel consumption.

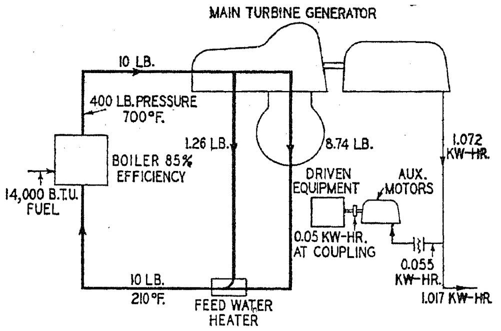


Fig. 4—Station Efficiency Diagram with All Auxiliaries Motor Driven

Overall thermal efficiency of station 24.8 per cent

Not as a principle unique to either turbine or motor drives, but as one common to both, it should be realized that the station efficiencies shown in Figs. 3 and 4 could and should be increased by heating the feed water further than shown in these examples by extracting steam from one or two high temperature points of the main turbine so as to raise the water to a higher temperature. Moreover the 1¾ per cent advantage attained in the station shown in Fig. 4 would have been slightly greater if, instead of using one point of extraction, two had been used to heat the water to the same temperature; and due alone to the stray losses encountered in actual operation, the advantage might be as high as 2½ per cent.

The difference in the efficiencies of the two stations compared above would have been very much less if, in the comparison, the high efficiencies now obtainable in auxiliary turbines had been assumed instead of the low efficiencies actually taken as better representing common practise. However, it should be understood at this point that in the two stations compared, their relative efficiencies were estimated for only one condition of station load: both stations were operating at their maximum output and the auxiliary turbines and motors were performing at their maximum and most efficient loads.

POWER CONSUMPTION

In order to ascertain the total difference in power consumption between steam and electric driven auxiliaries over the whole working range of the boilers and turbine generators, and hence the net annual saving in fuel, it is necessary to determine the power taken at partial loads by both types of drives. This has been done by analyzing separately the power input to each essential auxiliary when driven with turbines, squirrel cage, and adjustable speed wound rotor motors. The results of these analyses are given in Figs. 5 to 9, in each of which the curves are plotted to show, at any gross output of the boiler or main turbine generator, what percentage of that output is consumed by the particular auxiliary with each type of drive. In the case of those curves based on power output of the boiler, that output has been reckoned as the power which the main turbine generator would generate with the quantity and quality of steam delivered by the boiler at its different loads. The power consumed by the turbine-driven auxiliaries in all cases has been calculated as the power which the quantity of steam used in these auxiliaries would have produced if used in the main turbine generators instead.

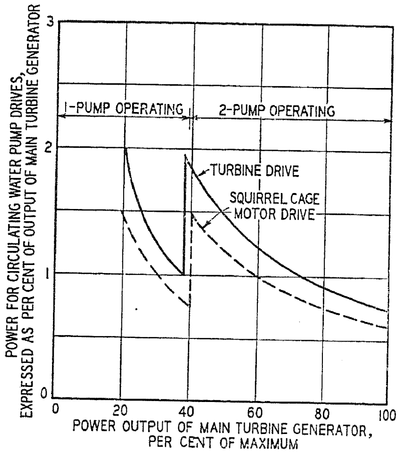


FIG. 5—CIRCULATING WATER PUMP DRIVE

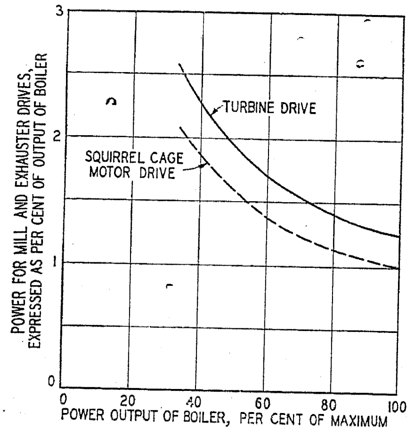


FIG. 6—UNIT PULVERIZING MILL AND EXHAUSTER DRIVES

These curves represent the performance of equipment in stations having large boilers and turbine generators, where the auxiliary turbines also would be large and slightly more efficient than those often used. However, turbines having higher efficiencies than assumed in this analysis are now in operation on auxiliary service.

It will be seen from the curves that as the boiler and turbine generator loads decrease from a maximum, the relative power consumptions of steam and motor driven auxiliaries vary in the following manner:

1. For such auxiliaries as circulating water pumps and fuel pulverizing equipment, for which the output is substantially constant throughout a wide range of load on the main units, and for which constant speed squirrel cage motor drive is suitable, the electric drive maintains its relative efficiency over the turbine drive, throughout the whole range of load.

2. For such auxiliaries as boiler feed pumps and forced and induced draft fans, in which the output varies with the load on the main units:

(a) The squirrel cage motor rapidly loses the advantage in power saving which electric drive gives at maximum load, and below about 85 per cent load on the main units it is less efficient than the turbine. At lower loads, the initial relationship between the two drives can be reestablished by transferring the load to a lower speed motor, but on reducing the load still further this motor will in turn become less efficient than the single turbine.

(b) The superior economy over turbine drive which the adjustable speed wound rotor motor gives at maximum load diminishes greatly as the load decreases and would vanish at low loads unless operation is transferred to an additional and lower speed motor.

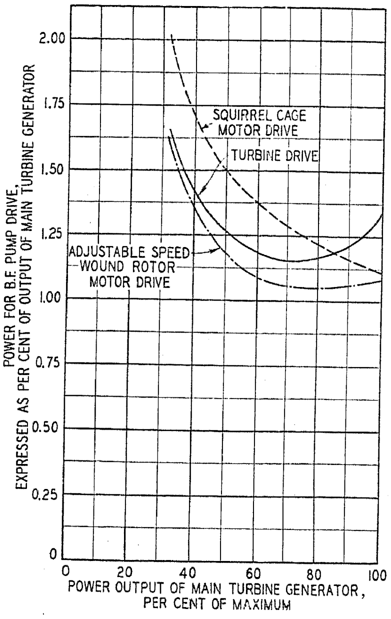


FIG. 7—BOILER FEED PUMP DRIVE

Relative Operating Costs

These conclusions indicate that from considerations of power consumption, the most favorable application for turbine drive is on equipment like draft fans which, having to traverse a wide range of speed and load, use more than one motor when electrically driven.

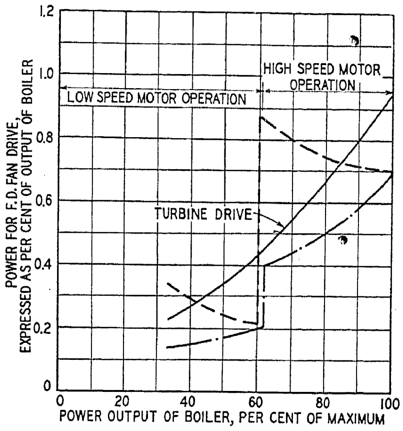


Fig. 8—Forced Draft Fan Drive

— — — — — — Two squirrel cage motors

— · — · — Two adjustable-speed wound rotor motors

Even on this application the electric drive saves power; however, it involves a larger investment in both driving and generating equipment.

Regarding the investment in driving equipment, it is estimated that for large boilers having a maximum output of the order of one-half a million pounds of steam per hour and with two wound rotor motors per fan, the installed cost of motors, controllers, and wiring for forced and induced draft fans would be about $10.00 per rated fan horsepower more than for turbines with their valves and piping.

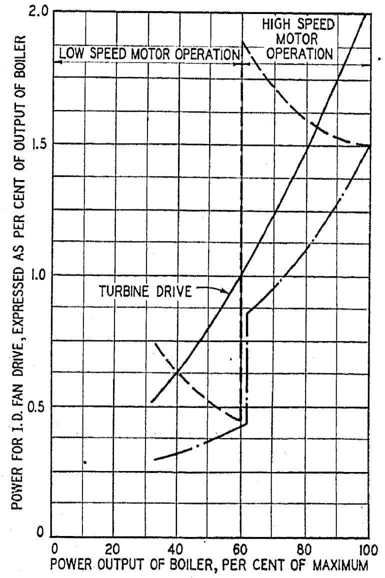


Fig. 9—Induced Draft Fan Drive

— — — — — — Two squirrel cage motors

— · — · — Two adjustable-speed wound rotor motors

Regarding the investment in generating equipment assessable against the fan drives, if the fans are electrically driven and assessment is made of all the equipment usually provided for normal and emergency power supply, it will be found that the turbine room cost increases about $49 per installed fan horsepower. If the forced and induced draft fans are steam driven, 0.7 per cent more total evaporating capacity is needed than if motor driven, as is shown in Figs. 8 and 9; as a result the boiler room in this case costs about $12 more per installed horsepower than in the other case. Those investment costs are based on a total station cost of $95 per kw. output.

As determined above, the total investment incident to the electric driving of fans will be $47 higher per installed fan horsepower than for the turbine driving. If the annual charges on this higher investment are less than the cost of fuel per installed horsepower saved by the electric drive, then motors would be more economical than turbines; if not, not. Seldom, however, would motors be more economical. Even with coal at $4 per ton and a capacity use factor of 100 per cent, the combined fuel cost and fixed charges would be less for the turbine drive than for the motor drive.

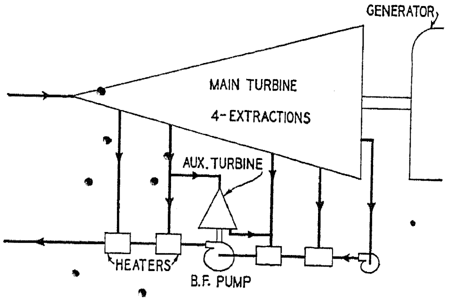


Fig. 10—Steam and Exhaust Connections for Auxiliary Turbine in High-Pressure Station

In plants designed to deliver large quantities of steam for industrial uses, or for heating, the economic balance is usually in favor of turbine drive regardless of load factor and fuel cost.

Future Developments

To fully realize the advantages obtainable by using auxiliary turbines in conjunction with large main units, more efficient designs and methods of application become desirable, practicable, and commercially feasible. With the conventional designs and applications of these machines, more stages and larger diameters can be used to raise the efficiency at maximum load; however, care must be taken that the gain so obtained is not at the expense of inefficient partial load operation.

Novel means for securing very high efficiency have been adopted in turbines recently installed in one large central station; there, not only have high rotative speeds and many stages of highly refined design been introduced, but the steam is passed in series through two turbines: first, through the induced draft fan turbine and then through the forced draft fan turbine.

For stations of high steam pressure, about 600 lb. per sq. in. and over, still other means can be invoked to improve the efficiency of auxiliary turbine operation, as indicated in Fig. 10. Here the improvement is derived through using conventional designs of machines, but novel steam connections. In this case the boiler feed pump is shown driven by a turbine receiving steam from one extraction point on the main turbine and exhausting it to a heater connected to a lower extraction point. Thereby, as the pressure drop through the auxiliary turbine is small, this being a factor giving good efficiency, the machine can have relatively few stages and slow wheel speed; moreover, as the throttle steam is of relatively low temperature and pressure, the construction can be simple. All these features contribute to giving a low initial cost of machines and piping, and low maintenance. In adopting systems of this character, much care and some ingenuity of design are required to protect continuity of operation without undue complications of piping, and to achieve the simplicity of operation which is quite as essential as simplicity of design.

Although the gears with which turbines are equipped usually perform quietly, it is desirable that still more attention be given in their design and manufacture so as to remove completely the objection to turbine auxiliaries still offered in some quarters where objectionable noise has been experienced with geared turbines.

While it is indicated that the immediate development of the auxiliary turbine will be along the lines of higher efficiency and greater refinement of mechanical detail, the development of electric drive should be directed toward greater simplicity and lower installation cost. Greater sturdiness of adjustable-speed motors is desirable, as well as wider speed ranges, but the greater field for improvement is in distributing systems and control equipment. It is desirable to retain the efficiency and convenience of motor drives, without sacrifice of reliability, at substantially lower cost than the systems now in common use require. While it is not clear at the moment how this end may be accomplished, the problem is receiving the attention which it deserves. The great variety of systems adopted in recently built stations indicates that power plant designers are far from satisfied with the conventional auxiliary power systems, and that they are striving to produce something better than has been designed before.

CONCLUSIONS

It will be inferred from the foregoing discussion that in the present state of the art the turbine constitutes the simplest and most reliable drive for steam power station auxiliaries. For conditions requiring extreme measures in the protection of auxiliary power supply its simplicity and reliability may warrant the selection of turbine drive for all essential auxiliaries.

For maximum station load conditions there will be an improvement in over-all fuel economy of the order of 2 per cent incident to the use of motor drive instead of turbine drive for all essential auxiliaries: with both types of drive, steam would be extracted from the main turbine for feed water heating, more being extracted in the case of the motor-driven station.

For the auxiliaries taking constant power and driven at constant speed the economy due to motor drive is even greater at partial than at maximum station loads.

For the auxiliaries driven at adjustable speed the advantage in economy with motor drive is slight at lower loads.

Under most conditions the total annual expense for forced and induced draft fans and perhaps for boiler feed pumps, including fuel costs, maintenance, and capital charges, will be less with turbine drive than with motor drive.

There is an element of convenience in starting and stopping, which commends the motor drive to the average operator, when other considerations are approximately equal.

There seems to be little justification for the use of turbine drive for non-essential auxiliaries, (those which may stop for a short time without inconvenience) these auxiliaries being in the main driven at constant speed.

Future developments may change the relative status of these two types of drives, but it would appear that progress in the immediate future will tend to decrease the advantage in fuel economy which the motor now possesses, and to increase the turbine's advantage in reliability.

Discussion

For discussion of this paper see page 341.

Electrically Driven Auxiliaries for Steam Power Stations

BY LYLE W. SMITH[1]

Member, A.I.E.E.

Synopsis.—There is a definite trend toward the use of all electrically driven auxiliaries for steam power stations. This paper reviews the reasons for this trend, summarizes the advantages of electric drive and discusses in a general way some of the economic factors in the selection of an auxiliary system. A companion paper covers steam-driven auxiliaries.

INTRODUCTION

THIS paper reviews the advantages of electric drive for steam power station auxiliaries in light of present conditions and outlines some of the economic factors that enter into the selection of an auxiliary system. The subject will be discussed in a general way because it is apparent that each problem has to be worked out to meet the local requirements.

A review of the history of auxiliaries for steam power stations shows a gradual change from complete steam drive to the other extreme of complete electric drive except for a few reserve steam driven auxiliaries such as boiler feed pumps and service pumps. During an intermediate period, dual steam and electric drive was used quite extensively for some of the essential auxiliaries, but the number of dual drives appears to be steadily diminishing.

REASONS FOR TREND TOWARD ELECTRIC DRIVE

The higher efficiency of electrically driven auxiliaries receiving their power from the efficient main turbine or house turbine, as compared with small inefficient turbine-driven auxiliaries, has always been attractive to the station designer. However, lack of confidence in the reliability of the early forms of electric drive for essential auxiliaries retarded its general adoption until more operating experience was obtained and other reasons than efficiency of the auxiliary drive appeared.

The change from steam to electric drive is largely due to an increasing feeling of confidence in the reliability of electric drive, an increasing appreciation of the simplicity and flexibility of electric drive, and important changes in boiler and turbine practise. Confidence in electric drive has been strengthened by improvements in the manufacture and application of electrical equipment, the proved value of station and system interconnections, and a record of successful operating experience with electric drive. The convenience and flexibility of electric drive are generally known; however, they will be discussed more in detail elsewhere in the paper. The important changes in boiler and turbine practise that have influenced the use of electric drive are multi-stage bleeding of turbines for feed water heating, the use of higher steam pressures and temperatures, the development of larger boilers and turbines with their multiplicity of similar auxiliaries, and to some extent, automatic combustion control.

RELIABILITY OF STEAM AND ELECTRIC DRIVE

Reliability has always been considered the chief requirement for essential auxiliaries. Steam is considered more reliable than electric drive chiefly because it is a more direct method of obtaining power. Electric drive introduces additional apparatus such as generators, transformers, switching and control between the boilers and the auxiliary to be driven. If the electrical driven auxiliary is connected directly or indirectly to the main bus, it is also exposed to the effects of line disturbances. The performance of the intermediary equipment required for electric drive will now be considered from the standpoint of reliability.

The main turbines and house turbines are probably more reliable than efficient auxiliary turbines. Furthermore, the main turbines and house turbines receive much closer attention than could be given economically to each auxiliary turbine. Statistics show that generator failures are extremely rare, especially since closed ventilation systems have been used. In the case of a failure of the main generator, there is usually no immediate need for the associated auxiliaries of the generator and boilers. Auxiliary transformers have an equally good record for freedom from trouble. Modern switching and control, when selected and installed with the usual liberal factors of safety, have given relatively little trouble in recent years. A large proportion of the motors used for electric drive is of the squirrel cage induction type, which is unusually sturdy and readily permits full voltage starting with its simplicity of control. The other types of motors when correctly applied and maintained have given good service. Improvements in insulation and bearings have made motor coil failures rare. The extensive use of bus and feeder reactors, and diversity of power feeds to the electrically driven auxiliaries have all tended to minimize the effects of line disturbances. The present trend toward faster operating oil circuit breakers will also improve this situation. It seems apparent that reliable service can be expected from electric drive when correctly applied.

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SIMPLICITY OF ELECTRIC DRIVE

Simplicity is an important requirement for auxiliary drive. It is difficult to conceive a simpler drive for rotating equipment than the squirrel cage motor, or a simpler starting operation than pushing a button. Developments in control have made varying and adjustable speed motors equally simple to operate. A motor can be started in a fraction of a minute whereas the ordinary small auxiliary turbine usually requires a preliminary warming up period. Convenient control of auxiliaries from a few central points often permits a reduction in the size of the operating staff and contributes toward better efficiency in station operation.

FLEXIBILITY OF ELECTRIC DRIVE

Flexibility is likewise an important requirement for auxiliary equipment, chiefly from the standpoint of station design. The flexibility of electric drive is almost unlimited as evidenced by a modern boiler combustion control system. The absence of steam piping to the turbine room and boiler room auxiliaries results in less congested basements, thereby facilitating design, operation and maintenance. Electric drive provides greater flexibility in obtaining the station heat balance.

EFFECT OF STAGE BLEEDING CYCLE

The general adoption of the multi-stage bleeding cycle for boiler feed water heating is one of the most important changes in boiler and turbine practise that has tended to increase the use of electrically driven auxiliaries. The stage bleeding cycle, of course, does not require complete electric auxiliary drive. However, the use of complete electric auxiliary drive does have the following advantages:

1. It decreases the B. t. u. per kw-hr. rate of the station approximately 2 per cent.
2. It simplifies the heat balance system of a station.
3. It simplifies operation because the steam extracted for feed water heating varies automatically with change in load.
4. No provisions are required for transferring auxiliary load to obtain a heat balance with varying station load as is the case when a separate house turbine is used.

EFFECT OF HIGHER PRESSURES AND TEMPERATURES

The increasing use of higher steam pressures and temperatures has made steam driven auxiliaries less desirable than electrically driven auxiliaries. Some of the more important reasons for this condition are as follows:

1. The cost of small high-pressure turbines used for auxiliaries is considerably higher than for small low-pressure turbines. The initial cost of a 1,200-lb., 750 deg. fahr. auxiliary turbine may amount to approximately 40 per cent more than auxiliary turbines for 400-lb., 750 deg. fahr. operation, with a gain in efficiency of but 10 per cent.
2. The efficiency and capacity of small high-pressure turbines fall off more rapidly than small low-pressure turbines with the accumulation of a slight amount of scale in the steam passage, because high pressure requires smaller volumes of steam and therefore smaller clearances. This characteristic also results in higher maintenance cost.

EFFECT OF LARGER BOILERS AND TURBINES

The trend toward larger boilers and turbines has favored the use of electrically driven auxiliaries for the following reasons:

1. Large boilers and turbines permit the economic use of two or more similar auxiliaries per unit which allows diversity of auxiliary power supply, thus increasing the reliability of electric drive.
2. It justifies the use of the more reliable unit system for auxiliary power supply in which all of the essential auxiliaries for a turbine generator and its boilers are supplied from a source connected directly or indirectly to the turbine generator. It also tends to lighten the interrupting duty of the auxiliary switching equipment and therefore reduces the cost.

ECONOMICS OF AUXILIARY DRIVE

The selection of the type of drive for a station auxiliary is not usually made as a result of a direct comparison between the cost and performance of a motor and a turbine alone, but is also determined to a large extent by the general design of the station. Some of the features of station design that influence the type of auxiliary drive have already been mentioned. A comparison between steam and electric drive introduces two factors that are subject to considerable debate. One of these factors is the cost of an electric auxiliary power supply that can be considered equally reliable as steam drive, and the other is the actual water rate of small steam turbines.

Some engineers feel that an electric auxiliary power supply equivalent in reliability to steam driven auxiliaries should make use of auxiliary generators connected to the same shaft as the main generators. Reserve capacity in this case would also be supplied by transformers connected to the station bus and a separate house turbine generator. Other engineers feel that a sufficiently reliable auxiliary power supply can be obtained from transformers connected to the generator leads ahead of the generator oil circuit breaker, or connected to sections of the main bus, if the station forms part of an interconnected system. The use of bus and feeder reactors and the type of system interconnections influence the valuation that can be placed on the reliability of the various methods of auxiliary power supply.

Operating experience with small steam turbines indicates that their efficiency falls off very rapidly unless they receive considerable maintenance. Cases have been reported where the efficiency of small turbines has dropped to half of the guaranteed efficiency, because of lack of maintenance. It is apparent that any comparison between electric and steam drive should take this characteristic of the small turbine into consideration, although it may be difficult to evaluate it.

Some of the factors that affect the economics of an electrically driven auxiliary system will now be considered.

Power Required for Electrically Driven Auxiliaries

The amount of power required for an electrically driven auxiliary system is relatively small and depends to a large extent on the type of fuel-burning equipment used. The station load factor and nature of condensing water supply also affect the amount of auxiliary power to a lesser extent. A few typical values of auxiliary power with different types of fuel burning equipment are given below:

Fuel burning equipment	Per cent of gross generated auxiliary power
Pulverized coal	5.5—6.5
Stokers	4.0—5.0
Gas and oil	3.5—4.0

The subdivision of auxiliary power for a modern 650-lb. steam generating station of approximately 200,000-kw. capacity, entirely equipped with electrically driven auxiliaries and using the unit system of pulverized fuel equipment as expressed in per cent of the total auxiliary power is approximately as follows:

Coal preparation	22.0	per cent of aux. power
Boiler feed pumps	22.0	" " " " "
Turbine auxiliaries	22.0	" " " " "
Forced and induced draft fans	20.0	" " " " "
Heater drain pumps	3.8	" " " " "
Service pumps	2.6	" " " " "
Station lighting	2.4	" " " " "
Coal handling	1.6	" " " " "
Miscellaneous auxiliary power	1.5	" " " " "
Ash sluice pumps	1.0	" " " " "
Air compressor	0.7	" " " " "
Dust precipitators	0.4	" " " " "
5.5 per cent of gross generated power	=100.0	per cent of aux. power

A study of the above tabulation indicates that the possibilities of further saving in the amount of auxiliary power appears limited to one or two per cent of the gross generated power at the best.

Distribution of Cost for Electrical Auxiliary System

The following tabulation shows the major subdivision of investment cost for the electrical part of the electrically driven auxiliary system of the same 200,000-kw. generating station for which the subdivision of auxiliary power has been given. This station utilizes two 4,000-kw. shaft generators, and two 9,000-kva. transformer banks connected to the main bus for auxiliary power. The main switching equipment and the 2,300-volt auxiliary switching equipment are of the vertical lift metal-clad type. Air circuit breakers and magnetic switches are used for 440-volt motors. The boilers are equipped with full automatic control. The investment costs given below are expressed in approximate percentage of the total station cost.

Prorated share of cost for turbo-generator units used for auxiliary power	0.8 per cent
Auxiliary power distribution system, including transformers	1.0 per cent
Motors	1.2 per cent
Control for motors	2.0 per cent
Cable and conduit for motors and control	2.0 per cent
Total for electrical part of station auxiliary system	7.0 per cent

In the above tabulation only the prorated share of the cost of turbo-generator units used for auxiliary power is included, because the boilers and the balance of the station equipment are common to both steam and electrically driven auxiliary systems. No allowance has been made for space occupied by electrical auxiliary equipment because it would be approximately the same for either type of drive. The main auxiliary transformers and high-tension switching in this particular case are installed out of doors. The flexibility of electrical control equipment enables it to use space that might be otherwise wasted.

A study of the tabulation indicates that the greatest possibilities of saving in the cost of an electrical auxiliary system are the control and cables for motors. However, it seems apparent that a radical reduction in the investment cost of the complete station will not be the result of major modifications to the electrical part of an auxiliary system, but will have to come from a series of small savings. The possibility of making some of these small savings in the electrical auxiliary system will now be discussed in a general way.

Electric Auxiliary Power Supply

A review of station designs with respect to source of electric power for auxiliaries shows quite a variety of systems, all of which are modifications of two fundamental systems; power from main bus or power from a separate generator. The advantages and disadvantages of the principal modifications of these fundamental systems are covered very completely in N.E.L.A. Publication 088, April 1930.

It is interesting to note something of a trend back to the original system of supplying auxiliary power from the main bus after passing through a cycle of house turbines and shaft generators. The system of taking auxiliary power from the generator leads ahead of the oil circuit breaker is virtually the same as taking the auxiliary power direct from the main bus, since the generator oil circuit breakers are usually non-automatic except for a fault in the generator. Both of these latter systems are subject to the same criticism of being affected by bus voltage disturbances unless the station has been designed to minimize the effects of line disturbances.

Operating experiences with various types of electric auxiliary power systems appear to justify in many instances the use of the simpler and more efficient system of obtaining the auxiliary power from transformers connected to the generator leads, or from sections of the main bus, for large stations forming part of a large interconnected system. Power from the transformers connected to the generator leads is the least expensive of these two systems because it eliminates the need for auxiliary transformer high-tension oil circuit breakers. The disadvantages of the generator lead system are substantially the same as for an auxiliary shaft generator. Both are subject to interruption if the throttle of the turbine trips on overspeed, and both will cause the auxiliary system to be subjected to a high voltage if the throttle does not trip on overspeed. The shaft generator is, of course, free from main bus voltage disturbances. Improvements in turbine governors and the increasing use of generator voltage regulators for system stability purposes tend to minimize the possibilities of trouble with the auxiliary power system from overspeeding of the turbine.

If an auxiliary system is being selected for an isolated generating station, it is apparent that a more elaborate system of electric auxiliary power supply will be required. This type of station will also require more steam-driven reserve auxiliaries for starting up purposes than are now considered necessary for stations forming part of a large interconnected group.

The economical use of house turbine generators appears to be confined to the stations that used d-c. adjustable-speed motors or where there is use for exhaust steam other than for feed water heating. Many engineers feel that a house turbine is justified for use as a reserve to the other systems of power supply and for starting up the stations. The type of house turbine generator that can be started up almost instantly from a remote point appears well adapted for a reserve power supply when local conditions justify its use.

The unit system of auxiliary power supply, whether it be from house turbines, shaft generators, bus transformers, or transformer on generator leads, appears justified for all large stations. As already mentioned, the chief advantages of the unit system are greater reliability and lower interrupting duty on the auxiliary power control equipment.

ELECTRIC AUXILIARY POWER DISTRIBUTION SYSTEM

The selection of the voltage of an electric auxiliary power distribution system is usually determined by the standard motor voltages available, together with a study of the over-all cost of the motors, cable, and control. In many of the earlier stations, the lengths of cable runs were relatively long and the interrupting duty on the control relatively light, with a result that motors of 50 hp. and above were usually rated 2,200 volts. However, with the present trend toward larger stations with greater concentration of auxiliary power, the cost of control has become a large item of the total cost of the electrical auxiliary system and deserves considerable study. The 1926 report of the A.I.E.E. Power Generation Committee pointed out this condition and suggested that a study be made of the possibilities of using a 440-volt auxiliary power distribution system with carbon air circuit breakers for control. It is interesting to note that a 460-volt auxiliary power system using carbon air circuit breakers was adopted for the Gould Street Generating Station at Baltimore as a result of a thorough study of investment costs and performance of motors and control. The Gould Street Station is designed for four 36,000-kw. units.

In the case of larger stations designed for the use of units with a capacity of 50,000 kw. and upwards, it usually works out that two or more auxiliary distribution voltages are required. The common auxiliary distribution voltages are 2,300 volts and 460 volts; however, 550 volts is being used in preference to 460 volts in some stations. In many instances an appreciable saving can be made in the over-all cost of motors, control, and cable by extending the horsepower range of the low-voltage motors up to 100 hp. or more, and establishing low-voltage power distribution groups central to the group of motors fed by the distribution point. The performance of carbon air circuit breakers for low-voltage control has been very satisfactory.

A further saving can sometimes be made in the auxiliary power switching and control equipment by diversified grouping of duplicate unit auxiliaries. For instance, half of the auxiliaries required for one boiler can be supplied from one bus section and the other half of the auxiliaries from another bus section. This not only makes a more reliable power supply, but eliminates the need for double buses, because it is usually possible to isolate at least one-half of boiler auxiliaries during off peak conditions for maintenance of the switching and control equipment.

AUXILIARY MOTORS AND CONTROL

A review of motor applications for recent large stations indicates a decided trend toward the use of more squirrel cage motors and fewer slip-ring motors. Machinery designers are becoming reconciled to full voltage starting of squirrel cage motors and in many cases, have withdrawn their demands for slip-ring motors merely to obtain smooth starting conditions. Coal pulverizer mills are now available that do not require adjustable-speed motors. Satisfactory results have been obtained with multi-speed squirrel cage motors driving fans equipped with adjustable vanes in the fan inlet. Station designers appear to be more willing to give up small gains in plant efficiency obtained by adjustable-speed motor drives, in order to reduce the investment costs for motors and control. One exception to this trend toward squirrel cage motors appears to be boiler feed pump motors for high-pressure boilers.

It is apparent that if the present high investment cost of auxiliary motors and control is to be reduced, every auxiliary drive application calling for adjustable-speed motors must be investigated and justified by an appreciable saving in over-all cost. The use of adjustable-speed motors increases maintenance and space requirements and to some extent, reduces the reliability of the electric drive. N.E.L.A. Publication No. 088 contains very complete data on the selection of motors and control for station auxiliaries which should be helpful in making decisions of this kind.

Considerable progress has been made by the manufacturers in supplying self-contained motor control groups completely fabricated at the factory. In many cases where the field labor costs are high, a substantial saving can be made by using factory fabricated control groups.

An appreciable saving can often be made in conduit work for motors and control by using armored cable installed in metal troughs or large pipes indoors, or parkway cable outdoors. This method of handling cables has given satisfactory results in the middle west during the past few years.

SUMMARY

The chief advantages of electric drive over steam drive for station auxiliaries are:

1. It facilitates an efficient station design by permitting all of the steam required for feed water heating being taken from the main turbine by stage bleeding.
2. It may be approximately twice as efficient in terms of B. t. u. per kw-hr. as steam drive with small turbines, depending upon the amount of maintenance received by the turbines.
3. It is uniformly efficient, because lack of maintenance does not materially affect the efficiency of electrical equipment, although it may reduce the reliability of the equipment somewhat.
4. It requires less maintenance.
5. It is simpler to operate.
6. It improves the physical design of a station and its operation by reducing the amount of auxiliary piping required, particularly in the basement.

Some of the possible methods of reducing the cost of an electrical auxiliary system are:

1. Obtain auxiliary power supply from transformers connected to the main generator leads ahead of the generator oil circuit breakers, and reserve supply from bus, if reliable interconnections are provided with other stations and station bus is protected by reactors or the equivalent.
2. Establish both high-voltage and low-voltage auxiliary power distribution groups central to location of auxiliaries instead of in electrical bay, thereby reducing length of cable and conduit runs.
3. Increase the horsepower range of low-voltage motors in view of relatively high cost of high-voltage control equipment.
4. Diversify feeds to all duplicate auxiliaries where possible, thereby permitting simpler switching layout for adequate maintenance.
5. Use squirrel cage motors and full-voltage starting equipment wherever possible even at sacrifice of a small percentage of station efficiency.
6. Use factory fabricated control equipment wherever possible.
7. Use armored cable, grouped in metal troughs or large pipes for auxiliary power and control when installed indoors or parkway cable when installed out of doors.

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Discussion

AUXILIARY DRIVE FOR STEAM POWER STATIONS

(HOLLISTER)

STEAM-DRIVEN AUXILIARIES FOR POWER PLANTS

(DRYER)

ELECTRICALLY DRIVEN AUXILIARIES FOR STEAM POWER STATIONS

(SMITH)

A. H. Kehoe: The experience of the New York Edison System with steam and electric drives for power station auxiliaries is as follows:

1. Continuity of Service. Continuity of station output with either steam or electric auxiliary drive has been accomplished in our stations by the following provisions:

All essential steam auxiliaries are supplied from steam headers which are provided with sectionalizing valves and fed from two or more sources.

Each essential auxiliary motor circuit has supply available from several sources.

Auxiliary supply circuits and buses are so sectionalized and diversified that electrical trouble at any one point will not affect more than a portion of the auxiliary service.

2. *Flexibility of Control.* Flexibility of control in small steps has been found to be necessary only for hoists and for the coal, air, and water supply to boilers.

Satisfactory flexibility of control has been obtained either with steam turbines or with electric motors equipped for speed variation in small steps, or with other types of motors where the driven unit can be properly controlled by throttling the output.

The tendency in modern boiler plant design to provide a minimum of storage for coal, water, and steam within the boiler unit has added importance to the desirability of so controlling the flow of fuel and air to the furnace and of water to the boiler that at all times the correct quantities will be present.

Two-speed and three-speed induced draft motors are operated with throttling control on fans, our practise being to pass the speed-change points on motors by the simultaneous hand control of motor speed and vanes or dampers. Automatic equipment designed to accomplish this purpose is being provided on the units now being erected.

At the fans speed-change points, pulverized fuel fires have been found particularly sensitive to the large increment changes in the flow of fuel or air, due to the difficulty under such conditions of so synchronizing the mechanism causing the flow of fuel and air to the furnace as to maintain the fuel-air ratios at all times consistent with stable combustion.

On stoker fired boilers having a common forced draft duct for a number of boilers, single-speed motors with throttling control on fans have been found to give smooth and efficient operation.

We have experienced serious overloading of some motors driving fans and centrifugal pumps when the efficiency or discharge resistance of the driven unit was materially different from that specified. This was due to a difference in the speed torque characteristics of the motor and the driven auxiliary.

3. *Heat Consumption.* No comparative heat consumption performance results are available on steam and electric auxiliary drives. The metering of the energy consumed by electric drive is accomplished with comparative ease, but the metering of the energy consumption of steam drive is impractical.

Two recent engineering studies have been made comparing steam drive and electric drive of boiler fans. At our Hell Gate Station, where a boiler deficiency in an existing station was being corrected, the top bleed point in the existing heat balance was so low that it was possible to condense all the steam from the fan drive turbines without exceeding a practical back pressure. For these conditions, the steam and electric drives showed substantially the same net heat rate. From a recent and very thorough comparison of steam and electric drive for induced draft fans only for Hudson Avenue, which case can be considered as much more representative of usual conditions than the circumstances at Hell Gate, it was determined that the net heat consumption of the turbine generator installation under consideration would have been increased approximately ½ per cent had this one set of auxiliaries been driven by steam turbines instead of motors.

4. *First Cost of Auxiliaries.* We have found some saving in installed cost of auxiliaries with electric drive instead of steam, especially where the simpler types of electric motors are adaptable, such as a-c. squirrel cage motors of either constant speed or two or three-speed pole-changing construction.

5. *Effect on Cost per Kw. of Station Output Capacity.* The choice between steam and electric drives has been influenced by the effect on net station output capacity where there has been a limitation in available capacity of either the main turbo-generators or the boilers. Such considerations have been highly local in character, and have been of major importance.

6. *Policy Considerations.* It has been recognized that—other things being equal—there is an advantage to an electric supply company using electricity instead of steam for its own power needs. This advantage is partly in its advertising aspect and partly in the increased familiarity of the company's personnel with problems of utilization of electric service.

I. E. Moultrop: Auxiliary power requirements of the stations built in the future will probably be considerably in excess of the auxiliary power requirements at present and in the past.

Boilers will be operated at much higher ratings than in the past, with greatly increased power requirements for forced and induced draft fan drive.

The use of higher steam pressures has materially increased the power requirements of the boiler feed pumps.

The radical decrease in the surface of condensers per kilowatt of turbine capacity has also increased the power required to pump the circulating water per million pounds of steam condensed. The use of higher steam pressures and the regenerative cycle have fortunately more than offset this increase in power and have actually resulted in a reduction in the power required for circulating water pumps per unit of station capacity.

A comparatively few years ago all essential auxiliaries in our steam-electric generating stations were steam-driven. The development of the small auxiliary turbines was backward because of the false impression that high economy was not necessarily desirable because the exhaust steam was used to heat the boiler feedwater.

The reliability and efficiency of the a-c. motor was improved and within a very few years we had changed to practically all-electric drive for auxiliaries with a very few necessary stand-by turbine-driven auxiliaries.

In the light of recent experiences it now becomes evident that we may have gone too far with a good thing. Both steam-driven and electric-driven auxiliaries have their place in the ideal layout. Now that we have tried both extremes, we will probably find a compromise that will use both steam and electric drives, each in its proper place.

As a starter, we all seem to be agreed that reliability is the primary requisite. This point cannot be too strongly stressed. The reliability of the service from a station can never exceed the reliability of the station auxiliaries.

Unfortunately, we have another requirement that is diametrically opposed to our major requirement. This second requirement is low unit cost. The question of lowering the unit cost of construction is one of the most important facing the station designer today and we cannot forget it in our consideration of the subject under discussion.

The development of the turbines for auxiliary drive in the past few years has been very satisfactory and we can now say that auxiliary turbines of high reliability and very good economy are available.

The feature that still requires improvement in connection with the auxiliary turbine is the valve gear arrangement. The complicated and expensive valve gear designs used for the main generating units are neither justified nor warranted for auxiliary turbines. They should be equipped with an overspeed trip operating a separate valve and a suitable valve chest designed for remote control by the several control devices now in general use. The valves in the valve chest should be designed to give a constant increase in the output of the driven auxiliary for a given impulse from the control system. The failure to meet this basic requirement has been the source of many operating difficulties and the complicated and improperly designed mechanisms often offered for this simple service are very undesirable and unsatisfactory.

Reduction gears have been improved, as pointed out in the paper, to a point where we can safely say that suitable gears are available that will give years of service with reasonable maintenance.

One of the most important auxiliaries in a steam-electric generating station is the boiler feed pump. With the small water storage in modern boilers, 100 per cent reliability of the boiler feed service is a necessity. Turbine-driven boiler feed pumps are generally installed but they are usually used as stand-by units. The boiler feed pump appears to be the auxiliary where turbine drive for normal service seems to be most desirable.

Furthermore, suitable adjustable-speed motors for large size high-pressure boiler feed pumps are not available. To meet this contingency many of these boiler feed pumps have been built to suit the motors available with very unsatisfactory results both from the standpoint of first cost and of maintenance.

The use of turbine drive for high-pressure boiler feed pumps permits the use of the best pump design and the horsepower of the turbines usually required permits of a highly economical design. They can be worked into the heat balance of the station in a number of ways. The turbine drive also permits of proper pressure control which is somewhat unsatisfactory with motor drive.

Electric motor drives for station auxiliaries have many advantages over turbine drives but the one serious drawback is the lack of satisfactory adjustable-speed a-c. motors.

The adjustable-speed a-c. motors and controls available today are abnormally expensive, do not have the desirable range in speed control, and are inefficient when operated at speeds considerably below synchronous speed.

To obtain the necessary range in speed for forced and induced draft fan drive it is often necessary to use two motors on the same shaft.

The lack of suitable adjustable-speed a-c. motors has resulted in the development of a complicated vane control for forced and induced draft fans so that a constant-speed motor can be used.

These makeshifts are a challenge to the motor designer but so far no satisfactory answer has been produced.

It is obvious that more development work must be done on adjustable-speed a-c. motors with regard to wider range of speed variation, particularly for stoker motors, draft fans, and boiler feed pumps. A possibility under consideration is a system based on induction frequency changer equipment in which the stator element is rotated at varying speeds thus controlling the frequency of the auxiliary motor supply.

The application of power tubes is also being investigated. With increased efficiency of rectifying equipments such as phanotrons, rectifiers, etc., it may become practical to provide direct current at the individual motor or group of motors, thereby obtaining the recognized advantages of the adjustable-speed d-c. motor. Power tubes may also be found useful in providing more steps, more evenly graduated, for controlling speed.

The many parts, such as rheostats, contactors, control relays, etc., required to control the adjustable-speed a-c. motor seriously reduces the reliability of the apparatus as a whole.

When the a-c. adjustable-speed motor is brought to approximately the same degree of reliability as the constant-speed motor we shall still have the problem of reliable supply at the motor.

The supply becomes less important as the unit treatment of turbine generator, boilers, and auxiliary equipment is approached. In this connection, it should be recognized that the auxiliaries may be the weak link. More systematic maintenance and the present trend toward increase in size, making necessary the subdivision of auxiliaries into units of half the total capacity, tend to reduce the possibility of complete shut-down of the units. Unit treatment of the auxiliaries further increases reliability where the station contains a number of units.

With regard to the auxiliary buses, it may be well to consider the use of reactors between bus sections, to insure continuity in the source of supply for driving the auxiliaries. When important bus sections are operated together, they should have bus relay protection.

In the matter of protection, surer and faster relaying systems are needed to insure proper selection of circuit or motor at fault. Attention should also be given to time-delay features on low-voltage releases.

The motors available today are really remarkable when compared with the motors available twenty years ago, but it should be distinctly understood that they are far from being developed to a point where they can be regarded as satisfactory. This is especially true of the adjustable-speed a-c. motor.

It may be necessary for the station designers to swing back to all-turbine drive for station auxiliaries in order to spur on the motor designers to produce a motor that will more nearly meet the requirements of the service.

The designers of motors and controls and the designers of station auxiliaries must work together so that the motor and control designers will be more familiar with the present and future requirements, and at the same time the designers of equipment requiring motor drives will design with the problems of the motor and control designer clearly in mind.

Francis Hodgkinson: The importance of reliability of the drives for the essential auxiliaries of a power station cannot be overemphasized. As far as possible, they should be immune from disturbances either within or without the power station. During disturbances to operation, whether they are electrical or otherwise, is a time when the operators are occupied with the difficulty. This is the one time when uninterrupted operation of the auxiliaries is most essential, at least until it has been concluded that the related main generating unit is no longer operable.

These are reasons which, years ago, justified the separate steam drive for auxiliaries, and later the development of separate house turbines with independent electric systems for the operation of motors driving the auxiliaries. It would appear nowadays, however, that a greater degree of reliability of general power station service, in some cases at least, justifies a reversion to the oldest practise of taking the energy from the prime mover either by means of the station bus or by a direct-connected house generator.

There is discussion in the papers of the relative reliability of steam turbines and electric-motor drives. Considering merely the drive itself, there would seem to be little to choose between them. Perhaps the choice depends upon the experience of the individual expressing an opinion. The simple and relatively inefficient types of turbines usually employed for auxiliary drives, together with reduction gears, may be highly reliable. If, however, high efficiency of the steam path is required, the gain in efficiency may, in instances, be secured at some sacrifice of reliability. With the types of small-capacity turbines used for auxiliary drives, efficiency and reliability are sometimes incompatible.

Certain auxiliaries, the boiler house fans and feed pumps, preferably operate at variable speed, as has been pointed out in the papers, when the steam turbine would seem to have the advantage because of the ease with which automatic speed controls may be provided.

Much of the benefits derived from operating the condenser circulating pumps at variable speed may be obtained by the installation of two pumps operating at constant speed, with the added advantage of reliability. One pump only is operated during cold weather, or in hot weather during an emergency, with a loss of about 0.2 to 0.3 in. vacuum from a normal vacuum of 29 in.

The choice of the type of auxiliary drive turns more on the general station layout, as pointed out by Mr. L. W. Smith. The problem is intimately associated with the feed-heating problem, including the feed-heating to be done by the economizer, and preheating of air for combustion, besides the question of the desirability of variable speed in instances previously discussed. Years ago, steam-driven auxiliaries were regarded as imperative in order that their exhausts might preliminarily heat the feed water, before admission to the economizer, to as high a temperature as possible consistent with not too great a loss of exhaust steam to the atmosphere during periods of light load on the main units. Admitting water to the economizer without this preliminary heating introduced difficulties with the economizer because of the deposition of tarry compounds and sulfurous acids on the cold surfaces and the deposition of some of the scale-forming compounds within the economizer instead of in the heater. With the advent of the more economical principle of extracting steam in stages from the prime mover for heating feed water, it became no longer necessary to employ auxiliary exhaust steam for this purpose, and the electric motor operation of auxiliaries came to be generally employed. Just as much a reason, however, for this change in practise was the desire to eliminate the complexity of steam and exhaust piping, the mess incidental to drains, etc., and to reduce the heat incidental to poorly-ventilated basements, where the auxiliaries are generally located.

When air pre-heating is employed, the greater portion of the feed-heating is accomplished by extracted steam from the prime mover, only enough economizer surface being installed to reduce the flue-gas temperature to the desired degree and provide air to the furnaces at the desired temperature. There are instances where the whole of the feedwater heating is accomplished by bleeding the prime mover, no economizer being furnished.

More energy per unit weight of steam is to be had from steam taken in stages at progressive pressures from the efficient main unit for feed heating than from the less efficient auxiliary steam units. A further advantage of the former method lies in the fact that some 20 per cent of the steam admitted to the main unit may be extracted, leaving that much less to go to the condenser. Therefore, the steam turbine may be given a correspondingly higher rating with given dimensions of low-pressure blading. This is important when one considers the high capacities of turbine generators at given speeds that are now available, which results in a problem for the turbine designer to secure sufficient low-pressure blade area with appropriate leaving loss commensurate with available capacities of generators.

House turbines have frequently been employed for furnishing steam for feedwater heating by means of their exhausts. An elaboration of this worthy of note is that employed in the Klingenburg Station of the Berlin Electricity Works, where the house turbines are arranged for stage bleeding. They exhaust into the first feed heater and are bled at two higher-pressure stages. The advantage is that inasmuch as the auxiliary load is nearly constant regardless of the load on any one prime mover, the feedwater temperature will vary much less than in the case of bleeding the main prime mover. In this instance, the house turbine generating set is coupled to an electric motor arranged to give and take energy to and from the main bus. By this means, an equable feedwater temperature may be easily maintained by the switchboard operator, by means of distance control of the house turbine governor.

It would seem that the foregoing considerations, in addition to the statements of the authors, generally lead to a preference for the electric drive for all essential auxiliaries except boiler feed pumps and fans.

H. M. Cushing: The study of the best method for driving power station auxiliaries has been a fascinating one. The trend of the development is clearly depicted in the three papers by Messrs. Hollister, Dryer and Smith. Mr. Dryer, in handling the subject of steam-driven auxiliaries, has made a very favorable case for the electric drive. Our points of view are naturally biased by our own personal experiences. The Huntley Steam Station, with which I am most familiar, started out with a preponderance of steam-driven auxiliaries. The stokers, forced draft fans, boiler feed pumps, hotwell pumps, circulating pumps, hurling water air pumps and exciters, were all equipped with turbine drives. The induced draft fans, a few miscellaneous ventilating fans and pumps, and the coal handling equipment were the only auxiliaries supplied with electric drive.

Although our engineers and operating men had been brought up where steam drive was used almost exclusively, they are now so convinced of the superiority of electric drive, that we have used it exclusively for driving the auxiliaries in the new 60-cycle steam plant, known as Huntley Station No. 2. All boiler feed pumps are motor driven. The only steam-driven auxiliaries in the plant are the small steam-driven lubricating oil pumps, which were furnished by the turbine manufacturer as a part of the equipment.

The general use of electric drive for auxiliaries has been greatly delayed by a number of causes, mostly attributable to the designers themselves:

1. They accepted motor accessories offered by the manufacturer which were of too flimsy design.
2. They accepted standard commercial equipment, using standard insulation where moisture proof insulation should have been used, and used open frame construction where totally enclosed motors would have better met the requirements. Most of the electrical auxiliary equipment in a power house should be drip proof, if not entirely moisture proof, so as not to be damaged by leaks in steam and water lines.
3. They made inadequate provision for protection of bus bars, from the standpoint of insulation and rupturing capacity. We will not permit the use of bare bus bars or bare bus connections for the main power buses, or the auxiliary buses in the station. All must be taped sufficiently to withstand the full operating voltage.
4. They followed the commercial practise of protecting their motors against overload. It is the duty of an operator to see that his equipment is not overloaded. The necessary instruments should be provided for this purpose. It is better to burn out a motor occasionally than to run the risk of a service interruption due to faulty relay operation.
5. They delayed the adoption of the across-the-line starting of motors with its simplified control.
6. They used the holding coil type of contactor for starting and controlling electric motors.

As late as 1928 a manufacturer with engineers specializing on power station controls recommended the holding coil type of contactors in the supply circuits to electric motors. These are utterly unsuitable for a service of this character. Many engineers, realizing the defect of the holding coil type of contactor, have used oil switches on their 220- and 440-volt motor circuits to gain the equivalent of the latched-in contactor. The oil switch is mussy; its contacts are not easily inspected. Small quantities of oil are to be avoided where possible. We believe that the recent development of the latched-in type of contactor is one of the big factors in improving the reliability of electrical equipment for power station auxiliary drives.

I do not understand Mr. Hollister's statement, to the effect that where the bin system of pulverized fuel supply is used, exhaust steam is usually required to dry out the coal, thus justifying the use of some steam-driven auxiliaries. The need for exhaust steam does not necessarily justify the use of steam-driven auxiliaries, for this steam can often be obtained more effectively when bled from the main turbines. It is true that with the bin system, pulverization peaks do not usually coincide with load peaks, and, therefore, the addition of some live steam might be required. Otherwise the question of the bin or the unit system of pulverized coal firing does not enter into the picture. It has been the writer's experience that steam is occasionally used to dry coal in both the bin and unit systems. It is used at Huntley No. 1, on the unit system, as an expedient, on account of the physical layout of the plant. It should not be a deciding factor in the choice between steam and electric drive for auxiliaries.

As Mr. Dryer says, reliability is the most outstanding requirement in the choice of drive for power station auxiliaries. He calls our attention to a point which is often overlooked in considering steam-driven auxiliaries; that is, the sudden dumping of a large block of load on the steam plant may seriously interfere with its ability to sustain proper operation of the steam-driven auxiliaries. This is especially true where the steam plant is on peak-load stand-by service. We disagree with his arguments, that a house alternator is necessary to get the proper reliability for the supply of electric power to the auxiliaries. We do agree that two or more separate sources of supply are necessary. These sources, however, may be generators in the same station, or widely separated stations.

In conclusion No. 1 Mr. Dryer states that "The turbine constitutes the simplest and most reliable drive." Our experience at Huntley Stations shows that electric drives are more reliable and easier to operate and maintain than the steam drives. The governors and overspeed devices of small turbines need frequent attention. Many of the original steam drives have been replaced by motors to reduce operating and maintenance expenses. Those adopting electric drives must realize that the class of service required by power house auxiliaries demands that the motors and their controls be built to rigid specifications, and the best of material and workmanship used in their installation.

We follow the practise of using special lock washers on all current, potential and control circuits, and use special insulation on the control wires, instead of the N.E. Code Standard. We thoroughly insulate all buses for the operating voltage.

Conclusion No. 5 advocates turbine drives especially for the induced and forced draft fans. Comparative figures for these two types of drive would be of especial interest if available. We believe that the electric motor lends itself more readily to the adoption of centralized controls, which many operators feel are necessary in our larger plants.

The designers of pulverized fuel plants realize the advantages to be gained by a system of complete interlock between the pulverizers, coal feeders and fans, which will prevent an operator from starting or stopping his auxiliaries in the wrong sequence. This might produce an explosive mixture of the coal and air, which is not present under normal operating conditions.

Conclusion No. 8 predicts that higher efficiency turbines may increase the turbine's advantage over the electric motor drive. We do not think so. The higher efficiencies are obtained by operating with smaller clearances, and we would expect that such a machine would require rather more than less of the operator's attention.

Mr. Smith advises that State Line has provided two 4,000-kw., shaft alternators and two 9,000-kw. banks of transformers (26,000 kw.) for the supply of their electric-driven auxiliaries, for 200,000 kw., of generating capacity. We would like to ask how much additional mechanical power is being supplied by the steam drives of the steam-driven auxiliaries.

At Huntley Station No. 2, there are seven 3,750-kva. transformers (26,250 kva. total) for supplying all the auxiliaries of this 160,000-kw. plant.

I would like to ask whether the percentage figures in Table I of Mr. Smith's paper are in per cent of installed horsepower capacity, or in per cent of their power consumption for a certain station loading, and if the latter, for what loading? Also, whether the figures include the spare equipment or just that which is necessary to carry the load?

The designer who believes in centralized control quite naturally turns to electric drive in preference to steam. With the large units, turbines and boilers which are going into our new power plants, it is not practical for the operator to go around to the different auxiliaries to determine how they are operating, so the designer has to bring the information which is required for successful operation of the equipment to some central point. It is then a very natural development to bring the controls to this same point. Having gone to the expense of bringing the boiler controls to a central point, it becomes a rather simple and inexpensive step to provide automatic control for the boilers. Several features regarding the auxiliary drives at Huntley Station No. 2, may be of interest in this connection. The substation supplying the power house auxiliaries is placed at the center of the load, between the boilers and turbines. The auxiliaries for each unit are split into two sections—one-half being supplied from one bus and the other half from another. Either bus may be interrupted without reducing the output of the unit more than 30 per cent even if the boilers in service are carrying maximum load at the time. The transformers serving these two supplies are connected to the generator leads between the generator and its main oil switch. These supplies are backed up by the auxiliary bus, which is supplied from the main bus bars. These main bars are connected to the outside system by means of two 110-kv. transmission lines and to the adjacent 25-cycle station, by means of a 20,000-kw. frequency changer set.

All auxiliaries in Huntley Station No. 2, are electric-driven except for the two auxiliary oil pumps, which were furnished by the turbine manufacturer as part of his equipment. All auxiliary motors of 15 hp., or larger, are supplied with 550-volt, three-phase 60-cycle current through air break circuit breakers and latched-in contactors.

The adjustable-speed drives for the coal feeders are supplied by Ward-Leonard sets. Miscellaneous small motors are supplied at lower voltages with their stepdown transformers, of the dry type, located near the motors.

F. C. Hanker: The discussion of the relative merits of steam-driven and electrically-driven auxiliaries has continued for a number of years. A review of the conditions as presented in the group of papers discussing the various types of drive is timely and will give the different groups of engineers the opportunity to analyze carefully the characteristics of the available types of apparatus and adopt those most suitable for the conditions existing on the various applications.

The requirements of the different auxiliaries vary from constant-speed operation to adjustable-speed conditions with ratios ranging from 2 to 1 for some applications to 5 to 1 for others and from a service that requires the highest order of reliability to that of the less essential needs.

For the constant speed applications all will agree that suitable designs of motors can be obtained. The development of the squirrel cage induction motor to its present state of efficiency, reliability and simplicity makes it well fitted for the most important applications. The use of full voltage starting has greatly simplified the control problem and made it possible to fulfill any specification that operating conditions impose.

When it comes to adjustable-speed drives the problem is not so simple. The simplest type of a-c. motor is inherently constant speed and a change from this characteristic necessitates the introduction of wound rotors with secondary control, the use of commutator type a-c. motors on some form of mechanical speed changing device where the a-c. power supply is adopted. The use of direct current enables the designer to take advantage of varying speed control through the use of this type of motor. This necessitates the use of commutators with the resultant maintenance of brush rigging and current collecting parts to which some operating engineers are opposed. The d-c. motor, however, has likewise been developed to a high degree of reliability but will usually be found to be higher in cost and difficult to justify economically. Any one of these alternates introduces complications that may be classed as undesirable but nevertheless can be made dependable. Close cooperation between the station designer and the designers of motors and control can greatly simplify the combined equipment over that installed in the past. A corresponding cooperation would be of advantage between the mechanical and electrical equipment designers to secure the best compromise in the inherent characteristics of each type.

I feel that a great deal of the criticism of electric drive is undoubtedly due to the effort to apply motors and control developed for industrial applications to the more rigorous duties imposed by central station conditions. In industrial work there is usually frequent opportunities for minor adjustment of control devices and current collecting parts. The motors are installed where the operating conditions are more suitable and the source of supply usually of smaller capacity. These conditions result in materially lower rupturing duties on the contact devices so that less sturdy equipment is needed to give successful service.

In comparing the industrial requirements with those found in the central station field, it will be found that the motors and control are required to function for weeks or even months at a time without a shutdown. For instance consider the auxiliaries of a large turbine unit; it will be found that such a unit may be in continuous service and obviously it would be undesirable to operate the control equipment at frequent intervals and interfere with the service of, for example, a 60,000-kw. prime mover. This condition is being rapidly improved by developments in contact details. It is also frequently desirable to provide special treatment for the winding insulation to make the equipments suitable for operation in places where coal dust or moisture results in more difficult operating conditions. The specifications for the equipment should be carefully reviewed by engineers familiar with the electrical conditions to insure satisfactory apparatus.

A great deal of attention has been given to the power supply for auxiliary drives. These have frequently been considered in connection with the heat balance of the station and have resulted in the installation of various schemes for controlling the demand on the main unit or some auxiliary source. Many operating engineers have required undue insurance in the question of power supply and this has necessitated complicated control layouts that have increased the cost of electrically-driven auxiliaries as compared with steam. The trend toward the unit system of operation should permit of simplification of both the power supply and the auxiliary drive as it will avoid complication in the sources of supply and thus simplify the control for the auxiliaries as well as minimize the duplication of auxiliaries as is frequently done in practise.

Robert Baker: In the papers by Messrs. Dryer and Smith it is brought out that for the plants on which they base their conclusions, and under maximum load conditions, an improvement in overall fuel economy of the order of 2 per cent will result incidental to the use of motor drive instead of turbine drive for all essential auxiliaries. As plants operate under maximum load conditions for a very small portion of the time, it would seem that comparisons should be made on the basis of between 30 and 40 per cent capacity factor, which would represent more nearly the actual operating conditions met by most generating plants. A comparison on this basis would probably show a reduced saving.

In Mr. Smith's paper there is a tabulation showing the approximate per cent of total station cost for the electric equipment to drive the auxiliaries in a modern station. No figures, however, are given as to the cost of the steam-driven equipment required for a station using steam-driven auxiliaries. Without these figures comparisons can only be made between steam and electric drive on the basis of fuel cost. Comparisons of this nature give only half the picture as an analysis of the overall operating costs of the average plant shows that the cost of fixed charges is approximately equal to the fuel cost. It is also obvious that the capital cost of steam drive for auxiliaries is less than the cost of an equivalent electric drive, when all factors are taken into account. The case then becomes one of balancing the saving in fuel cost obtained with electric drive, against the saving in fixed charges secured with steam drive.

Table II in Mr. Smith's paper gives the cost of electric equipment to drive auxiliaries as 7 per cent of the total station cost. On the basis of a 2 per cent saving in fuel when operating with this type of drive under maximum load conditions, which percentage would probably decrease when corrected to cover average operating conditions, and assuming that the cost of fixed charges on capital investment is about equal to the fuel cost, it can be seen that if the cost of steam equipment to drive auxiliaries is such that it reduces the station cost by 2 per cent, the savings in fuel realized by the more efficient electric drive are eaten up by a like increase in fixed charges. In fact a thorough study of a particular case might show that the saving in fixed charges realized with steam drive would amount to more than the saving in fuel with electric drive. On the basis, however, of a balance in overall operating cost between steam and electric drive the decision should be made in favor of the less costly installation as there would be no increase in economy to justify an additional capital outlay.

In order to compete with steam drive, the first cost of systems of electric drive must be reduced, and this can only be accomplished by a simplification of such systems.

Due to inexperience with electric drive, the earlier systems of auxiliary drive were necessarily complicated, having reserves pyramided on reserves, and still further complicated by automatic throw-overs, interlocking relays, dual drives and what not, all in the name of reliability. Experience has shown, however, that equal reliability could be obtained with a much simpler system of supply. I would point out, however, that the pendulum should not swing too far toward simplification as to affect reliability. In fact an auxiliary system should never be simplified to the point where it becomes less reliable than the station bus, but should be in every case of somewhat greater reliability, the degree to which it approaches this condition being controlled by the importance of that particular station with relation to the system as a whole.

Philip Sporn: To my mind the aim in auxiliary layouts for future power plants should be towards further simplification of the power supply and control equipment. When designers first went to electrical drives, it was natural to feel some hesitancy about their use and to resort to elaborate interconnections and steam stand-by drives. With increased confidence in this type of equipment, fully justified by experience, and with the reliability of supply due to interconnection of large systems, a great deal of this elaboration can be dispensed with. Interconnection particularly has not been taken full advantage of and many station designers are still thinking of individual power stations as if they were the lone forts in a deserted area which have to be capable of being self-sustaining under all conditions of system disturbances. As soon as we get away from that standpoint a great deal of duplication of facilities can be done away with to the ultimate advantage of the system.

There are certain fundamental considerations to be kept in view in making an auxiliary layout, and it seems to me that the requirements in most power plants are sufficiently similar so that only a very limited number of systems need be considered. In looking over various auxiliary layouts, one is struck by the fact that in a dozen different stations there are almost that many methods of providing for the auxiliary supply. Perhaps this has been due to the rapid development of electrical auxiliary systems in the past few years. The merits of the systems are now, however, fairly well recognized so that further standardization and simplification can take place. Designers have perhaps too often been tempted to make their layouts a little different from existing ones. In many instances this has resulted in marked improvement but very often originality was all that could be claimed for this procedure.

A great deal remains to be done toward the development of a simple sturdy adjustable-speed a-c. drive for those few station auxiliaries which require speed variation. Multispeed or double motor drives, adjustable-speed wound rotor motors and special combinations all have disadvantages due to elaborate special switching, poor efficiency, maintenance expense, and entirely too frequent breakdowns due to difficulty with the insulation. For example, when thirty breakdowns are experienced on six motors in a single plant in two years it surely indicates that something is still to be desired in the art of motor insulation. As regards the attainment of a simple and sturdy adjustable-speed a-c. motor, great hope is being placed now in the possibilities of development of vacuum tube devices, but of course it is too soon to tell whether the successful development of what appears a theoretical possibility will actually materialize.

The difficulties with insulation have, without a doubt, been brought about primarily by the fact that not rigid enough attention has been paid by the designers to the problem and by the fact that they have all too frequently been satisfied with making a design without being fully familiar with the conditions of service that the equipment had to encounter. More familiarity with true operating conditions would go a long way toward eliminating many of the difficulties.

It is interesting to note that while the electrical auxiliary drive started originally in low voltages and was gradually pushed up to the point where the great majority of the auxiliaries in large stations were being served at 2,300 or 4,000 volts, there has been a definite swing toward the lower voltages within the last few years in American practise. From studies we have made it appears that the horsepower limit for 440-volt auxiliary motors can be considerably raised over that now considered good practise. We have found that 500-hp. motors, for example, may be economically used at this voltage and only at 750 hp. is the use of 2,200 volts definitely indicated. The reason for that, of course, is primarily the increase in cost of the high-voltage motors. We have recently looked into the question of higher voltage for steam station auxiliary motors, considering voltages as high as 6,600 volts, but the price of motors for this voltage is so much in excess of that for the lower voltage motors that for the present at least it does not appear that any economies can be effected by their use. It is interesting, however, that in Europe 6,000 volts for the larger motors is quite standard practise and the engineers sponsoring that voltage claim that definite economies have been realized.

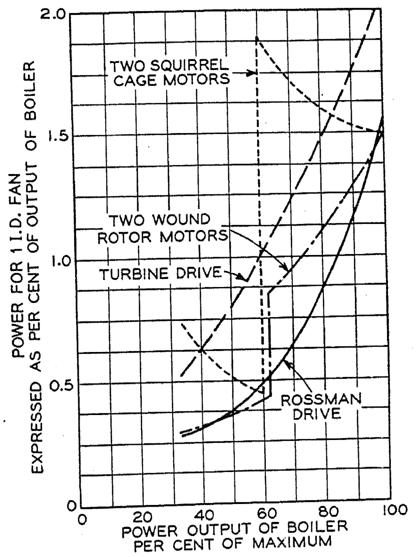

Fig. 1

S. M. Dean: It seems to me that in attacking the problem of developing a low-cost, sufficiently reliable auxiliary system, it would be well to look first to the source of power. If the main generating station switch-house electrical layout were such that each generator section was fairly well protected against voltage disturbances caused by major faults occurring in the other sections, and there were fortunately high-speed relaying and switching, very serious thought should be given to feeding the essential auxiliaries of each individual main turbine unit from transformers directly connected to the generator terminals. In doing this it would be recognized that should faults occur on a feeder or a bus of the section served by the particular generator that the voltage on the generator terminals would probably go so low that the induction motors driving the essential auxiliaries would slow down. But if the switching and relaying were of such character that a fault would be removed from the system in 15 to 20 cycles, the motors would probably come up to speed with sufficient promptness so that the operation of the main turbine unit would not be impaired. Of course, one would have to be very sure that disturbances on a given generator occasioned by troubles on the electrical section connected to its neighbors would not pull the voltage low enough to affect the auxiliaries.

It seems to me that such a method of attack shows distinct possibilities from the point of view of low first cost and acceptable operation. At least, that is the first attack that I should make on the problem.

I should also like to point out that while the papers are confined to steam and a-c. drives the possibilities in d-c. drives should not be overlooked. When account is taken of the extent to which the station rating is reduced (some 5 per cent), the smoothness and ease of operation, the absence of serious short-circuit problems, etc., it does not compare unfavorably with the other methods.

F. H. Hollister: Mr. Dryer has shown some interesting curves comparing the performance of steam turbines and induction motors when driving various types of auxiliaries. He has also pointed out the objection of transitional disturbances which accompany the use of slip-ring motors for driving draft fans.

There are, of course, efficient types of electric drive which do not have this objection, such as the d-c. motors, the a-c. brush shifting motor and the Rossman drive. The use of d-c. motors for fans is largely confined to stations having a d-c. auxiliary power system. Where such is the case the d-c. motor provides a satisfactory method of operating fans. The a-c. brush shifting motor has likewise given satisfactory service in a number of stations, but is limited in size to approximately 25 hp. per pole which would mean 200 hp. at 900 r.p.m.

The Rossman drive, which is a combination of a-c. and d-c. drive,* has been in successful operation driving fans at the Powerton Station for the past year. This drive has also recently been placed in service at the Philo and Sheboygan stations. Two 2,500-hp. Rossman drive units are now under construction for boiler feed pumps.

Fig. 1 has been prepared from Fig. 9 of Mr. Dryer's paper on which the performance of the Rossman drive has been superimposed. The performance of d-c. motors and a-c. brush shifting motors would be similar to the Rossman drive.

Fan power data have recently been received from the Operating Department of the Powerton Station, which may be of interest in view of the Rossman drive being relatively new. A comparison has been made between the power input to the fans for two similar boilers, one equipped with two-speed squirrel cage motors, and the other with Rossman drives. For a run of 30 days the boiler equipped with two-speed squirrel cage motors averaged 639 lb. of steam for one kilowatt-hour fan input, and the boiler equipped with Rossman drives for the same period averaged 853 lb. of steam for one kilowatt-hour fan input. This represents a saving of approximately 33 per cent power input to the fans with Rossman drive, and confirms estimated figures given in Mr. Rossman's paper. The cost for each of these types of fan drive and control, when installed at Powerton, is approximately $30.00 per horsepower.

F. M. Stoddard: All auxiliaries at the Columbia Power Station are electrically driven except two boiler feed pumps. The power for the essential auxiliaries is supplied from shaft generators connected to each main turbine unit. Transformers connected to the main bus supply the non-essential auxiliaries and serve as a reserve for the shaft generators. In six years of operation there has been no trouble with the auxiliary system from

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*A *New System of Speed Control for A-C. Motors*, by A. M. Rossman, A.I.E.E. Trans., Vol. 50, March, 1931, p. 162.

loss of auxiliary power or fluctuation of bus voltage from external faults. On the whole, the operation of electrically driven auxiliaries has been very satisfactory.

The use of electrically driven auxiliaries facilitates operation of a power station in the following manner:

1. It provides convenient control of boilers from one central place.

2. It provides a simple and convenient method of interlocking pulverized fuel equipment for proper starting sequence or other forms of interlocks.

Steam provides a somewhat more reliable power supply particularly for isolated stations. A station using all steam-driven auxiliaries would require a separate turbine-driven lighting generator to be consistent.

Wellen H. Colburn: Probably no part of a steam generating station is so subject to difference of opinion, or open to discussion and variation in design, as the auxiliary drive. Both complete steam and complete electric drive, as well as a wide range of combinations between, and several methods of providing steam and electric supply have their protagonists.

As Mr. Hollister mentions, too little is known about relative maintenance costs of the various schemes but as pointed out these data are difficult to get on a comparable basis. However, maintenance costs are not all and data on overall operating costs are equally desirable and equally difficult to obtain on a comparable basis. More comprehensive costs and the detail of their makeup is desirable.

Mr. Dryer makes an attractive case for "all turbine drive" of auxiliaries but appears to over-emphasize the importance of reliability of auxiliaries, unless his paper is to apply to isolated stations only. However, duplication of generating equipment or interconnection with other stations or systems, as Mr. Smith points out, makes loss of a generating unit from failure either of itself or auxiliaries of lessening importance.

Marginal gains in economy in steam-electric generation are henceforth to rule and these, to be of value, must be gained without a complete offset in increased capital costs.

This factor is reflected in the main unit as well as in the auxiliaries. No analysis is complete which considers the economics of auxiliaries without taking account of the effect of the resulting heat balance diagram upon the initial cost and efficiency of the main turbine. Neither Mr. Dryer nor Mr. Smith appear to have given this consideration.

One of the overlooked items relating to turbine-driven auxiliaries both in design and operation appears to be the steam loss necessary to warm up turbines of larger sizes and better efficiencies required with auxiliaries for modern large generating units. Items of this nature account for discrepancies between practise results and design figures.

Mr. Smith makes a less positive case for electric drive apparently because he recognizes more variables in accomplishing the desired ends.

The point that failure of the main unit makes the auxiliaries unnecessary deserves a corollary. If the main unit can be spared when it fails, we may conclude that it can be spared when its auxiliaries fail. This permits much economy and simplification by non-duplication of auxiliaries and station piping. With such unit design still further economies in electric operation become evident by elimination of house sets and complicated switching equipment. At the same time the growth in size of units, which Mr. Smith points out results in auxiliaries of such size or so located as to require duplicate drives of half the total capacity, tends to minimize the possibility of complete shut-down of a unit by loss of an auxiliary.

That the adjustable-speed a-c. motor leaves much to be desired is too well recognized to be debatable. New designs and methods of adjustable-speed operation with a-c. power supply, possibly with vacuum tube or mercury rectifier applications, are and will be forthcoming which will obviate some, if not all, of the present objections.

It is fairly well recognized that the pendulum swings too far. It is probable that the trend to electric drive of auxiliaries has gone too far and that the economic mean lies between all steam and all electric drive. However, the correct proportion cannot be evaluated without more careful study of the economics of design and operation, taking into account the whole cost of all related factors. Such data are only partially available and undoubtedly will require modification from time to time as equipment designers provide new and more adaptable electric and steam drive and utilization equipment.

John H. Lawrence: There cannot be one general answer to the problem of making a decision between electric and steam drive, any more than we can say that 400, 600 or 1,400 lb. is the correct steam pressure for any station. For instance, a steam plant to act as standby for a hydro plant must be treated differently from a steam plant which operates continuously. Likewise a station to supply an Edison system such as in New York must be treated differently from one supplying the average a-c. system.

There is no doubt about the electrical system gaining the supremacy immediately following the introduction of stage bleeding. This was the logical conclusion as was evidenced by the number of stations which adopted the electrical drive. However, the manufacturers of auxiliary turbines, seeing their markets being cut down, awakened to the situation and as a result the efficiency of the small turbine has been considerably improved, so much so, that today it is a mooted question as to whether steam or electric drive should be used. I would say that the greater the reliability is desired the more steam auxiliaries should be used. The less the reliability, then more electrical auxiliaries should be used. Not that one system is more reliable than the other, but because to get equal reliability the electrical system is more costly.

The heat efficiency of a station with steam auxiliaries can be brought within 1 per cent of the station with electrical auxiliaries, or about one-half the difference quoted by Messrs. Dryer and Smith.

Assuming that many of the auxiliaries, such as circulating pumps, coal pulverizers, etc., should run at constant speed, the advantages coming with adjustable speed are lost. The circulating pumps use considerable power and a large saving can be made by slowing down the pumps as the temperature of the water drops. Also it has been found that variations in characteristics of coals justify a variation of speed to get the best results from some coal pulverizers. If the advantages of adjustable speed are neglected, the efficiency of the electric drive may show up very favorably, but the heat efficiency of a station with properly designed steam drive should show very little loss over electric drive, certainly not over 1 per cent.

I believe the cost of the electric drive with its prorated share of the station cost will be so much in excess of the station with the steam drive, that for the average generating plant steam will be found to be more economical in overall costs.

Assume that the auxiliary system for each system will have the same cost beyond the bus with electric drive or beyond the boiler with steam drive. To be fair to the electrical system assume that the capacity of the boiler plant with the steam drive system must be increased 1 per cent, although it is likely that the units would be overloaded by this amount. This would increase the station cost by about one-half of one per cent. On the other hand if power is taken from the main bus for the auxiliaries, the prorated cost of the turbine room and electrical gallery with equipment should be charged against the electric drive. If this part of the station is worth $50.00 per kilowatt, then if the auxiliaries require 5.5 per cent of the total output, this percentage should be charged against electric drive, for with steam drive this power is available for sending out on the system. In other words, 2¾ per cent of the total station cost, if the total cost is

$100 per kw. is chargeable against increased cost of the turbine room and electrical gallery with equipment.

Deducting the ½ per cent excess cost of boiler plant with steam drive, the excess first cost of an electric drive station is 2¼ per cent more than a steam drive station.

With equally good designs of the two systems, I do not believe the efficiency of the electric drive can be improved to an extent that this extra cost can be justified unless the capacity factor is higher than usually found in a central station.

H. L. Wallau: The Ashtabula Plant of The Cleveland Electric Illuminating Company exemplifies the use of both steam and electric drive for the power supply to station auxiliaries.

Referring at this time, only to those auxiliaries which are directly linked with the operation of the boilers and turbines, the drives used are as follows:

Boiler feed pumps, steam turbine drive.

Primary air fans, squirrel-cage induction motor drive.

Induced draft fans, two-speed induction motor drive.

Fuel feeders, adjustable-speed d-c. motor drive.

House service pumps, two turbine drive and two induction motor drive.

Circulating pumps, two per unit, induction motor drive supplemented by steam turbine (dual) drive on one unit of each pair.

Condensate pumps—two per unit, same types of drives as circulators.

Exciters, (steam) shaft drive from main unit.

Spare exciter, dual turbine—induction motor drive.

Fuel feeder motor supply, two generators, induction motor drive, one supplemented by steam turbine (dual) drive.

All dual-driven apparatus is set to cause the turbine to pick up the load on the equipment at a 3 per cent reduction in speed. Electric power for the motor drives is obtained from bus transformers.

Boiler feed pumps are steam turbine driven to obtain maximum reliability of power supply, regulation being accomplished by excess pressure governors. All of the other essential auxiliaries are normally motor driven. However, as noted, the next most essential auxiliaries have been partially provided with ancillary steam drives, so that failure of electrical supply will not result in total loss of load.

The relatively small amount of steam taken by the feed pumps, neglecting radiation losses, is used at 100 per cent thermal efficiency, the exhaust being used to heat the feed water by means of a jet type barometric heater condenser. Except for the additional heat furnished the condensate by the generator air coolers and steam air ejectors, the balance of the heat supplied (about 75 per cent at full load) is obtained by bleeding at the 13th and 15th turbine stages. This combination of drives is a compromise between Figs. 3 and 4 of Mr. Dryer's paper, more nearly approaching Fig. 4, and has yielded a net annual overall thermal efficiency somewhat in excess of that set up by him. The total use of electricity for all plant purposes was 4.2 per cent of the amount generated.

In our plants the turbine-driven and dual-driven pumps and d-c. generators are all located in the turbine room basement. This simplifies the piping and facilitates operation. If the second set of condensate and circulating pumps connected with each generator had been equipped with steam instead of motor drive, the heat balance would be upset when these pumps are operated on account of the additional exhaust steam supplied to the heater condenser.

Since draft fans cannot be efficiently grouped at one location, piping for these, if steam driven, would have to be carried to many points. This diversity of location is disadvantageous to steam drive as compared with electric drive. Another disadvantage of steam drive, though not a major one, is the necessity of using reduction gearing. Reliability is of less importance for draft fans than for boiler feed pumps, because the loss of a draft fan, while possibly jeopardizing service, does not threaten the destruction of expensive equipment as well.

In our newer plants, the reliability of any given draft fan is of lesser importance than that of a condenser pump, because either of the two boilers associated with a given generator can furnish steam enough to carry 80 per cent of full load on that unit.

The use of turbo-vane draft fans coupled to two-speed pole-changing induction motors furnishes all necessary variations in draft and permits the use of the low-speed low-power rating of the motors for the greatest percentage of the operating time at good efficiencies. The simplicity of full-voltage starting, induction motor drive for the primary air supply is an obvious advantage. Steam drive for the pulverized fuel feeders was at no time considered.

The turbine auxiliaries have been referred to previously. Except at high loading coupled with high summer temperatures of condensing water one set of pumps only is needed, and the dual-driven one employed. Our operating experience over many years has proved that constant-speed equipment is entirely satisfactory for this service.

Of the four house service pumps, two are steam driven for maximum reliability. Had dual-drive been used on all or part of these, the first cost would have been higher, and if only three pumps had been installed, the reserve less.

Mr. Smith has pointed out the relatively small percentage of total investment involved in the cost of electric equipment for auxiliary drive and outlines some of the possible methods for reducing these costs still further.

One of these is the use of house transformers connected to the generator leads ahead of the generator breakers. This does away with breakers on the high side of the transformer and also lends itself admirably to the strictly unit method of operation of main units and their auxiliaries. A supply from another source is, of course, necessary at the time of starting a unit. This method requires one bank per generator and hence more space than if a smaller number of banks of larger individual capacity is used, which latter scheme requires bus transformers and consequently breakers.

At Ashtabula one transformer bank, bus connected, is used to supply the auxiliaries for two turbines and four boilers. Normally each bank feeds an independent section of truck type board. These sections, four in ultimate number, are physically separated one from another and adjacent ones may be electrically joined through two-section tie breakers, one on each board, connected together by a relatively short cable run. Thus the failure of a tie breaker cannot involve more than one section of board. No provision has been made to ring the bus by connections between the first and fourth sections. Spare trucks are provided for all positions so that maintenance or repair work does not interfere with the functioning of any group of auxiliaries.

A transformer bank connected to a given bus section normally supplies the dual-driven condensate and circulating pumps of each of the two turbo-generators associated with it, while the straight motor-driven sets of pumps are supplied from another section. Hence, even when both sets of pumps are in use, a failure of electric supply results in the loss of but one set on each generator.

The four boilers associated with a group of two generators have their primary air fans and induced draft fans fed from a sectionalized 2,300-volt truck type boiler supply board, with two boilers fed from each section. Each section is fed by means of one circuit capable of carrying the load of the four boilers. One of these circuits issues from the section of main supply board associated with the turbines of this group, and the other from another section. Normally both circuits are energized and the tie breaker open. In case of deenergization of a circuit, it is cleared and the tie breaker closed automatically, thus maintaining continuity. Interlocks prevent more breakers from being closed at one time.

Another 2,300-volt truck board, not sectionalized but having duplicate feeds from two independent sections of the main board, supplies the coal handling and preparation equipment. Normally one circuit is open at the receiving end, and loss of the other results in a temporary interruption of power supply to these nonessential auxiliaries until manual throw-over is accomplished. The supply breakers on this board are adjacent and interlocked so that but one may be closed at any time.

Motors below 40 hp. are supplied at 440 volts and distribution centers adjacent to the groups so supplied are established. This is also done with respect to the d-c. fuel feeder motors.

Circuit breaker and stop-valve motor control is obtained from a d-c. 120-volt control bus on which two batteries (468 ampere-hours each at one hour rate) float, trickle-charged through diverter-pole generators.

Normal lighting service is alternating circuit three-wire. In case of failure, a number of key circuits is automatically thrown over to d-c. supply, a neutral for the 230-volt d-c. system being at all times maintained through balancer-sets. A further automatic throw-over to battery supply keeps one-half of these key-circuits operative in case of simultaneous or subsequent failure of the normal d-c. source, (d-c. dual-driven generators).

The above methods of auxiliary supply have been in use for several years at the Avon Plant, Ashtabula including some refinements, providing greater flexibility for inspection and maintenance.

Note the following points:

The sectionalized operation of both the main, 2,300-volt supply and the boiler group boards.

The use of a single bus in all boards.

The use of factory fabricated switching equipment, truck type boards for 2,300 volts and Urelite factory built board for 440 volts. This too is sectionalized.

The availability of spare trucks for any position.

The physical isolation of the various sections of the main 2,300-volt supply board, and of the boiler group supply boards.

The use of two section-tie breakers in series in connection with the main board sections.

The flexibility and reliability obtained for both operation and maintenance by the interlacing of supply circuits and the use of dual-driven equipment at critical locations.

With a few exceptions, use full voltage starting for all a-c. motors.

We believe that the supply scheme used results in a high degree of reliability, operating flexibility, and low installation and maintenance costs.

R. C. Powell: In the matter of reliability, the writer's experience has shown that the reliability of electric drive depends almost entirely upon the reliability of the auxiliary power source. A house generator is impracticable in a station operating at 1,250 lb. steam pressure where all the boilers are reheat boilers, that is, a plant of unit power design, and for such a plant the most reliable auxiliary supply is the shaft-driven auxiliary generator. This would be very reliable were it not that the main turbine is very liable to go out on overspeed following a heavy disturbance. Prevention of tripping on overspeed is a matter of such importance both in the matter of main and auxiliary power supply that manufacturers and operators of steam turbines should make every effort to correct this defect.

The electric drive has considerable advantage in steam plants operated as part of a system which at certain seasons is supplied principally from water power plants, but when even during the plentiful water season most of the steam units are floated on the line to insure service. In such older plants with several boilers per turbine, it is possible to practically shut down the boilers and operate the auxiliaries from water power. In the newer plants with very large boilers and larger turbines, a minimum load of 6 or 7 per cent of maximum must be carried per turbine unit in order to insure safe operation and a fast pick-up of load. At certain times, then, more steam power is generated than is actually required and water that could otherwise be used is wasted. In this case the electric drive is preferable to steam since about one-third of this minimum steam power will be required to operate the auxiliaries, with an appreciable saving in fuel as against steam drive.

As mentioned in the papers, most of the drives may now be at constant speed. This will enable the station designer to use synchronous motors and make some savings in cost with a gain in efficiency.

There is no doubt that the steam drive has attractive features, but the writer agrees with Mr. Smith that the electric drive has on the whole inherent advantages even if there is much to be desired in the matter of cost.

F. W. Gay: The impression left from the papers relating to station auxiliaries is that steam-driven auxiliaries have a more reliable source of power than electric-driven auxiliaries. It would appear that insufficient stress has been laid on the fact that electric-driven auxiliaries may receive power from their own generating station or from electric networks having in themselves great inherent reliability. On the other hand, it is very seldom possible for steam-driven auxiliaries to receive steam from any other station and the transfer of a steam-driven auxiliary from one steam service to another in the same station is not generally so conveniently accomplished as with electric drive.

Supplementing this inherent advantage peculiar to electric drive is the additional advantage with the unit system of supply that upon a prolonged failure of any one generator the auxiliaries for that unit are no longer required. The point has been raised by inference that short circuits on generator buses or feeders, especially in the case of the unit system, may reduce the voltage on the auxiliaries for that unit so as to require their being thrown over to another source of power. No mention has been made of the fact that systems have been devised whereby auxiliary power may be taken from a generator and the voltage of the auxiliary source of supply may be maintained at substantially 100 per cent, while the voltage on the station bus is dragged down to zero by a short circuit. Such a system was thoroughly investigated by Professor Nims and described in the *Electrical World* for April 25, 1931. In the system described by Professor Nims, it is necessary to employ a large amount of transformer capacity and since this transformer capacity is a substantial item for electric auxiliary drive this system is expensive, although probably not as expensive as a shaft end generator. On the other hand, if the unit system of auxiliary drive is employed and the generator supplies its power to a high-voltage bus through a direct-connected transformer it becomes possible to take the auxiliary power from tertiary windings on the main transformer bank.

With substantially no additional expense it is possible to assemble the secondary winding and the tertiary, each in separate inductive relation with respect to the primary, so that the primary has a high equivalent negative impedance with respect to the tertiary for loads on the secondary. Under these conditions the tertiary voltage is over compound, that is, it rises with heavy inductive loads on the secondary and with primary voltage maintained. With a proper design the rise on the tertiary windings may be made to compensate for the drop in generator voltage so that for all loads on the transformer secondary windings up to and including a dead three-phase short circuit on the secondary windings, the voltage on the tertiary winding will be maintained at a serviceable value.

While such a transformer design will not seriously affect the straight through impedance between primary and secondary, it will give a relatively high impedance between primary and tertiary windings and it may be found desirable to equip the tertiary windings with induction voltage regulators and control equipment to maintain a substantially constant voltage on the auxiliary supply bus for all loads on the transformer secondary windings up to and including a dead three-phase short circuit

While induction voltage regulators will not be fast enough to avoid lamp flicker on auxiliary power circuits for sudden changes in generator loading, they should be able to correct for minor changes in the auxiliary supply voltage so fast that the speed of auxiliaries even when driven by slip-ring motors with control resistance in circuit should not vary much over a fraction of one per cent.

A. T. Brown: The writer believes that greater progress could be made if instead of dealing in generalities, careful and detailed studies could be made. After proper reliability has been secured, choice as to the type of drive to be selected for various auxiliaries in a power station, which are intimately connected with the main units, should be determined from the overall costs of operation, including all factors.

It is obvious, if cost is not considered, that from an operating point of view electric drives present many advantages. However, the writer believes that it would be a great advantage to the art of designing power stations if someone would undertake a complete and detailed study covering all-electric, part-steam and part-electric direct steam drive, and great steam drive for auxiliaries for, say, stations of 400 lb., 650 lb., and 1,500 lb. of steam. I have never seen such a detailed study, and it would not be an easy thing for busy engineers to prepare, as it would require practically the designing and plotting performance curves for several entire power stations.

If such data can be secured, I believe points at issue between the advocates of electric and steam-driven auxiliaries can be reduced.

In the last part of Mr. L. W. Smith's paper, it is apparent that he realizes the great cost of motor drives. This is apparent from an inspection of the table on page 393 where it is stated that of the total station cost, the motors are 1.2 per cent of the total, and this appears at the site of the auxiliary drive application. What does not appear at the site is the 5.8 per cent and this apparently does not include the cost of the building space occupied by the control panels, switches, etc., which is often considerable.

If the building cost mentioned above is included, it is safe to say that each motor carries behind the scenes approximately five times its cost in equipment to serve it.

In a large modern power station, which burns, say, $2,000,000 worth of coal annually, a fuel saving of 1 per cent is well worth accomplishing.

If the fixed charges on the capital invested in a power station may be taken as roughly of the order of 90 per cent of the fuel cost, a saving in capital cost of the order of 1.10 per cent will accomplish the same saving as a 1 per cent fuel saving.

If then we take the table referred to above as a fair statement of the cost of electric auxiliary drive, we can see that in order to accomplish this result we must reduce the cost of the behind-the-scenes auxiliary electric equipment in the neighborhood of 18 per cent.

It seems as though this could be done by careful planning and we urge electrical engineers to see what can be done in this direction.

In Mr. W. P. Dryer's paper, the statement is made that if the draft fans are steam driven the boiler evaporating capacity must be increased 0.7 per cent over electric drive. Was this figure obtained from a carefully calculated heat balance, carried all the way through? For unless the loss of heat to the circulating water was increased in the case of steam-driven draft fans, as compared with electric drive, it seems that this figure of 0.7 per cent may give us an erroneous impression.

George A. Orrok: Early practise in this country has always inclined towards the steam auxiliary, while European practise has been always along the lines of bus-driven auxiliaries. The middle ground between the two systems where the auxiliary power is furnished by an auxiliary turbine not connected to the main bus except in emergency was developed to attain the advantages of both systems.

The introduction and development of the regenerative system with its almost universal adoption at the present time has again emphasized the essential differences of the two types of drive. We must also differentiate between types of plant such as the larger industrial installations, with the demand for process steam at low pressures and the central station plant where, due to the regenerative system, no large use of exhaust steam is possible.

The question of reliability is so intimately bound up with the operating policy of the plant, the size and character of the load, the outside interconnections, and the number of units in the plant that it is impossible to give any general solution for the question. The difference in economy between the two types of auxiliary apparatus is not large, possibly of the order of one-half per cent on the overall thermal efficiency of the station, and it has become increasingly evident that all of these factors must be taken into consideration, and the solution for any given plant will not necessarily be a general answer in any sense.

It is of course evident that the smaller number of links between the coal pile and the user represents the most reliable operation, and it is also evident that no solution in which the operation of the auxiliaries is likely to be upset by a short circuit on the outside system is of advantage to a central station.

Modern auxiliary units have been developed to such a state that their reliability in all cases is equally good and the task of the designer has been to use the smallest number of links which with equal cost will give a substantially equal efficiency.

Nevertheless, individual problems must still be settled in an individual way taking into account all of the many factors which make for successful operation and such papers as these, which set forth in such a detailed manner the advantages of competing types of apparatus, serve a most useful purpose.

L. W. Smith: Messrs. Kehoe, Cushing, and Stoddard have brought out two additional advantages of electric drive, namely, it provides a convenient and accurate method of measuring auxiliary power and a simple method of obtaining interlocks for the operation of pulverized fuel equipment.

In answer to Mr. Cushing's questions concerning the station used in the paper as an example of electric drive, it may be noted that there are no steam-driven auxiliaries except boiler feed and fire pumps which are used only as reserves. There is a total connected load of approximately 25,000 hp. in motors including reserves which is equivalent to approximately 9.3 per cent of the station output. The per cent power consumption given for each class of auxiliaries is the average for a period of two weeks during which the station had a maximum net output of 189,000 kw. and average net output of 126,500 kw., which gives a station load factor of 0.669. The apparent excess in auxiliary transformer capacity at the station is largely due to the fact that all of the coal handling equipment is served from one transformer bank. The other transformer bank serves as a reserve for the shaft generators and auxiliary transformers. Subsequent units will not require proportional amounts of transformer capacity. Incidentally, the auxiliary power for units 2 and 3 will be served from transformers connected ahead of the main generator oil circuit breakers instead of by auxiliary shaft generators.

The comments by Messrs. Baker, Lawrence, and Brown are related in that they all touch on the relative investment cost of steam and electric drive. Numerous cost tabulations could have been prepared for steam and electric drive in connection with these papers, but it is doubtful if they would have any value except for the assumed conditions. There are so many variables entering in station design that each case must be worked out on its own merits. The tabulations in the paper were included chiefly for analyzing the distribution of auxiliary power and the cost of the component parts of an electric auxiliary system.

It has been pointed out in the paper that the relative economics of auxiliary drive hinge to a large extent on what constitutes a reliable system of auxiliary power supply and what is the actual water rate of auxiliary turbines. The paper on the East River Generating Station by Messrs. Grady, Lawrence, and Tapscott referred to in the bibliography is of interest in connection with this phase of the problem. The paper on the East River Station states that a comparison of the investment costs between various auxiliary power schemes showed that an a-c. electric supply with shaft generators, reserve house generator, and bus transformers would cost 35 per cent more than steam turbines, and that a d-c. system with storage battery reserve would cost 120 per cent more than turbines. A capitalized credit of 2 per cent is given for fuel saving in the case of a-c. supply and 3 per cent for d-c. supply. It is also interesting to note from this paper that the steam consumption for the auxiliaries at the East River Station is somewhat higher than the original calculations. The conclusions arrived at by the study of the various schemes of auxiliary power for the East River Station are no doubt correct for the particular conditions that confronted the designers, but these conclusions can not be applied to other station designs without a similar analysis. In most cases the number and type of reserve auxiliary power sources could be reduced with an appreciable saving in investment cost. Likewise any major change in the station design might alter the picture as far as the type of auxiliary drive is concerned.

The comparative reliability of electric drive has not been emphasized in this paper because the real question is whether or not electric drive is sufficiently reliable for central station service. The increasing use of electric drive with its increasing simplification of power supply methods would indicate that electric drive does provide a reliable, convenient, and economical method of operating steam power station auxiliaries.

Standard Decrement Curves

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Synopsis.—*For circuit breaker and relay application, it is frequently necessary to determine the decrement in power system short-circuit currents. For this purpose certain "standard decrement curves" approved by the National Electrical Manufacturers Association have been in general use. The original curves appeared in 1918 and a revised set was prepared in 1923. New decrement curves given in the present paper supersede those formerly in use. The paper also includes a discussion of the need for new curves, the underlying assumptions upon which they are based, the method of application and their limitations.*

INTRODUCTION

For circuit breaker and relay application, it is frequently necessary to determine the magnitude of power system voltages and currents under fault conditions for various intervals of time measured from the instant of the application of a fault. The decay of fault current from initial to sustained values is commonly determined by means of decrement curves. In 1918 a standard set of decrement curves was published for the first time.[1]

The standard decrement curves of 1918 were derived on the basis of the available information on the decrement characteristics of machines and systems. By 1923 considerable additional data had been collected, and accordingly, the curves were revised. These curves, published in the *N.E.L.A. Relay Handbook* (1926) and in various manufacturers' publications are superseded by the new standard decrement curves, Figs. 1 and 2, published for the first time in the present paper. They have been approved by the National Electrical Manufacturers Association. The presentation of these curves is the principal object of this paper, but it also reviews the need for new curves, the underlying assumptions on which they are based, their application, and finally their limitations.

In the past eight years, there has been considerable progress in the analysis of transients in synchronous machines. Furthermore, the method of symmetrical components has come into general use for the calculation of unbalanced faults. These developments in theory, together with the increased knowledge of machine constants, have made desirable the preparation of a new set of decrement curves.

The 1923 decrement curves assumed a single decrement for the a-c. component of fault current. The new decrement curves take into consideration the fact that the a-c. component is made up of two exponential terms and the fact that the transient time constant varies with system reactance. The method of using the curves includes a means to give more accurate results in case the time constants of machines differ appreciably from the assumed average figures upon which the curves are based.

The 1923 curves include three separate sets, one for each type of fault, namely, three-phase, line-to-line, and line-to-ground. The new curves include only a single set which, however, is applicable to the same three types of faults. This is feasible because the new curves are based on the method of symmetrical components. The old curves assume equal initial values of line-to-line and three-phase currents whereas the present curves are plotted for the more usual ratio of 0.866. The old curves for line-to-ground faults were restricted to the particular case of a fault on the terminals of one or more generators, all the neutrals of which were solidly grounded; the new curves are not restricted in this respect.

In the 1923 curves, no distinction was made between machines with and without damper windings. The curves included in the present paper have been designed to apply particularly to salient pole machines with amortisseurs and to turbo-alternators. Therefore, they give inaccurate results for the first 0.2 seconds for salient pole machines without damper windings and are supplemented by a method of calculation which gives the initial value of short-circuit current accurately for these machines.

DERIVATION OF THE STANDARD DECREMENT CURVES

The important differences between the new and the old decrement curves have been outlined above and the derivation of the new curves is given in Appendix I. The basic assumptions, being of more general interest, are given below.

1. The generators are assumed to be operating at rated voltage and kva. at 80 per cent power factor immediately preceding the short circuit.
2. No automatic voltage regulator is used.
3. The actual system subjected to fault may be represented by a single equivalent generator of the same total rating and an external reactance.
4. The load is assumed to be located at the machine terminals, and the machine reactance is taken as 15 per cent unless the total reactance is less than 15 per cent. In this event all the reactance is assumed to be in the machine.

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*General Electric Co., Chicago, Illinois.

†Westinghouse Electric & Mfg. Company, East Pittsburgh, Pa.

[1] For references see Bibliography.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

5. The short circuit is assumed to occur on an unloaded feeder.

6. The short circuit is assumed to occur at the point of the voltage wave which corresponds to maximum possible instantaneous current.

7. All resistance in the circuit including the resistance of the fault is neglected.

8. All the machine e.m.f.'s are assumed to be in phase.

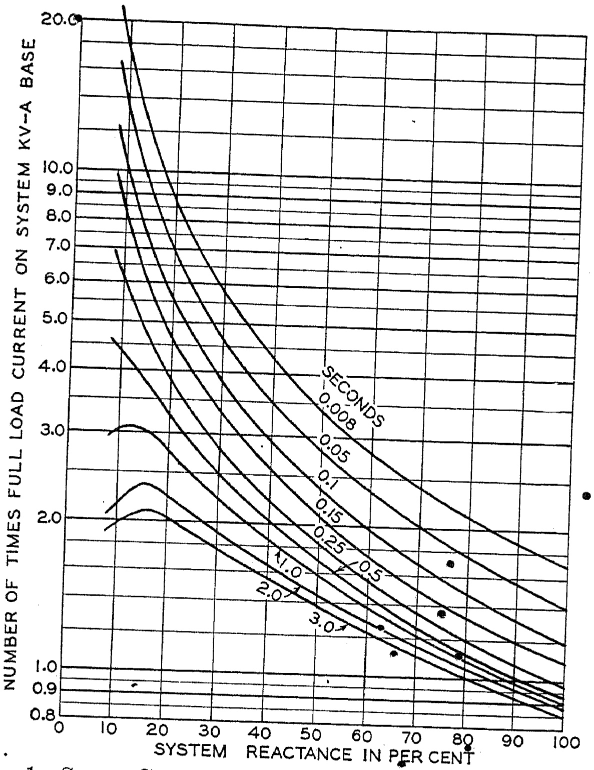


FIG. 1—SHORT-CIRCUIT CURVES FOR SYNCHRONOUS MACHINES

System reactance *must be* based on system kva. and *not* on the particular kva. chosen as a base for calculations.

System kva. is connected synchronous capacity in kva.

Note: These curves are based on $Tdo' = 5$ sec.

For other values of Tdo', multiply actual times by $\frac{5}{Tdo'}$, to get equivalent times to use in curves

How to use curves:
- For three-phase short circuit—
  - Reactance—Use system reactance to the point of fault
  - Times full load (normal) scale—use scale reading
- For line-to-line short circuit—
  - Reactance—Use two times the system reactance for the three-phase fault
  - Times full-load (normal) scale—multiply scale reading by $\sqrt{3}$
- For line-to-ground short circuit—
  - Reactance—Use $(2x_1 + x_o)$ or

$$\frac{3 \times \text{(initial three-phase short-circuit current)}}{\text{(initial line-to-ground short-circuit current)}} \times \text{system reactance for three-phase fault}$$

  - Times full-load (normal) scale—multiply scale reading by 3

9. The machine reactances and time constants are taken as representative of modern machines. Of particular importance is the relation between transient and subtransient reactance of machines. (See Fig. 3.)

APPLICATION OF STANDARD DECREMENT CURVES

Description of Decrement Curves. The decrement curves of Figs. 1 and 2 give the r.m.s. total current expressed in terms of "times normal current" with total connected synchronous capacity in kva. as a base. In Fig. 1 reactance is plotted as the abscissa with elapsed time from the beginning of the fault as a parameter for the curves. In Fig. 2, the elapsed time is the ordinate and reactance is the parameter.

The reactance to be used with the curves for any kind of a fault may be obtained either by means of analytical calculations, or by use of the calculating board, or indirectly by the substitution in the formulas on Figs. 1 and 2, if the initial values of symmetrical short-circuit currents happen to be known from other sources. In order to choose the proper decrement curve, it is essential that the reactance used to select a curve in Fig. 2 or a line in Fig. 1 be expressed as a per cent of the *total connected synchronous capacity in kva.* rather than as is common practise, an arbitrary value conveniently chosen to expedite system calculations.

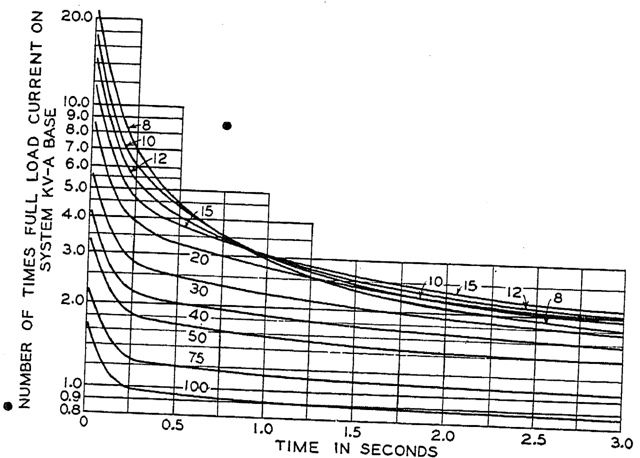


FIG. 2—SHORT-CIRCUIT DECREMENT CURVES FOR SYNCHRONOUS MACHINES (REACTANCE = 8 TO 100 PER CENT)

How to use curves—see Fig. 1

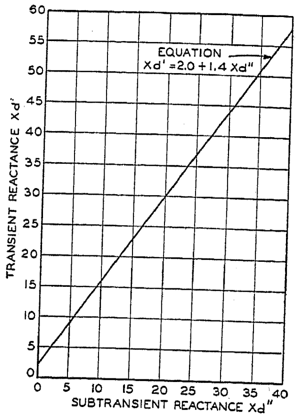
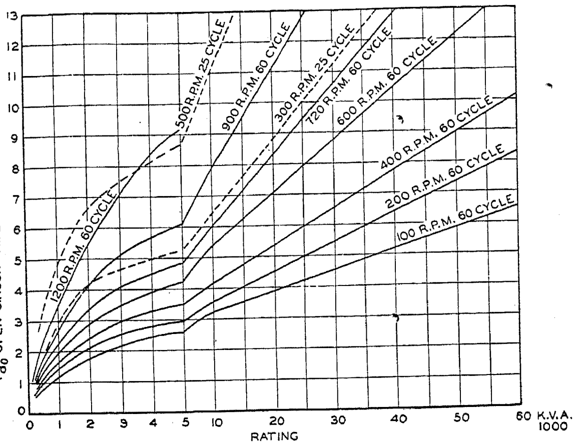


FIG. 3—RELATION BETWEEN Xd' AND Xd'' (SATURATED VALUES) FOR THREE-PHASE SYNCHRONOUS MACHINES

If the system open circuit time constant referred to the point of fault is known to be other than 5.0 sec., the time scale of the curves may be corrected to conform to y means of the formula given on the curve sheets.[4,5] The problem in Appendix II illustrates the use of the ves when the initial short-circuit kva. and connected chronous kva. are known.



G. 4—Open-Circuit Time Constants of A-C. Generators and Motors

current have been fully developed in other papers and are not discussed here.[6-11] In case synchronous machine reactances or time constants are not known Table I and Figs. 4 and 5 may be used as a guide in selecting reasonable values.

The system reactance referred to the point of fault enables the proper decrement curves to be selected which gives the three-phase r.m.s. short-circuit current or kva. in "times normal" at any time after the occurrence of the short circuit, subject to the assumptions previously given.

Line-to-Line Short Circuit. For a line-to-line short circuit the fault current is determined by the use of the standard decrement curves of Fig. 1 or Fig. 2 in the same manner as the three-phase short-circuit current by using a reactance equal to twice the system reactance (the value used for three-phase short-circuit calculation) and multiplying the resulting current in times normal read from the curves by $\sqrt{3}$.

The above rule results from the theory of symmetrical

TABLE I*—TYPICAL THREE-PHASE 60-CYCLE SYNCHRONOUS MACHINE CONSTANTS

	x_d	x_d''	x_2	x_0†
urbo generator	$\frac{1.10}{0.95-1.45}$	$\frac{0.12}{0.07-0.17}$	$= x_d''$	$\frac{0.03}{0.01-0.14}$
alient pole motors and generators (with amortisseurs)	$\frac{1.10}{0.60-1.45}$	$\frac{0.22}{0.13-0.35}$	$= x_d''$ (nearly)	$\frac{0.06}{0.02-0.20}$
aterwheel generators (no amortisseurs)	$\frac{1.10}{0.60-1.45}$	$0.85\ x_d'$	$\frac{0.50}{0.30-0.70}$	$\frac{0.07}{0.04-0.22}$
ondensers	$\frac{1.80}{1.50-2.20}$	$\frac{0.25}{0.18-0.38}$	$\frac{0.24}{0.17-0.37}$	$\frac{0.08}{0.02-0.15}$

Note—x_d' for waterwheel generators $= \frac{0.35}{0.20-0.45}$ (saturated)

x_d' for turbo generators and salient pole machines with amortisseur from Fig. 3 (saturated)

*For test data of specific machines, refer to Bibliography item 14.

†x_0 varies so critically with armature winding pitch that the average values given are not very dependable.

Three-Phase Short Circuit. The reactance to be used o select the proper decrement curve for a three-phase ault, is what is commonly spoken of as the "system eactance" referred to or viewed from the point of fault. t is the reactance used with the decrement curves ormerly published. When determining this reactance t is important that synchronous machines be represented by their subtransient* reactances, and also that *all loads other than synchronous be neglected.* Proper account has been taken of these loads for the average system in deriving the decrement curves.

Means for obtaining the system reactance referred to the point of fault, and the corresponding short-circuit

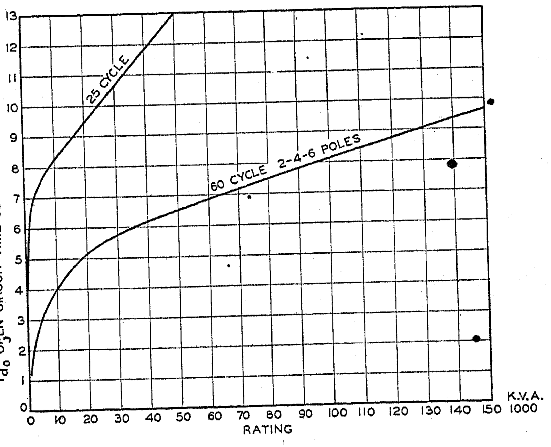


Fig. 5—Open-Circuit Time Constants of Turbine Generators

components,[7,8,9,11] assuming the negative phase sequence reactance of the system as measured from the point of fault to be equal to the system reactance x'. Where the negative phase sequence of the system is known, more accurate results may be obtained by using a reactance equal to the sum of the system reactance x_1 and the negative phase sequence system reactance x_2 instead of twice the system reactance x_1. The negative phase sequence reactance of the system is obtained in the same manner as the system reactance x_1 except that the negative phase sequence reactance of the machines is substituted for the subtransient reactance of machines.

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*The "subtransient" reactance is the reactance to use in determining the r.m.s. initial value of the a-c. component of short-circuit current. It includes the effect of induced currents in damper windings, etc., which have a very rapid decrement. The transient reactance on the other hand determines the a-c. component when the rapidly decaying component is neglected while the synchronous reactance is used to determine sustained values of current.

Line-to Ground Fault. For line-to-ground faults, the decrement curves of Fig. 1 and Fig. 2 may still be used, provided that the ratio of the instantaneous values of three-phase to line-to-ground fault currents or the proper value of reactance is known. For this case, the reactance may be taken as three times the system reactance $(3\, x_1)$ multiplied by the ratio of the three-phase to the line-to-ground short-circuit currents. Ordinarily the line-to-ground short-circuit current must be calculated by the method of symmetrical components or the reactances for the different sequences used as outlined below. The line-to-ground current at any time will be equal to three times the value of "times normal current" read from the decrement curves using the above values for system reactances.

The value of reactance to be used with the curves for a line-to-ground short circuit may be determined, using the method of symmetrical components, the reactance being taken as the sum of the positive, negative and zero sequence reactances of the system $(x_1 + x_2 + x_0)$ as measured from the point of fault. For ordinary calculations it is sufficiently close to use a reactance equal to $(2\, x_1 + x_0)$, where x_1 is the system reactance used for three-phase short-circuit calculations and x_0 is the zero phase sequence reactance to the system. The zero phase sequence reactance may be obtained in a manner similar to that used for the determination of the system reactance for three-phase short circuits by changing the reactances in accordance with the following rules.

Ungrounded transformers...... = ∞
*Grounded Δ-Y transformers. = ordinary reactance to neutral on Y side and open-circuited on Δ side
Neutral impedance............. = 3 × actual impedance in neutral
Single-circuit aerial lines (no ground wires).................. = 3.5 × ordinary reactance
Double-circuit aerial lines (no ground wires)................. = 5.5 × ordinary reactance of double line
Single-circuit aerial lines (copper or aluminum ground wires).. = 2.0 × ordinary reactance
Double-circuit aerial lines (copper or aluminum ground wires).. = 3.0 × ordinary reactance of single line
Cables, three-phase........... = 3 to 5 × ordinary reactance
Cables, single-phase.......... = 1.0 × ordinary reactance

For auto-transformers, grounding transformers and other special apparatus, refer to papers and articles which discuss the determination of the zero phase sequence impedance in more detail.[8,12,13]

Minimum Fault Current. For certain relay applications it is sometimes necessary to know the total r.m.s. alternating fault current, *i. e.*, with no d-c. component, rather than the maximum current at any time as given by the standard decrement curves which are based on an asymmetrical fault. The initial r.m.s. alternating (symmetrical) currents for values of system reactance are given in Table II.

TABLE II—INITIAL R.M.S., ALTERNATING (SYMMETRICAL) CURRENT

Reactance on decrement curve..........	8	10	12	15	20	30	40	50	75	100
Initial symmetrical current.............	12.3	9.96	8.38	6.88	5.11	3.37	2.52	2.01	1.33	1.0

As the asymmetrical component is negligible after 0.3 second, the r.m.s. alternating (symmetrical) current for any time less than 0.3 second may be obtained by interpolating between the initial value obtained from the above table and the curve values at 0.3 second. Satisfactory results can readily be obtained by sketching in a curve starting from the initial r.m.s. alternating (symmetrical) current and making it tangent to the corresponding standard decrement curve at 0.3 second.

R.M.S. Total Current for Salient Pole Machines without Dampers. Where salient pole machines without damper windings supply the short-circuit current the standard decrement curves give good results after 0.2 second, provided that for these machines the equivalent subtransient reactances are obtained from the known transient reactances by means of the relation given in Fig. 3, but the initial values indicated by standard decrement curves may be 30 per cent too high. If more accurate values of initial current are desired, it is necessary to make a separate calculation of the system reactance using the actual value of the subtransient reactance of the salient pole machines without damper windings instead of the equivalent value obtained from the transient reactance by means of the relation given in Fig. 3. A curve may readily be sketched between the new initial point and the 0.2 second point on the standard decrement curve corresponding to the equivalent subtransient reactance of Fig. 3.

The subtransient reactance of machines without damper windings has generally been taken as equal to the transient reactance. However, there is an appreciable subtransient component in the short-circuit current of such a machine due to the damping action of the pole rivets and other closed circuits and also due to the rapidly disappearing effects of saturation. The test data available indicate that the subtransient reactance of a machine without damper windings should be taken as about 85 per cent of the transient reactance.

Limitations of the Standard Decrement Curves. There are certain limitations in the application of standard decrement curves that should be recognized. The decrement curves with the supplementary methods will give safe values for circuit breaker application, except possibly in unusual cases. However, for relay application, it is frequently insufficient to know that the current does not exceed a particular maximum value but instead it is important to know the minimum current values under given fault conditions. In deriving the curves, it was assumed that the system could be replaced by a single equivalent generator. If the initial currents supplied to the fault by the several machines are not in proportion to their ratings, then the division of the sustained currents will not be proportional to the initial current division. Furthermore, the total current will actually be considerably smaller, for example, if a large part of the initial short-circuit current is supplied by a machine whose rating is a small fraction of the total kva. the curve may give a sustained value that is more than 100 per cent too high. It has also been assumed that the system is fully loaded, so that, if the decrement curves are used in the case of a system which is only partially loaded, it is evident that the sustained current will be less than the values given by the curves. It has further been assumed that all of the generated e.m.f.'s are in phase. Departures under operating conditions from this assumption result in smaller values of current.

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*Replace by circuit which permits current to flow from Y side to ground but not on Δ side.

When voltage regulators are used there will be a tendency to increase the current over that given by the decrement curves. For the first 0.2 second the fault current will ordinarily be unaffected but the sustained current may be increased in proportion to the ratio of the maximum exciter voltage to the nominal exciter voltage corresponding to rated voltage and kva. at 80 per cent power factor. For ordinary excitation systems, *i. e.*, where quick response systems are not used, the sustained currents may be increased as much as 50 per cent by the action of the voltage regulators.

In general, for systems whose characteristics differ radically from the assumptions upon which the curves are based, it may be necessary to use the more accurate methods of calculation such as described by W. C. Hahn, and C. F. Wagner and S. H. Wright.[2,3]

ACKNOWLEDGMENTS

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Appendix I

DERIVATION OF CURVES

In deriving one set of decrement curves to represent the decrement of a large number of different systems, it it necessary to assume a typical system as a basis for the curves. The typical system assumed is shown in Fig. 6. The value of the load impedance is that which corresponds to full load at 80 per cent power factor on the system.

The initial r.m.s. value of short-circuit current is usually calculated for a system assuming that the system is operating at no-load and that the terminal voltage of each machine on the system is unity. However, the decrement curves apply to a fully loaded system. Thus the decrement curves must include the effect of system load though based on a reactance corresponding to the no-load condition. In order to properly include the effect of load in the decrement curves the amount of internal and external reactance in the system must be known. Neglecting the load impedance the system reactance x_1 is

$$x_1 = x_d'' + x_b \tag{1}$$

For the previously published decrement curves[1] it was assumed that all of the reactance was in the generator for values of x_1 less than 15 per cent. For values of x_1 greater than 15 per cent, 15 per cent was assumed to be in the generator and the rest to be external. This assumption was intended to represent the average system condition and is believed by the authors to be as accurate an assumption as can be made. The same assumption is, therefore, made for the present decrement curves.

The initial r.m.s. a-c. component of short-circuit current may most easily be calculated by employing the superposition theorem. This theorem applies to a network composed of impedances which are independent of the voltage across them and the current through them. It states that the sum of the currents at any point in the network due to two or more sets of terminal conditions applied one at a time, is equal to the current at that point with all the terminal conditions applied together. In the present case, the curves are calculated for a three-phase symmetrical short circuit. By proper conversion factors, (given in the body of the paper, Figs. 1 and 2) the line-to-line and line-to-ground short-circuit currents may be obtained from the three-phase short-circuit curves. The symmetrical three-phase short circuit is assumed to take place on an unloaded feeder which is at normal voltage. In order to satisfy this condition before short circuit and to supply the load on the bus to which the feeder is connected, normal voltage must be applied in series with the short circuit and sufficient voltage applied behind the reactance of the generator to produce normal voltage at the feeder bus. The current which flows under this condition is the load current, no current being in the feeder to be faulted.

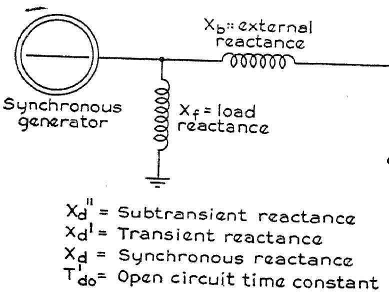
X_b = external reactance
Synchronous generator
X_f = load reactance
Short circuit
X_d'' = Subtransient reactance
X_d' = Transient reactance
X_d = Synchronous reactance
T_{do}' = Open circuit time constant

FIG. 6—TYPICAL SYSTEM

All these voltages are now removed and a voltage equal to normal but opposite in direction is applied at the point of fault to determine the short-circuit current flowing in each branch of the network. The sum of the short-circuit current and the load current in each branch is the total current in that branch under short-circuit conditions.

In the derivation of the curves only the current in the faulted feeder is required. Since this feeder is unloaded the subtransient current under short circuit has no-load component and can be calculated in the following way. The load impedance is combined in parallel with the generator subtransient reactance and added to the external reactance to the point of fault. The reciprocal of this total reactance is the current desired since normal (unity) voltage is applied at the point of fault to determine the short-circuit current.

Let i'' = subtransient current in the fault

then

$$i'' = \frac{1}{x_b + \dfrac{x_d'' x_f}{x_d'' + x_f}} \tag{2}$$

The transient a-c. component may be calculated in the same manner by replacing x_d'' by x_d'.

Let i' = transient current in the fault

then

$$i' = \frac{1}{x_b + \dfrac{x_d' x_f}{x_d' + x_f}} \tag{3}$$

The load impedance, x_f, should include both resistance and reactance to represent the load, but in the present case, the inclusion of the resistance affects the result less than 1 per cent, so that x_f is taken as a pure reactance of 1.66 representing the reactance part of an 0.80 power factor load. For the 8, 10, 12 and 15 per cent curves $x_b = 0$. For the rest of the curves, $x_b = x_1 - 15.0$ in per cent. Values of i'' calculated from equations (2) and (3) are given in Table III. Values of x_d' corresponding to assumed values of x_d'' are calculated from the relation.

$$x_d = 1.4\, x_d'' + 2.0 \text{ in per cent} \tag{4}$$

which is based upon a large amount of data on turbine generators and salient pole synchronous machines with amortisseurs.

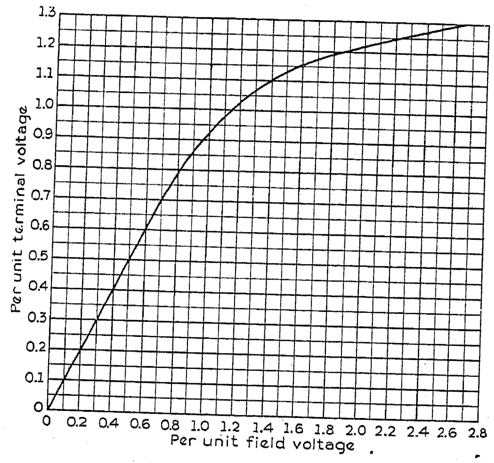

FIG. 7—SATURATION CURVE

The sustained currents (i) are calculated from the Potier diagram, correcting for saturation by adding the difference in excitation taken from the actual no-load saturation curve and that taken from the air-gap line to the excitation obtained neglecting saturation. The saturation curve used is given in Fig. 7. The value of Potier reactance was taken as 1.3 times the subtransient reactance of the machine, as representing an average condition. The load resistance was included in calculating the excitation as it has an appreciable effect. These values of (i) are also given in Table III.

The short-circuit time constant of a machine without saturation is given by Park and Robertson as:

$$T_d' = T_{do}' \frac{x_d' + x_e}{x_d + x_e} \tag{5}$$

In the present case, the external reactance x_e is to be taken as x_d in parallel with x_f.

Oscillograms of short circuits on actual machines show little departure from a true exponential after the effects of amortisseurs and the d-c. component have died away. Therefore, the effects of saturation, if any, are included in the subtransient effect. Inasmuch as the transient reactance is found by extrapolation, back to zero time, from the latter part of the curve, no correction for saturation need be made on the above time constant.

The open-circuit time constant T_{do}' is assumed as five seconds. The formula for the curve connecting transient and sustained current is then

$$i_t = i + (i' - i)\, e^{-\frac{(x_d + x_e)}{(x_d' + x_e)} \frac{t}{5.0}} \tag{6}$$

The time constant assumed for the subtransient component of current for all the curves, 0.05 second, is an average value corresponding to $T_{do}' = 5$ sec. The formula for the subtransient component of current is

$$i_{st} = (i'' - i') e^{-\frac{t}{0.05}} \tag{7}$$

The sum of the subtransient component (7) and the current from equation (6) is the r.m.s. a-c. component (symmetrical part).

The curves are plotted for the maximum possible offset or d-c. component. The representative time constant assumed for this component is 0.15 sec. Then the d-c. component at any time is

$$i_{do} = \sqrt{2}\, i''\, e^{-\frac{t}{0.15}} \tag{8}$$

The square root of the sum of the squares of this and the r.m.s. a-c. part is the r.m.s. total current, values of which are given in Table IV. It will be noticed that instead of values for zero time, values at $t = 0.0083$ second are given, corresponding approximately to the current at the end of ½ cycle, inasmuch as the value at zero time has no practical significance.

ERROR CAUSED BY ASSUMPTION OF EQUIVALENT SYSTEM

In some cases, the decrement is wanted for a short circuit on a bus to which generators as well as a tie line from a large system are connected. In most cases, the current from the generators can be separated from the current from the large system and the decrement curves applied separately. This gives fairly accurate results. In case a reactance is between the bus and the short circuit, separation of the currents may be difficult. An approximate method which may be used is to apply the decrement curves to the generator and tie line currents separately as before and add them. This results in less error than applying the decrement curves to the total short circuit, but may still be considerably in error. It is relatively difficult to predict the actual error in a large number of cases, such as, the division of short circuit between several machines not on the same bus. Better approximation in cases of this kind will be given in other papers.[2,3]

ASSUMED DISTRIBUTION OF LOAD

The decrement curves are based upon the assumption of a short circuit occurring upon an unloaded feeder as shown in Fig. 6. That this assumption is not productive of a large error is evident from the following:

Since zero external reactance is assumed for the first four curves, for these curves the division of the load between the faulted feeder and the other line feeders is immaterial, as the fault is in effect a generator bus fault with all the load connected directly to this bus. For the

TABLE III

x_s in per cent	8	10	12	15	20	30	40	50	75	100
x_b in per cent	0	0	0	0	5	15	25	35	60	85
x_d'' in per cent	8	10	12	15	15	15	15	15	15	15
x_d' in per cent	13.2	16.0	18.8	23.0	23.0	23.0	23.0	23.0	23.0	23.0
i''	13.1	10.6	8.9	7.3	5.3	3.5	2.6	2.0	1.35	1.01
i'	8.2	6.8	5.9	5.0	4.0	2.84	2.20	1.81	1.24	0.95
i	1.86	1.90	1.92	1.97	1.85	1.61	1.44	1.29	1.01	0.83
T_d'	0.57	0.69	0.82	1.00	1.16	1.43	1.64	1.80	2.11	2.31

TABLE IV

X_s in per cent	8	10	12	15	20	30	40	50	75	100
time in seconds										
0.0833	21.4	17.3	14.5	11.9	8.73	5.79	4.28	3.35	2.23	1.69
0.05	16.6	13.3	11.2	9.37	6.92	4.68	3.49	2.80	1.87	1.40
0.10	12.3	10.2	8.66	7.20	5.56	3.82	2.90	2.34	1.57	1.19
0.20	8.03	6.94	6.07	5.19	4.18	2.98	2.33	1.91	1.32	1.01
0.40	5.18	4.83	4.44	4.02	3.39	2.56	2.06	1.72	1.21	0.935
0.70	3.73	3.72	3.60	3.44	3.00	2.36	1.94	1.64	1.18	0.92
1.00	2.97	3.09	3.08	3.06	2.74	2.22	1.86	1.59	1.15	0.91
1.5	2.32	2.48	2.54	2.63	2.42	2.04	1.75	1.52	1.12	0.89
2.0	2.06	2.18	2.26	2.37	2.22	1.93	1.67	1.46	1.10	0.88
2.5	1.94	2.05	2.10	2.20	2.09	1.83	1.61	1.42	1.08	0.87
3.0	1.90	1.97	2.02	2.11	2.01	1.76	1.56	1.39	1.06	0.86

rest of the curves, the assumed external reactance is high enough in most cases so that only a small proportion of the total load can be carried through it on account of regulation. More exact methods are given in other papers[2,3] for cases in which it is believed that the effect of load position should be taken into account more accurately.

ALTERNATE FORMS OF DECREMENT CURVES

In Fig. 1, the decrement curves are replotted with system reactance as abscissas, number of times full load current as ordinates, and time in seconds as the parameter. In some cases, the choice of time is somewhat arbitrary while the system reactance is calculated, so that this form of presentation may be more convenient. The formulas for using the curves are the same as in Fig. 2.

Appendix II

SAMPLE PROBLEMS

The following numerical example is given to illustrate the way in which the general decrement curves are to be used. Let it be assumed that the following data have been obtained either analytically or by use of a calculating board and that the system voltage is 66 kv.

The total connected synchronous capacity is 5,500 kva.

The initial r.m.s. alternating component three-phase short-circuit current is found to be 480 amps. or 55,000 kva.

Initial r.m.s. alternating component of line-to-ground short circuit 570 amperes or 65,000 kva.*

The system reactance is then

$$\frac{5{,}500}{55{,}000} \times 100 = 10 \text{ per cent}$$

The three-phase short-circuit current at 0.3 sec. is desired.

Referring to Fig. 2 and following along the 10 per cent reactance curve to 0.3 second time it is found that at this time the short-circuit current or kva. is 5.6 times normal. Thus the short-circuit kva. at 0.3 sec. is

$$5.6 \times 5{,}500 = 30{,}800 \text{ kva.}$$

Since the short circuit occurred on a 66-kv. line, the corresponding short-circuit current is

$$\frac{30{,}800}{66 \times \sqrt{3}} = 269 \text{ amperes}$$

If the equivalent open-circuit time constant of the system is 3.0 seconds instead of 5.0 seconds, then the time to be used with the curves corresponding to 0.3 second is in accordance with the formula given on the curve sheet.

$$0.3 \times \frac{5.0}{3.0} = 0.5 \text{ sec.}$$

For this value of time a value of 4.3 times normal is read from the curve, and the three-phase short-circuit kva. at 0.3 second becomes

$$4.3 \times 5{,}500 = 23{,}600 \text{ kva.}$$

For a line-to-line short circuit the reactance to be used with the curves is twice the system reactance used to find the three-phase short-circuit kva. It is

$$10 \text{ per cent} \times 2 = 20 \text{ per cent}$$

This reactance at 0.5 sec. on the curve of Fig. 2 gives 3.3 times normal, which when multiplied by $\sqrt{3}$ is 5.72 times normal. The magnitude of the line-to-line short circuit at 0.3 second is then

$$5.72 \times 5{,}500 = 31{,}500 \text{ kva.}$$

The line-to-ground short-circuit reactance is found from the formula on the curve sheet as

$$\frac{3 \times 65{,}000}{55{,}000} \times 10 = 35.5 \text{ per cent}$$

The line-to-ground short-circuit kva. is found by interpolation for 0.5 sec. to be 2.2 times normal which multiplied by 3 gives 6.6 times normal as the line-to-ground short-circuit kva. at 0.3 sec., *i. e.,*

$$6.6 \times 5{,}500 = 36{,}300 \text{ kva.}$$

Since this fault occurred on a 66-kv. line the line-to-line ground short-circuit current at 0.3 sec. is

$$\frac{36{,}300}{66 \times \sqrt{3}} = 317 \text{ amperes}$$

As a second illustration, a system having only waterwheel generators without amortisseur windings will be considered. The system has a total synchronous capacity of 8,000 kva., a transient reactance of 38 per cent, of which 15 per cent is external, and an open-circuit time constant of 5 seconds. The three-phase short-circuit current at 0.1 and 2.0 sec. is desired.

The curves of Fig. 2 give the decrement after 0.2 second quite accurately if the proper transient reactance is used to select the curve. Referring to Fig. 3 the subtransient reactance corresponding to a machine transient reactance of 23 per cent is 15 per cent. The system equivalent subtransient reactance is

$$15 \text{ per cent} + 15 \text{ per cent} = 30 \text{ per cent}$$

Using the 30 per cent curve of Fig. 2, the times normal value at 2.0 sec. is found to be 1.9. Thus the three-phase short circuit is

$$1.9 \times 8{,}000 = 15{,}200 \text{ kva.}$$

By referring to Table I it is found that the subtransient reactance of a waterwheel generator without an amortisseur is about 85 per cent of its transient reactance. So this equivalent waterwheel generator machine has a subtransient reactance of

$$23 \times 0.85 = 19.6 \text{ per cent}$$

Adding to this the external reactance the system subtransient reactance which determines the initial short-circuit current is obtained.

$$19.6 + 15 = 34.6 \text{ per cent}$$

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*The short-circuit kva. for a line-to-ground fault is equal to $\sqrt{3}$ times the product of fault current and system voltage.

By interpolation between the curves of Fig. 2, the initial three-phase short circuit is found to be 4.8 times normal. From the curve sketched in connecting this value to the 30 per cent curve at 0.2 sec. it is found that the three-phase short-circuit kva. at 0.1 sec. is 3.7 times normal. It is

$$3.7 \times 8,000 = 29,600 \text{ kva.}$$

The short-circuit currents may be found as in the preceding problem.

NOMENCLATURE

x_d = direct axis synchronous reactance (unsaturated)

x_d'' = direct axis subtransient reactance (saturated)

x_d' = direct axis transient reactance (saturated)

x_2 = negative sequence reactance

x_o = zero sequence reactance (saturated)

T_{do}' = open-circuit time constant (unsaturated)

Discussion

R. G. Lorraine: The standard decrement curves presented by Messrs. Hahn and Wagner are based upon the reduction of the actual system subjected to fault to a system consisting of a single equivalent generator and an external reactance. Relations are assumed between the reactances of this equivalent system such that, when the total reactance of the system to the point of fault is known, all of the reactances necessary to determine the decrement curve are specified. In any system to which the decrement curves are applied, the relations between the reactances will probably not be those which were assumed for the standard decrement curves. Therefore, it seemed desirable to investigate the effect of changing these relations.

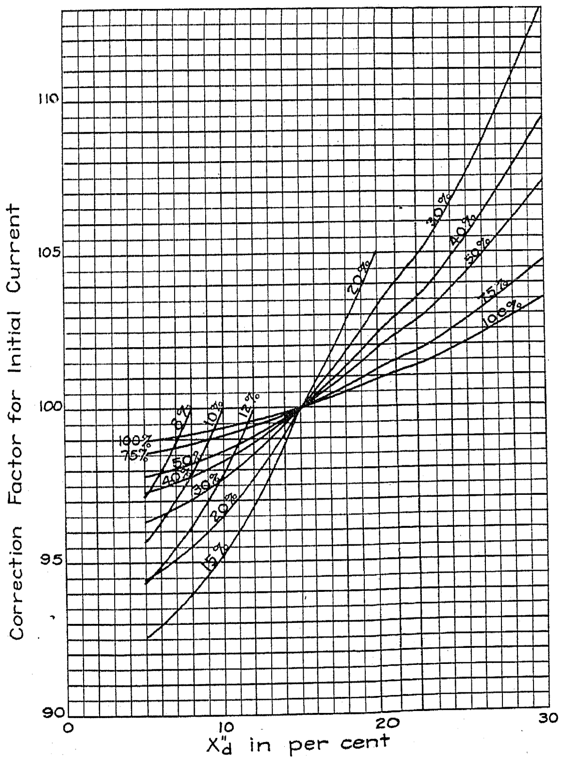

FIG. 1

Based upon the method of derivation given in Appendix I of the paper, three families of curves were developed. One family of curves, Fig. 1, shows the per cent initial current for a specific system based upon the initial current of the corresponding general decrement curve system. The variation in this initial current depends upon the relative magnitudes of the subtransient reactance of the equivalent generator and external reactance which make up the system subtransient reactance. This family of curves is used to determine the departure in the initial current.

Figs. 2 and 3 are of somewhat different character. They show values of transient and synchronous reactance respectively corresponding to a given subtransient reactance, which the equivalent generator must have in order to have approximately the same transient and sustained components of short-circuit current as the related standard decrement curve. An examination of these curves shows that there is a large number of system conditions in addition to those assumed which will produce the same transient and sustained short-circuit current as the standard decrement curves.

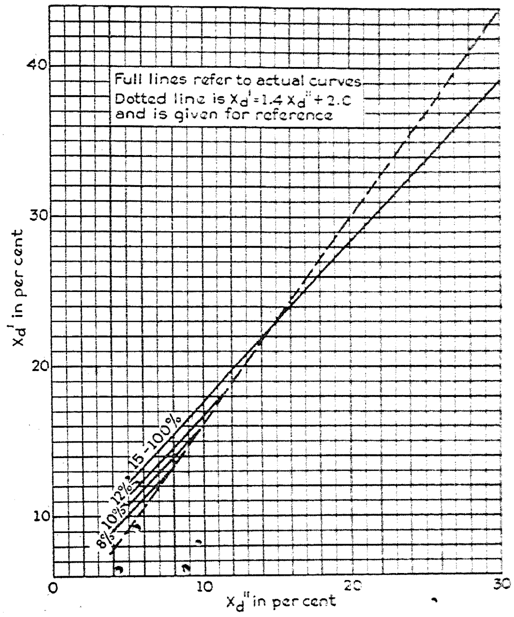

FIG. 2

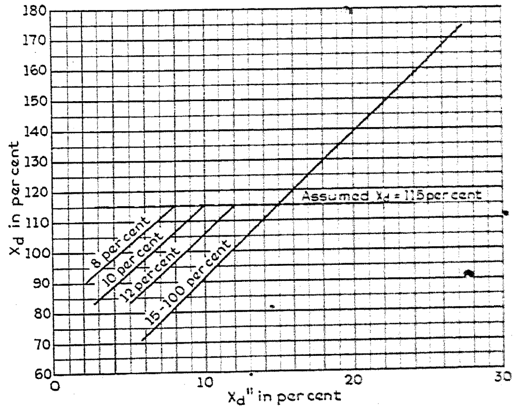

FIG. 3

An example will show how these curves may be used. Assume that the system under consideration has a reactance to the fault of 20 per cent; and that the equivalent generator has a subtransient reactance of 10 per cent, a transient reactance of 20 per cent, and a synchronous reactance of 100 per cent. This gives an external reactance of 10 per cent. The standard decrement curve for 20 per cent system reactance gives an initial current of 5.3 times normal. Referring to Fig. 1 the initial current is 96.5 per cent of this. From the curve of Fig. 2 the transient reactance is found to be 17.6 per cent to fit the standard decrement curve. As 20 per cent is the value for the system, and the external reactance is 10 per cent, the transient component of the standard decrement curve starts about 7 per cent too high. From the synchronous reactance curve of Fig. 3, it is found to be 91 per cent to fit the standard decrement curves. Therefore, the sustained current read from the decrement curve is about 8 per cent too high. Now, it has been found that the decrement curve reading for the specific system chosen is 3.5 per cent high initially, about 7 per cent high at 0.3 seconds and 8 per cent high at the sustained value. Therefore, the currents may be reduced by these percentages to give a more exact result.

A further inspection of the curves will show that while the general decrement curves are developed for a specific system based upon assumed relations between the various reactances, there are many other sets of relations between reactances which will satisfy the standard decrement curves exactly throughout part of their length and with reasonable accuracy throughout the remainder. Thus more confidence can be placed in the curves than one would suppose from an inspection of the assumptions made in their derivation.

Generalized Stability Solution for Metropolitan Type Systems

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***Synopsis.**—This paper presents general stability curves by means of which the transient stability during faults may be determined for any metropolitan type system. The results are obtained as the permissible fault duration in terms of certain indexes of the system and fault. These indexes may be found from a short-circuit study of the particular fault made on a d-c. calculating board or its equivalent. For a given fault only three readings are required to obtain the result.*

INTRODUCTION

THERE has been a feeling on the part of a number of engineers that there is no stability problem connected with the metropolitan type of power system. This feeling is not unwarranted because of the excellent operating record of metropolitan type systems.

The fact that the stability of the systems has not been a problem, however, does not necessarily mean that the study of stability would not be profitable. It is possible, for example, that these systems have been laid out very liberally because the factors affecting stability have not been very well known and the design is such that either, (1) greater stability than necessary has been obtained, or (2) the same degree of stability might have been secured in a more economical manner. In other words, it is not necessary for actual instability to occur to warrant a study of stability factors in metropolitan system design.

Other aspects of system operation, such as reliability of sources, flexibility of operation and reduction of short-circuit kva. sometimes suggest layouts differing from those at present in use, and it is desirable to be able to evaluate whether the desired changes will result in an increased or decreased stability, and whether the value obtained is within satisfactory limits.

Because of the foregoing, it is felt that in spite of the excellent operating record of metropolitan power systems from the stability point of view, there is nevertheless a real need for stability studies on such systems. However, if such studies are to find general application, it is necessary that very much simplified methods and short cuts be available. Fortunately, systems of the metropolitan type have many points of similarity so that generalized curves may be drawn using the principal characteristics as indexes. It is the object of this paper to present such curves and explain their application to any metropolitan type system.

STATIC AND TRANSIENT STABILITY

Investigation of many cases of so-called static instability has led the authors to the conclusion that *static instability is never experienced in the normal operation of metropolitan type systems.* In all cases in which sufficient information on the actual happenings was obtained it was found that some operating error or apparatus failure had caused a total or partial loss of excitation, excessive load or other abnormal condition, which were the actual causes of instability. Obviously such examples cannot be considered as instances of static instability. Analytical studies by the authors and others of static instability on metropolitan type systems have indicated that this is so far from a possibility that it may be omitted from consideration on all systems coming under this classification. Accordingly, the present paper is devoted entirely to transient stability, that is, action during faults.

THE METROPOLITAN TYPE SYSTEM

In this discussion a metropolitan type system is considered to be one in which the principal power sources are steam turbine-driven generators, located relatively close to their load centers, with distribution provided by a large number of moderate voltage circuits. The power supply to most metropolitan districts is of this character, but several large districts receive a major part of their power from distant hydroelectric sources which are quite different in character, and for these, individual studies are required. In some cases, short cut methods may be used for these studies also.[1] However, if the hydroelectric generating capacity is one-fourth or less of the total, the stability of the other generating sources for faults in their localities may still be analyzed by the methods presented without serious error.

Metropolitan type systems, as above defined, are similar in essential characteristics making generalized studies practicable. The principal features of similarity are as follows:

1. With equal per cent loadings the internal voltages of all generators on the system are practically in phase.
2. Because of the multiplicity of circuits the reactance of the connecting ties holding a generator or group of generators in step with the system does not change appreciably when the fault is cleared.
3. With steam turbine-driven generators and short transmission distances, the generator characteristics,

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Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

[1] For references see Bibliography.

no longer being affected by the speed of prime mover, line charging current requirements, etc., tend to become fairly uniform in their essential points, such as reactances, inertia and short circuit ratio.

This similarity makes it possible to calculate the stability of a number of hypothetical system layouts, plot the results of these calculations in the form of curves using the few remaining variable quantities as indexes and apply them to specific layouts by determining the values of these quantities for the specific case.

With a given system layout and considering stability under fault conditions, the measure of the stability is the permissible duration of the fault. Therefore, the curves in this paper are arranged to give as the result the permissible fault duration in seconds. Under the assumed conditions the most important factors affecting permissible fault duration are the initial generator output, synchronizing power and severity of fault. These factors may be expressed in a number of ways and in the curves described in the next section, the authors have chosen the method which appears to be the most useful.

General Stability Curves for Metropolitan Type Systems

Metropolitan type systems are usually laid out so that the generating units are not connected directly together at their terminals. The generating units may be individual generators, groups of directly paralleled generators in the same station, or the entire station, if all units are tied solidly in parallel. In what follows, the term "generator" will refer to a generating unit as indicated above.

Since a fault which affects all generators equally has little tendency to make them lose synchronism, the most severe fault from the stability standpoint is one which is electrically close to one of the generators. The stability problem is then one of holding this generator in synchronism with the remainder. The synchronizing power between this "faulted generator" and the remaining generators depends upon the connecting reactance between this generator and the others in parallel. This overall reactance includes the reactance of the system as well as the transient reactance of all machines and will be designated by X. The measure of the severity of the fault used by the authors is the fault current flowing from the faulted generator, expressed in number of times rated current modified if necessary by a factor to take account of fault location. Since the margin of stability is reduced with increased initial load, the curves are given for full load on all machines before the fault occurs, and later a factor will be introduced to take account of other values of initial load.

The fundamental premise underlying the use of the generalized curves for stability determinations is that setups giving the same values of the essential indexes will have approximately the same degree of stability. The problem then becomes one of determining the indexes from the actual system setup, and then obtaining the permissible fault duration of the corresponding hypothetical system setup from the general curves. These indexes were chosen in such a manner that they can be obtained from three readings on a d-c. calculating board.

It was necessary in the development of the general curves that minor influences be eliminated from consideration even though some sacrifice in accuracy was involved. Such influences include resistance in the lines and in the fault, and variations in load characteristics. In obtaining the indexes from the d-c. calculating board, only reactances are used, the results being obtained as reactances or reactive components of fault current. This was taken into account in drawing the curves, so that while the resistance effect of the loads was considered in computing the permissible fault duration, it was not considered in computing the indexes. Thus the difference is common to both the actual and hypothetical systems and is thereby compensated. This is equivalent to assuming that the load characteristics of the actual and hypothetical systems are the same.

Figs. 1 and 2 show the general stability curves for generator short-circuit ratios of 1.0 and 0.8 respectively. The two values of short-circuit ratio are used because of the consideration which has been given this factor in metropolitan system design, although the curves show that it has a minor effect on transient stability. Part A of each figure consists of a family of curves covering the range of overall reactance X, plotted in terms of short circuit current I_{FG} from the faulted generator and the permissible fault duration t. Part B is similar except that the power dropped by the faulted generator is used instead of I_{FG}. The latter curves, as will be shown later, provide a means of correction for resistance in the fault, and for other than rated initial load.

Part A of Figs. 1 and 2 applies directly to faults at locations electrically equivalent to the generator terminals, such as faults on feeders, just beyond the feeder breaker. To provide for faults occurring at other locations, a close approximation is secured by *dividing* the value of I_{FG} by a factor read from Figs. 1C or 2C, before entering Figs. 1A or 2A. This location factor is obtained as a function of the ratio of I_{FG} to I_F, the total fault current. This ratio will be denoted by r_{FG}.

Method of Application

The application of these curves will be demonstrated by an example. Fig. 3 shows schematically a typical metropolitan type system with reactances as indicated. A fault at point M will be considered. A three-phase fault at this location isolates generator A from the remainder of the system for the duration of the fault, and this generator is therefore the one most likely to fall out of step first. Therefore, machine A becomes the "faulted generator," and all other generators including those in the same group with A become the "remaining generators." Fig. 3 under these conditions reduces to Fig. 4. This reduction may be carried out analytically

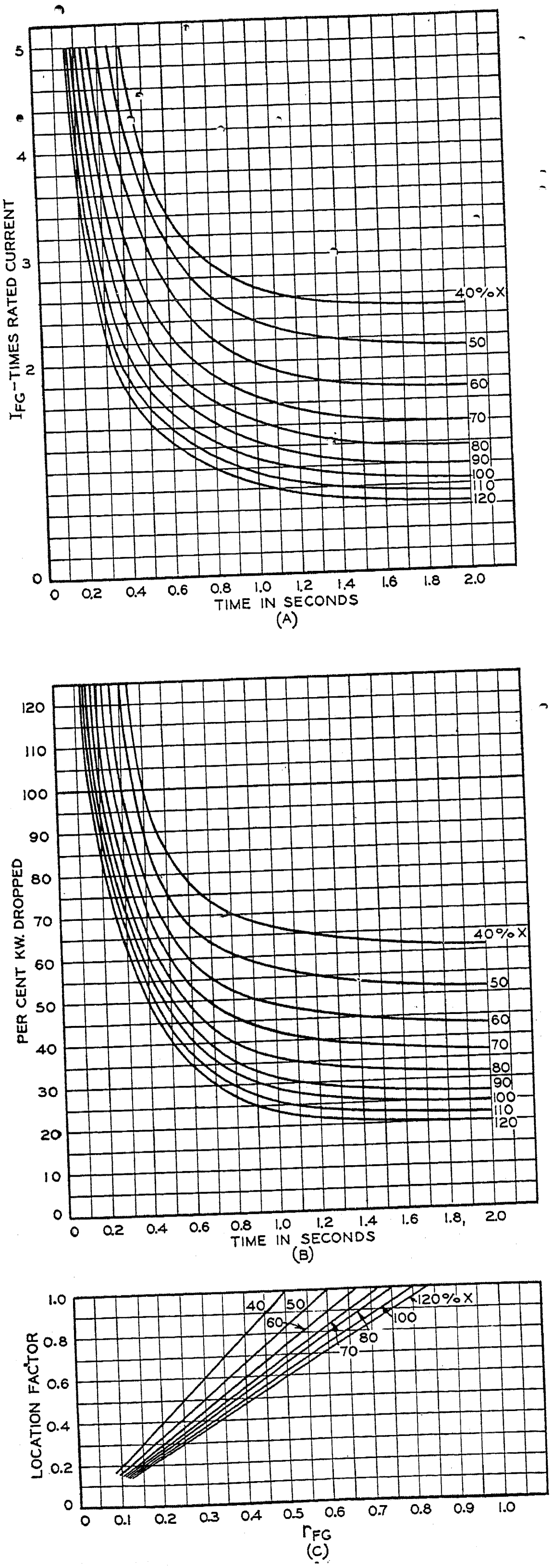

Fig. 1—General Stability Curves for Systems with Generators Having Short-Circuit Ratio 1.0

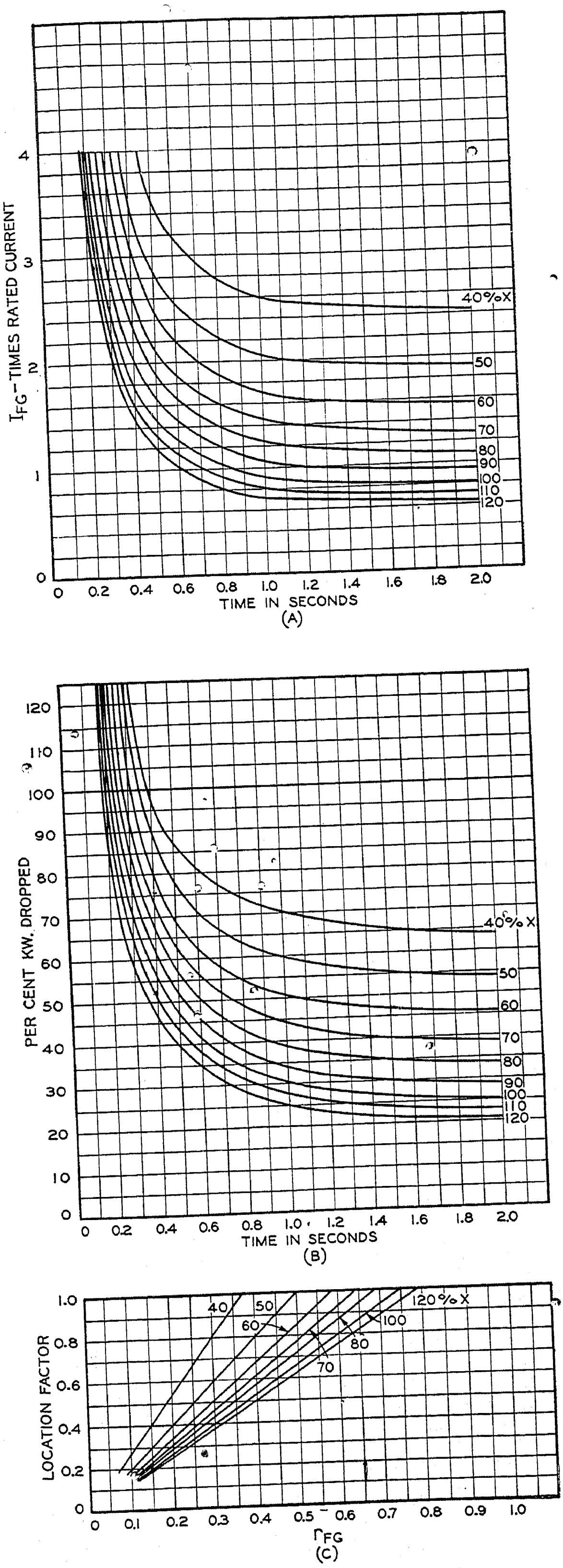

Fig. 2—General Stability Curves for Systems with Generators Having Short-Circuit Ratio 0.8

A. Principal curve. Short-circuit current I_{FG}, *vs.* permissible fault duration in seconds
B. Auxiliary curve. Per cent power dropped *vs.* permissible fault duration in seconds
C. Location correction factor

$$r_{FG} + I_{FG}/I_F$$

or by the use of the d-c. calculating board. In the latter case, the actual reduction to the simplified form need not be completed, as the necessary indexes may be obtained from the current readings necessary for the reduction. From the simplified figure, the overall reactance X between the faulted generator and the remaining generators is equal to $27 + 15 = 42$ per cent. This reactance must be based on the kva. rating of the faulted generator in all cases, and if the reactances are set up on some other base they must be converted to this base. To determine the total fault current I_F, the branches on each side of the fault are paralleled giving $(27 \times 15)/42 = 9.65$ per cent. The total fault current is therefore $1/0.0965 = 10.4$ times the rating of generator A. The fault current I_{FG} supplied by the faulted generator is $15/42 \times 10.4 = 3.7$ times rated current. The ratio of I_{FG}/I_F gives r_{FG} as 0.357. The essential indexes are thus

Overall reactance X .... = 42 per cent
Current from faulted generator I_{FG} = 3.7 × rating
Ratio I_{FG}/I_F, r_{FG} .... = 0.357

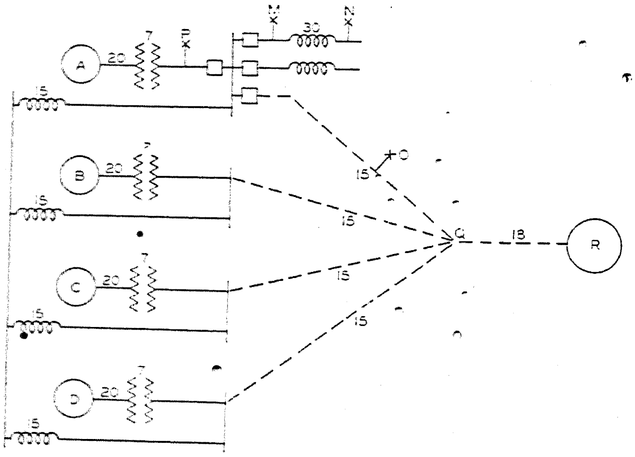

FIG. 3—SCHEMATIC DIAGRAM OF TYPICAL METROPOLITAN TYPE SYSTEM USED TO DEMONSTRATE APPLICATION OF CURVES

Assuming that the short-circuit ratio of generator A is 1.0, the location factor is obtained from Fig. 1C, entering the curve at $r_{FG} = 0.357$ and interpolating for $X = 42$ per cent, giving a value of 0.7. The adjusted value of I_{FG} is then found to be 3.7 divided by $0.7 = 5.3$. The permissible fault duration may now be read from Fig. 1A, for $I_{FG} = 5.3$ (extrapolated) and $X = 42$, and is found to be 0.39 seconds.

If the d-c. calculating board is used, the indexes may be found directly. With the system set up without the fault, generator A is connected to the positive bus, its transient reactance being included as part of the system, and the remaining generators connected to the negative bus in a similar manner. Rated voltage converted to the proper base is now applied between the two buses and the current flowing from generator A is read and converted to number of times rated current for this generator. For this case the value is found to be 2.38. The reciprocal of this value, expressed as a per cent, is X, and of course is 42 per cent as before. The remaining generators are now removed from the negative bus and connected to the positive bus and the fault applied by connecting point M to the negative bus. The fault current and current from generator A are then read and converted to number of times rated current of generator A. This gives the same result as was obtained from the simplified system of Fig. 4, that is, 10.4 and 3.7 times rated current respectively.

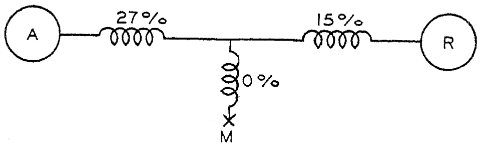

FIG. 4—SIMPLIFIED EQUIVALENT OF FIG. 3 FOR FAULT AT M

If the fault had occurred at point N outside the feeder reactor on a radial feeder, instead of at point M, the simple equivalent system would appear as in Fig. 5. The reactance of the feeder reactor is assumed as 3 per cent on the feeder rating, equivalent to 30 per cent on the rating of generator A, if the feeder rating is one-tenth that of the generator. If the faulted feeder had not been radial and had been tied into the system at the distant end, the equivalent system of Fig. 5 would have been slightly different, although the overall reactance would still be 42 per cent. It will be found for this case that r_{FG} remains the same since it depends only on the reactance branches adjacent to the generators. The total fault current is however greatly reduced owing to the presence of the 30 per cent reactance in series with the fault, and becomes 2.52 times rated current of generator A. I_{FG} is found to be 0.9 times rated current, and when adjusted for location becomes 1.29. When these indexes are referred to Fig. 1A it is found that the point is beyond the range of the curves and the permissible fault duration is over two seconds. Two seconds is about the upper limit of relaying time for most systems and the many variables render it undesirable to carry the curves much beyond this figure.

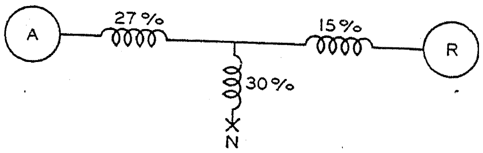

FIG. 5—SIMPLIFIED EQUIVALENT OF FIG. 3 FOR FAULT AT N

If the fault is at some point O so that it will affect all four of the generators in station A more equally, there is a possibility that this entire group may lose synchronism with the remainder of the system. In such a case, the group should be considered as the faulted generator, and a solution should be carried out expressing all indexes in terms of the rating of the group as a base. This solution should be compared with that for generator A considered as the faulted generator, and the shorter of the two values of permissible fault duration should be taken as the result.

One more variation will be considered, that of a three-phase fault on the leads of the transformer A at point P close to its bus. In this case generator A must obviously be disconnected from the system in order to clear the fault and the point of interest is how long this fault may persist without causing generators B, C, and D, to lose synchronism with the remainder of the system. Generator A is isolated from the system by the fault while the fault persists, and is isolated by its circuit breaker when the fault is cleared. Therefore, only machines B, C, and D, are considered as the faulted generator, and generator A is entirely eliminated from the calculations. The equivalent simplified system for this case is shown by Fig. 6, the reactances now being expressed in per cent on the combined kva. rating of generators B, C, and D. From this point the calculations proceed in the same manner as for the previous examples, leading to $X = 94.1$, $r_{FG} = 0.632$ and the adjusted value of $I_{FG} = 1.80$. This gives 0.53 seconds as the permissible fault duration.

If the fault had occurred on the bus the conditions during the fault would have been the same, but after the fault the tie between the synchronizing bus through the bus of generator A to the point Q is interrupted. This violates the fundamental assumption that the reactance after the fault is essentially the same as before the fault, so that the curves cannot be applied with the same degree of accuracy.

UNBALANCED FAULTS

The preceding analysis applies directly to three-phase faults. In many cases, however, investigations of unbalanced faults are of interest. The method of symmetrical components[2] shows that from the stability standpoint, an unbalanced fault may be represented by an equivalent impedance in series with the fault, the value of this impedance being a function of the negative and zero sequence impedances of the system viewed from the point of fault. These impedances may be found by reducing the negative and zero sequence network either analytically or on a calculating board.

CORRECTION FACTORS

The following influences were not considered in setting up the curves of Figs. 1 and 2:

1. Resistance in the connecting lines and in the fault.
2. Values of initial generator load other than normal rating.
3. Voltage regulators.
4. Systems not capable of reasonably accurate representation by two composite generators.

The correction to take these effects into account in all cases shows that the permissible fault duration is as great or greater than would be indicated if the factors were neglected. It is therefore felt that the curves of Figs. 1 and 2 can be used directly without any correction factor for most work. This view is held because, for most purposes, the limiting cases must form the basis of design. A fault of zero resistance, and operation at full load gives the greatest generator acceleration, and therefore lower indicated stability. Where systems do not reduce to two composite generators, the stability will be as high or higher than when they do. Voltage regulators, of course, increase the stability. Therefore, if the curves are used directly, without correction factors, the error will be on the safe side. It is appreciated, however, that in some cases, the effect of these variations on stability is of interest in system planning and operations, and approximate corrections have been set up to take care of these cases. Each of these variations has been investigated analytically, and in some cases, extensive calculations were necessary to determine their effect on stability. However, since it was considered most important that the correction factors be easily applied, some liberty in accuracy was taken in formulating a method of applying the corrections. It should be understood therefore that the correction factors now to be described differ somewhat from the theoretically correct values. They are arranged approximately in the order of their importance. Correction factors affecting the value of I_{FG}, if used, should of course be applied first.

1. *Effect of Resistance.* Resistances of the ordinary range have a minor effect on stability, with the exception of resistance in the fault circuit, that is, those branches carrying the greatest proportion of fault current. Due to the large currents in the fault circuit, a relatively small amount of resistance will result in a considerable power loss. This power loss compensates in a large measure the drop in load on the faulted generator due to drop in voltage, so that the tendency to pull out of step is minimized. Both parts A and B of Figs. 1 and 2 are plotted for faults at the generator terminals. Application of the location factor from part C converts a fault at any other location to its equivalent fault at the generator terminals. For each point in part A there is a corresponding point in part B, from which may be found the amount of power which would have to be dropped by the faulted generator for the equivalent fault at its terminals. If the amount of power dropped is reduced by the presence of resistance in the lines or fault, the effect would be nearly the same regardless of fault location. Hence the result may be found by reducing the equivalent amount of power dropped by the amount of the resistance losses taken by the faulted generator, and reading the corrected value of fault duration from the proper reactance curve of part B. In the example previously considered for a fault at M with $I_{FG} = 5.3$ and $X = 42$, a clearing time of 0.39 seconds is indicated by part A. The same clearing time must be indicated by part B, hence, for $X = 42$ and $t = 0.39$, it is found that 100 per cent power would have to be dropped by the generator during the fault if the fault were at its terminals. The curves have been drawn by

plotting this power as per cent of the generator kilowatt rating, which is 85 per cent of the kva. rating. Now suppose that generator A and its transformer have a total resistance of 1.5 per cent on their rating. With 3.7 times full load fault current flowing, the I^2R loss in the generator and transformer is $3.7^2 \times 1.5$ per cent or 20.5 per cent of the generator kva. rating, or 20.5 per cent divided by $0.85 = 24$ per cent of its kilowatt rating. Then, instead of the 100 per cent load dropped if the generator and transformer had zero resistance, the equivalent load dropped is 100 per cent minus 24 per cent or 76 per cent. The permissible fault duration, considering the effect of resistance, is then found from part B for $X = 42$ per cent, and power $= 76$ per cent, and indicates a permissible fault duration of 0.6 seconds.

In the example given above, the resistance in only the generator branch of the equivalent circuit was considered. The power loss was then easily determined, and was all supplied by the faulted generator. If the fault is located at some point on the system where there are a number of paths in multiple for the flow of fault current, the power loss may be determined by adding the I^2R loss in the various branches between the fault and the generator. The currents in the branches may be obtained by measurement of current in corresponding branches of the d-c. calculating board. They may then be squared and multiplied by the per cent resistance of the branches, and summed up to get the total power loss due to the fault.

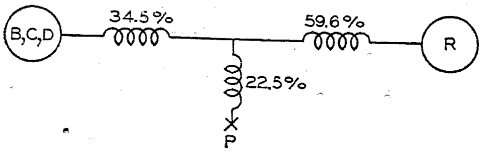

Fig. 6—Simplified Equivalent of Fig. 3 for Fault at P

If resistance in the fault itself is to be considered, the power loss in the fault must be apportioned between the faulted generator and the remaining generators. An approximate method of doing this is to multiply the total loss in the fault by the ratio r_{FG}, and add this value to the loss in the generator branch of the fault circuit, if this is considered. As an example, for a fault at M assume ½ per cent resistance in the arc of the fault, in addition to the resistance of the generator and transformer. With 10.4 times normal current, $10.4^2 \times 0.5$ or 54 per cent of generator kva. in power is created in the arc. This is equivalent to 54 per cent divided by 0.85 or 63.5 per cent of generator kilowatt rating. Multiplying by r_{FG}, 63.5 per cent $\times$ 0.357 = 22.5 per cent power in the fault, taken by the faulted generator. The power dropped is 100 per cent for $X = 42$ per cent and $t = 0.39$ seconds and when 24 per cent due to generator and transformer resistance and 22.5 per cent due to fault resistance are subtracted, 53.5 per cent remains as the equivalent power dropped. This value of power dropped is seen from Fig. 1B to give a permissible clearing time in excess of two seconds.

2. Initial Generator Load. For values of initial generator load other than 100 per cent of kilowatt rating, first find the value of kilowatts dropped, corresponding to X and t for rated initial load. Multiply this by the ratio of initial load to full load, and read the corrected clearing time from part B, for the curve corresponding to X. Example: with $X = 42$ per cent, $t = 0.39$ seconds, 100 per cent kilowatts is dropped; if the initial load had been 75 per cent instead of 100 per cent, the approximate fault duration may be read as $t = 0.61$ for kilowatts dropped = 100 per cent $\times$ 0.75 = 75 per cent and $X = 42$ per cent.

3. Voltage Regulators and Quick Response Excitation. Voltage regulators with a moderate rate of excitation response will give a certain amount of improvement in stability over the values given by Figs. 1 and 2. The precise amount is difficult to determine, but from calculations made, assuming a rate of response sufficient to maintain constant flux in the generator field, a reasonable idea of possible improvement may be obtained. The results of these calculations when compared with the standard curves without voltage regulators show that, over most of the range, the improvement can be approximated by multiplying the value of I_{FG} by 0.85 before entering the curves of Figs. 1A and 2A.

4. Different Types of Systems. The curves of Figs. 1 and 2 are theoretically applicable only to systems essentially connected in star form, typified by the synchronized at the load method of connection. This is because, as will be discussed later, the simplified systems which were analyzed were of the star form. However, analyses were also made of the ring connection, which represents the maximum possible deviation from the star connection, and the results of these analyses did not differ very greatly from those of the corresponding star systems. Hence, although certain variations will occur in special cases, no correction factor to take account of system connection is required for most work.

Part II

METHODS OF CALCULATION

In previous sections of this paper the general stability curves for metropolitan type systems were presented and the method of their use explained. The method of their derivation will undoubtedly be of interest to many engineers and the following discussion gives the essential features.

In recent years the concentration of large amounts of generating capacity on a single bus required by the larger metropolitan loads produced excessive amounts of short circuit capacity and greatly increased the danger of a system outage from the effects of a single short circuit. This led many utility engineers to consider other arrangements of system connections which would result in greater reliability with reduced short circuit kva., giving rise to the use of "synchronized at the load" and the so-called "loose linked" systems, including the use of a synchronizing bus connected to the individual generators through synchronizing reactors. To determine whether or not the new schemes would be stable, numerous stability studies of particular systems were made.[3,4] The results indicated that certain factors were of prime importance, but others of relatively much less. The similarity of the results indicated that satisfactory accuracy could be obtained with a great saving in labor, if the principal factors were included in a general study and the less important factors neglected.

The "synchronized at the load" scheme, if carried to the limit, would result in a star connection between the various machines of the system, as shown in Fig. 7. From the standpoint of a general study, the star connection is particularly advantageous, because it is the easiest to use in stability calculations. Also, for the same overall reactance, experience and calculations indicated that it would be the least stable, so that any error introduced would be on the side of safety, as the permissible fault duration indicated would be less than that which could actually be permitted. As a result, the star-connected system of Fig. 7, was chosen as the type of system to make the subject of a general investigation. Later other connections were investigated to determine the effect on the stability of the type of connection employed.

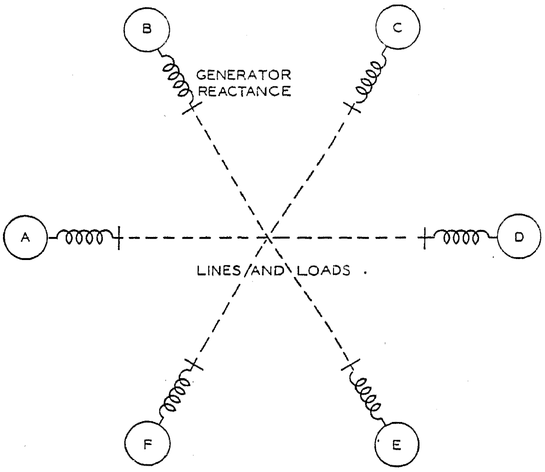

FIG. 7—SCHEMATIC DIAGRAM OF HYPOTHETICAL SYSTEM, USED FOR CALCULATIONS

An effort was made to choose the constants of the hypothetical system as near to the average of actual operating systems as possible. Typical machine constants and values of system reactance indicated by experience were taken. Because of the consideration which has been given in recent years to the subject of generator short-circuit ratio, two values of this constant were chosen, namely, 1.0 and 0.8. The short-circuit ratio affects the other constants of the machines to some extent, and this was taken into account in choosing the typical values of transient reactance and machine inertia. The following values were used:

Short circuit ratio	1.0	0.8
Transient reactance	20 per cent	25 per cent
Inertia	8.5 kw-sec./kva.	8.25 kw-sec./kva.

The lower value of short-circuit ratio will give a smaller machine, physically, which gives rise to the higher value of transient reactance and the lower value of inertia.

For a loosely coupled system to operate economically, each generating unit must supply its own load and the transmission of power from one generating area to another must be reduced to a minimum. Hence, it is here assumed that each generator carries its own load and that before the occurrence of a fault there is no transfer of power from one generating area to another. Furthermore, the load on a system of this type will generally consist of a large number of relatively small loads, so that the effect of the load in supplying short-circuit current is reduced to a relatively low value, and the load may be approximately represented by means of shunt impedances. Therefore, for the purposes of this analysis, all the load on a generating unit was considered as made up of shunt impedances, consisting of resistance and reactance, of an amount to give full load at the generator terminals at an average system power factor of 0.85. The location of the loads depends to a considerable extent upon the actual arrangement of the system and the distribution voltages, since a system distributing at generator voltage will generally have the load quite close to the generator, whereas a system stepping up to considerably higher voltages will have the load located more distant from the generator terminals. Although the actual location is not a factor of great importance in determining the stability, it is nevertheless desirable to satisfy as well as possible the conditions of all types of systems. What the authors consider to be the best compromise compatible with reasonable facility in calculation is to consider the load to be uniformly distributed along the lines connecting a generating unit to the others. Under the assumptions that each generating unit is operating at full load, 0.85 power factor, and that no load is being transferred from one generating area to another, the distributed admittance which will result in the required load on each generator may be easily determined. The method of treatment is quite similar to that of a line having distributed capacitance, so that each section of line may be represented by an equivalent Π or an equivalent T section, or it may be represented by the corresponding group of general circuit constants. Where the reactance between the terminals of the generators and the common star point is small, it is reasonable to distribute the load over the entire connecting link, but when this reactance becomes quite large, it does not seem reasonable to carry an appreciable portion of the load near the star point. Consequently, when the reactance between the terminals of a generator and the star point exceeded 40 per cent, the load was distributed only over the first 40 per cent of the reactance from the generator terminals.

On the actual system, faults may occur at almost any location. However, except at the generating station and at certain major substations a single short circuit will usually involve only one or perhaps two feeder circuits. Because of the impedance of the feeders, faults at intermediate locations will be much less severe than short circuits occurring at a point which affects all of the feeders to about the same extent. It can be shown, however, that a fault on any single feeder of a group of feeders connecting one generating unit to the remainder of the system is equivalent to a fault on all of the parallel feeders in series with an appropriate value of impedance, or reactance if series resistance has been neglected, as was done here. Thus, to take account of intermediate faults on single feeders, studies were made of faults at different locations on the lines of the hypothetical system, with a fault reactance connected between the point of fault and neutral. Various values of reactance were used, increasing from zero to an amount which would allow the system to remain in synchronism if the fault remained on the system up to two seconds.

With a hypothetical system set up in this manner, calculations were required of many different faults corresponding to variations in three principal factors, namely, the overall reactance between the faulted generator and the remainder of the system, the fault location, and the reactance in series with the fault.

The calculations were carried out in a conventional manner, making several of the simplifying assumptions which would usually be made in the corresponding individual study. Some of the assumptions have already been mentioned, but for completeness they will all be summarized here.

1. Resistances in generators and lines neglected.
2. Load considered as a distributed shunt admittance.
3. Load before fault, rated load on each machine.
4. Generator constants as tabulated above, except as given in 11.
5. Short-circuit losses in generators neglected.
6. All short circuits assumed to involve all three phases.
7. Problem solved as a two-machine problem.
8. Constant prime mover input assumed.
9. Damping factors neglected.
10. Combined inertia of system and "remaining generators" considered infinite.
11. Effect of cross-flux neglected.
12. Effect of voltage regulators neglected.
13. Six equal generating units assumed for purpose of calculation.

Certain of these assumptions may require further amplification. No. 7 indicates that the problem is solved as a two-machine problem. With a star-connected system as shown in Fig. 7 a short circuit on the feeders of any one machine will produce the same effect on all of the remaining machines, so that they will react similarly, and may, therefore, be considered as a single large machine in the calculations. Thus the problem to be solved reduces to the one shown in Fig. 8, comprising two generators connected by the system network which consists of the line with its shunt load and the fault reactance.

No. 8 states that the prime mover input has been assumed constant. This is equivalent to stating that the governors do not operate to change the steam valve position during the critical period of the fault.

No. 9 states that damping factors are neglected. Such damping factors would include the losses produced in the rotor by the asymmetrical direct current component of the short-circuit current, and the damping effect caused by the currents induced in the wedges and solid iron rotor.

No. 10 states that the combined inertia of the system and the remaining generators is considered infinite. In general the remaining generators will total several times the inertia of the faulted generator. Also the inertia effect of the load has been neglected by assuming the load as a shunt admittance, and including it will still further increase the ratio of remaining generator effective inertia to faulted generator inertia. When the ratio is large, the error introduced by considering it infinite becomes small and the calculations are considerably facilitated, since only the angular oscillation of the faulted generator must be computed. Whenever this assumption is valid, the actual number and rating of the machines comprising the remaining generator is of importance only insofar as the overall system reactance is affected.

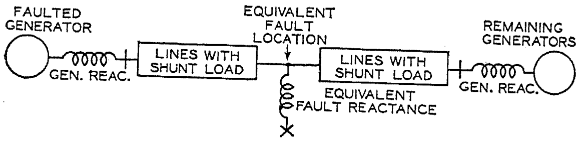

Fig. 8—Simplified Equivalent of Fig. 8 for Fault Near Generator *A*

No. 11 states that cross-flux was neglected. This means that only the flux in the direct axis of the field poles has been included in the calculations.

No. 12 states that the effect of voltage regulators was neglected. Many metropolitan type systems operate without automatic generator voltage regulators so that the short-circuit current causes a decrease in machine voltage and a reduction in stability. To include such systems the effect of decrement was considered, as described below. Later, calculations were made under the assumption of a quick response excitation system of sufficient rate of response and ceiling voltage to maintain constant flux in the fields, and the results made the subject of an approximate correction factor previously described.

No. 13 states that six generating units were assumed for purposes of calculation. As stated above, under the discussion of No. 10, a variation in this number is not of great importance except as it produces a change in the overall reactance.

It has been found by experience that calculations made with the above assumptions yield results which are pessimistic, that is, they indicate that the permissible fault duration is less than that which may actually be experienced without loss of synchronism.

The calculations for the duration of the fault were carried out by a step-by-step method, which assumes that the power output of the generators remains constant for a small interval, from which the acceleration and departure from normal frequency may be calculated, giving a new value of output power which is assumed to remain constant during the second interval, etc. At the same time the field circuit is being demagnetized by the action of the short-circuit current, and this was taken into account, calculating the decrement also step-by-step.

The permissible duration of the fault could have been determined by assuming that the fault was cleared at a certain time, and then making use of the conditions following the fault to determine whether or not the system was stable for the assumed time of fault clearing. This method is the one which must usually be used if three or more individual generating units must be considered in the stability analysis. However, when only two machines are involved, a simplified method known as the equal area method may be used. This latter method depends upon the fact that the energy absorbed in accelerating the machine rotors above synchronous speed must be restored to the system if the system is to remain in synchronism. That this condition is or is not satisfied may be determined by equating the energy absorbed by the generator rotors in accelerating during the period the fault is on the system to the energy which is given up to the system from the generator rotors in returning to synchronous speed after the short circuit has been cleared. This energy for each machine is proportional to the integral of the accelerating or retarding power with respect to the angular displacement of the rotor from its normal position. The representation of this integral as an area gives rise to the name applied to the method. In this method, as usually carried out, the assumption is made that the generator field flux does not decay, so that the generator voltages used remain constant. Under these assumptions the areas involved may be determined readily from the constants of the circuit during and after the fault. In the calculations carried out here, decrement was taken into account, as mentioned above, so that the area representing the energy absorbed during the fault had to be calculated by a step-by-step method and was carried forward for each case simultaneously with the remainder of the step-by-step calculations. It was assumed that when the fault was cleared by the opening of the necessary circuit breakers that the decrement ceased, so that the machine voltages would remain constant for the fraction of a second immediately following at the values which they had reached when the fault was cleared. Under this assumption the energy which could be absorbed by the system if the fault was cleared at a given time was calculated, and by trial a value of time was found which would make this energy equal to that absorbed in accelerating the rotors, as given by the step-by-step process.

The results of these calculations gave the permissible fault duration in terms of the overall reactance between the faulted generator and the remaining generators of the system, the fault reactance in series with the fault, and the fault location. This form was not found to be the most convenient for ready application to actual systems, so that the results were converted to the curves given in Figs. 1 and 2 as previously explained. In making the conversion, the loads were neglected and the indexes determined in terms of quantities which would be obtained if a short circuit study was made on the hypothetical system using a d-c. calculating board. The effect of faults occurring at different locations was found to complicate considerably the curves and render plotting them in convenient form difficult. An effort was therefore made to determine an empirical correction which would give the correct results if the faults were located at some other point than the generator terminals. The method previously described was found to give good results, and to allow convenient plotting of the curves, since only the results for a fault at the generator terminals are given directly.

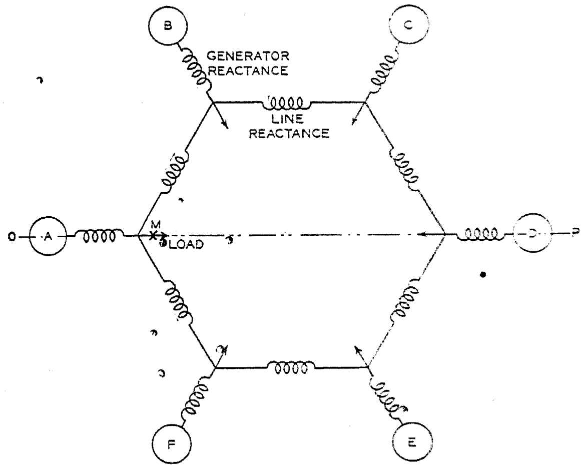

Fig. 9—Schematic Diagram of Hypothetical Ring-Connected System

In the preceding calculations the star connection was used, because it is the most "pessimistic" connection for a given overall reactance. Further calculations were made on a ring-connected system as shown in Fig. 9 to determine the limiting values of correction which might be expected with other forms of system connections, for, as explained previously, a ring connection is about as great a departure from a star-connected system as can be obtained. Six machines were again assumed for convenience in comparison. With this connection the load is assumed to be carried on a group of radial feeders emanating from each generator bus, and therefore for stability purposes may be assumed to be concentrated at that point. For facility in comparison, faults were considered on the radial feeders, for example, at point M in Fig. 9. With this setup the system is symmetrical about the center-line OP. Hence machines B and F and machines C and E will be affected similarly, and therefore may be considered as single generating units of double rating. Hence the problem can be analyzed as a four-machine problem. A calculation was made on this basis for a severe fault, and it was found that the angular displacement between machines C-E and D was so small that the three machines could be considered as a single unit in subsequent calculations. Therefore, the problem reduces to a three-machine problem, the first machine consisting of the faulted generator A, the second of generators B and F, and the third of generators C, D and E. The analysis followed the same general line as for the two machine problem previously discussed, except that the angular acceleration of the faulted generator now depends upon its relations to the two remaining machines instead of to just one machine as in the previous case. For this reason, the equal area method is no longer valid, and it was necessary to assume that the fault was cleared at a certain time and then determine if the system was stable by calculating the after-swing of the three machines. This process of estimating the clearing time was repeated until a point was found where the system was just stable, yielding a value of permissible fault duration for the assumed values of overall and fault reactance.

In order to take into account the effects of the angular displacement between the second and third machines, it was necessary to consider their inertia at its actual value instead of considering it infinite, as was done for the star-connected system previously discussed. In order to compare with the previous results and bring out clearly the effect of the type of connection, it was necessary to place both on the same basis. This was accomplished by recalculating the results for a few points of the star-connected system, assigning to the remaining generators their actual inertia instead of considering them of infinite inertia. The results were found not to differ materially from those obtained for the corresponding points of the ring connection. Thus with the indexes as set up, the star and ring connections have approximately the same degree of stability, and no correction factor need be employed. It is felt that this same similarity of results will hold when the inertia of the remaining machines is considered infinite in order to approximate the inertia effect of the load, as is done for the general curves in Figs. 1 and 2.

GENERAL COMMENTS

The curves presented in this paper provide a method of solving the transient stability problem on a metropolitan type system without the necessity of investigating the electro-mechanical oscillations as required in a conventional study of this problem. It is to be expected that the results obtained from these curves will have the same order of accuracy as results obtained by a specific analysis of the actual system using the assumptions conventionally made to simplify the work. Experience and test have shown that actual systems have greater stability than the usual calculations indicate. This fact should be borne in mind when using the curves for relay settings or system design, and in general no additional margins of safety need be allowed. The authors feel that the greatest value of these curves lies in the facility with which they allow comparative studies of different layouts to be made. When used in this way the margins between actual and calculated values of stability are largely compensating, and need therefore be of little concern except in extreme cases. Sufficient knowledge concerning load and machine characteristics, particularly those factors affecting damping, is not at present available to determine the margins with precision even as a result of the most careful and involved calculations, so that the curves presented here may be used in all cases where they apply until further data has been obtained on hitherto unevaluated factors.

ACKNOWLEDGMENT

The authors wish to acknowledge the encouragement and assistance of engineers of the operating companies, particularly Mr. S. M. Dean of the Detroit Edison Co., and the assistance of their associates, especially Mr. L. M. Olmsted, in making the numerous calculations.

NOTATION

I_F = total fault current, expressed in times rated current of "faulted generator."

I_{FG} = fault current from "faulted generator," expressed in times rated current.

r_{FG} = ratio I_{FG}/I_F

X = overall reactance between "faulted generator" and remaining generators in parallel, expressed in per cent based on rating of "faulted generator."

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Discussion

R. D. Evans: During the past seven or eight years, attention has been directed to the nature and importance of stability, methods of improving stability, and the calculation of stability. These discussions have been very voluminous and for the most part highly technical. For some time it has been feasible for a specialist to determine, as a result of tedious step-by-step calculations, that a particular system was or was not stable for a particular load condition and a particular system disturbance. From a practical point of view it is necessary to go further than this and to put a dollar value on the difference in the stability characteristics of various power system layouts. To do this has required the development of approximate or short-cut methods as obviously the tedious step-by-step methods are unsuitable. By short-cut methods I do not mean merely shortened stability calculations but instead methods which permit the determination of stability without actually making any stability calculations at all for the cases under consideration. More definitely these short-cut methods involve merely the determination of certain network constants of the particular system and the use of general curves.

The general stability curves given in the paper may be used in a manner similar to that of the "Standard Decrement Curves," see page 353 of this issue. These stability curves are the first that have been presented in connection with the closely connected or metropolitan type system. It is expected that their use together with previous curves developed for long distance straight-way transmission will lead to the development of similar methods for the remaining types of systems so that eventually short-cut methods will be available for all types of systems.

S. H. Wright: When applying the curves presented by Messrs. Griscom, Lewis and Ellis, cases may frequently arise where conditions of the system being studied are met by the conditions assumed in preparing the curves except in one respect, making it desirable to consider the inertia constant as an additional "index factor." This will be illustrated in the following:

Suppose we have a case where the "faulted generator" group consists of a large-capacity unity-short-circuit-ratio generator whose inertia constant is 5.0 kw. sec. per kva. Assume (as under item 10 in the paper), that the combined inertia of the system and "remaining generators" is infinite. The permissible fault duration varies as the square root of the inertia of the faulted generator, hence its value in seconds as read from the curves in the paper would be multiplied by $\sqrt{5.0/8.5}$ or by 0.767, to obtain the actual permissible fault duration; this means a reduction in time of 23 per cent.

As is stated, "The authors feel that the greatest value of these curves lies in the facility with which they allow comparative studies of different layouts to be made." In the spirit of the statement it is obvious that *comparisons* are valid regardless of such a time correction factor as illustrated above, *providing* the faulted generator characteristics are the same in all cases studied.

S. B. Crary: During the past few years, the extensive application of synchronous machine and circuit theory to the study of the transient stability limits of various systems has made it desirable to summarize and classify the results of the investigations already made. In doing this for the metropolitan type of system, the author's have shown some interesting results.

Any generalization or simplification of a problem which comprises as many factors as a transient stability study, must require numerous assumptions. The assumptions which are made, limit the field of application. Care must, therefore, be used in applying the curves of Figs. 1 and 2 of the paper to systems that do not adhere to the assumptions set forth. A few illustrations have been selected as the best means of illustrating this point.

The assumption is made that the reactance of the connecting ties holding a generator or group of generators in step with the system does not change appreciably when the fault is cleared. This assumption is valid when a fault is on a feeder, but is not when the fault is on one of the high-voltage ties between stations. Although the ring-connected system is considered as designed for no power transfer between stations under normal operation, conditions exist when these ties are used to transmit a considerable portion of a generating station load to another area. If a fault occurs on a tie line heavily loaded in this manner, the stability limit may be below that of the hypothetical star system used to determine the curves of Figs. 1 and 2 of the paper.

The assumption is made that the "remaining group" has an infinite inertia. This will probably be valid for most conditions, but the condition may exist, that the faulted group of generators represents more than one-third of the total connected capacity. Under such a condition, the required switching time in order to clear the fault and maintain stability may be reduced or increased an appreciable amount, depending upon the distribution of loads with respect to the "faulted and remaining groups."

The statement is made that "Where systems do not reduce to two composite generators, the stability will be as high or higher than when they do." This can not be taken as a general statement as cases do exist when the stability limit may be lower instead of higher. An example is a system in which "the remaining group" consists of a minor group relatively closely coupled electrically to the "faulted group," and a major group loosely coupled with respect to all of the other machines.

The severity of the fault as it affects stability is determined by the two indexes I_{FG} and I_{FG}/I_F. Could not both of these indexes be replaced by a single index based on the transfer reactance between the two machines when the fault is on? This reactance has been defined in a paper by Summers and McClure.* It is a simple function of the three branches of the simplified equivalent network and can readily be determined from the system set up on a d-c. calculating table.

S. B. Griscom, W. A. Lewis, and W. R. Ellis: In preparing the paper the authors felt that the range of inertia constants normally encountered would be sufficiently narrow that a separate correction or index for this factor would be unnecessary. Mr. Wright mentions an example in which the inertia departs materially from the average assumed, and the deviation is sufficient to warrant a correction. In general, the correction can be made in the manner he suggests. That is, the permissible fault duration as found from the curves and after the application of the other correction factors should be multiplied by $\sqrt{\frac{H}{8.5}}$ for unity short-circuit ratio machines or $\sqrt{\frac{H}{8.25}}$ for 0.8 short-circuit ratio machines, H being the inertia constant of the "faulted generator." It should be pointed out, however, that since decrement was taken into account, the correction obtained in this manner is somewhat too large. With the smaller inertia constant a given angular position is reached sooner and the voltage has a higher percentage of its original value. This higher voltage will give a higher power limit and hence a greater permissible fault duration.

Mr. Crary has emphasized that there are some cases which do not fall within the range of the assumptions made in the paper. From the nature of the problem, this of course is inevitable. However, the authors feel that the application of the curves may be extended to cases outside the range of the assumptions if care and judgment are used in interpreting the results. This will be illustrated by considering the examples selected by Mr. Crary.

If the reactance of the connecting tie holding a generator or group of generators in step with the system changes appreciably when the fault is cleared, upper and lower limits of permissible clearing time may be found by computing the "overall reactance" for both conditions and estimating an intermediate clearing time from the values read from the curves for the two reactance values.

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*"Progress in the Study of System Stability," A.I.E.E. TRANS., Vol. 49, Jan. 1930, p. 132.

If the "faulted generator" is a large proportion of the total system capacity, the assumption that the inertia of the "remaining generators" is infinite is not strictly valid. For the severe faults, which will be electrically close to the faulted generator, the permissible fault duration will be greater than the value given by the curves and the answer is, therefore, on the side of safety. For faults near the electrical center of the system, the deviation may be in either direction, but for such cases the permissible duration will usually be two seconds or more so that the error may be tolerated.

The reduction of the ring connection system of Fig. 9 of the paper for purposes of calculation gives a system which closely approximates the system mentioned by Mr. Crary in which "the remaining group" consists of a minor group relatively closely coupled with the faulted group and a major group loosely coupled with respect to the other machines. That is, machines *B* and *F* comprise the minor group and machines *C*, *D* and *E* comprise the major group. As stated in the paper the permissible fault duration for the case was found to be as great as or greater than the value read from Figs. 1 or 2.

It is appreciated that there may be cases in which accurate results cannot be obtained from the curves. However, it is felt that such cases would be the exceptions and will be recognizable from their special features.

An attempt was made to use as an index the equivalent of the transfer impedance as suggested by Mr. Crary. However, it was found that the driving point impedance was also required to fully represent the conditions, so that the form used was adopted as being more convenient.

The authors realize that as these curves are used certain additions and modifications may be indicated. It is hoped that a standardized set of curves for metropolitan and other types of systems will eventually receive the same recognition as the standard decrement curves referred to by Mr. Evans.

Stability of Conowingo Hydroelectric Station

Philadelphia Electric Company System

BY R. A. HENTZ* and J. W. JONES†
Fellow, A.I.E.E. Associate, A.I.E.E.

Synopsis.—*This paper gives a brief description of the Philadelphia Electric Company System with particular reference to the features influencing stability of the Conowingo Hydroelectric station, summarizes the various calculations and tests associated with the stability problem, and presents the record of operating experience from a stability standpoint during four years of service.*

Several cases of instability due to line faults have occurred. A thorough study was made of the matter, and as a consequence, changes were made in the equipment and system set-up involving the development and installation of higher speed circuit breakers and relays.

Detailed records are given for all faults on the parts of the system which are intimately associated with the Conowingo development. These records show the improvement of stability obtained by the changes in the system and indicate attainment of a generally satisfactory condition.

* * * * *

INTRODUCTION

THE matter of electric power system stability is becoming of more general interest and concern since the advent of large hydroelectric stations located at considerable distance from their loads and the general interconnection of large capacity power systems. A number of papers has appeared dealing with methods of calculating stability, but comparatively little information has been published showing the experience on actual systems. Such experience is of value as an indication of what may occur on other systems of similar character and is helpful in determining the basis for design of new projects.

The 252,000-kw. Conowingo hydroelectric station which is located 58 miles from the main system of the Philadelphia Electric Company and ties in with the 220-kv. Pennsylvania-New Jersey interconnection by means of two transmission lines, is a typical example of a generating station susceptible to instability under certain conditions of very severe line faults. Studies have been made of the stability of this system, and, during the four years the plant has been in service, considerable operating experience has been obtained including several cases of instability, with the result that several changes have been made in the system set-up and equipment to improve the performance. Automatic oscillographs are permanently installed to record system performance during faults and a large number of records has been obtained.

No operating tests have been made to determine stability, though several short-circuit tests on the 220-kv. system to determine high-speed relay and oil circuit breaker characteristics, intimately associated with stability, were made. A few high speed motion pictures of governor operation were taken, but gave little information.

DESCRIPTION OF SYSTEM

The system involved, shown in Fig. 1, consists of the Conowingo hydroelectric station with seven 36,000-kw. generators connected by two 58-mile, 220-kv. lines to Plymouth Meeting substation, which is a junction point for two lines of the Pennsylvania-New Jersey 220-kv. interconnection, and the location of step-down transformers for connection with the main part of the Philadelphia electric system by means of 66-kv. lines.

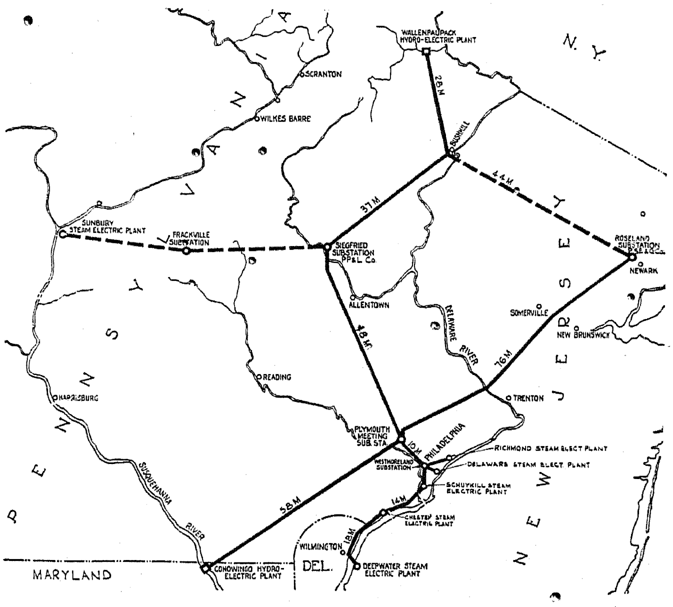
WALLENPAUPACK HYDRO-ELECTRIC PLANT
N. Y.
SCRANTON
WILKES BARRE
28 M
BUSHKILL
44 M
37 M
SUNBURY STEAM ELECTRIC PLANT
FRACKVILLE SUBSTATION
SIEGFRIED SUBSTATION PP&L Co.
ROSELAND SUBSTATION P.S.E.&G.Co.
NEWARK
ALLENTOWN
SOMERVILLE
NEW BRUNSWICK
48 M
DELAWARE RIVER
76 M
READING
HARRISBURG
TRENTON
PENNSYLVANIA
NEW JERSEY
PLYMOUTH MEETING SUB STA.
10 M
PHILADELPHIA
RICHMOND STEAM ELECT PLANT
WESTMORELAND SUBSTATION
DELAWARE STEAM ELECT PLANT
SCHUYKILL STEAM ELECTRIC PLANT
SUSQUEHANNA RIVER
58 M
14 M
CHESTER STEAM ELECTRIC PLANT
WILMINGTON
18 M
DEL.
MARYLAND
CONOWINGO HYDRO-ELECTRIC PLANT
DEEPWATER STEAM ELECTRIC PLANT

FIG. 1—MAP OF SYSTEM

The Pennsylvania-New Jersey interconnection as indicated consists of 220-kv. ties between the Philadelphia Electric Company system at Plymouth Meeting, the Pennsylvania Power and Light Company system at Siegfried, and the Public Service Electric and Gas Company system at Roseland. The 40,000-kw. Wallenpaupack hydroelectric station of the Pennsylvania Power & Light Company is connected to Siegfried with a 220-kv. line which it is planned to extend from Bushkill to Roseland in the ultimate development. The approxi-

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mate lengths of the lines are indicated on Fig. 1. The interconnected systems are all of large capacity, the Philadelphia Electric Company, having a peak load in 1930 of 699,000 kw., the Public Service Electric and Gas Company 540,500 kw., and the Pennsylvania Power & Light Company about 350,000 kw.

The connection between Plymouth Meeting substation and the main part of the Philadelphia Electric system where most of the Conowingo output is used, consists of three 10-mile 66-kv. aerial lines to Westmoreland substation, from which point lines radiate to the various load centers and generating stations as indicated. The Philadelphia Electric Company 66-kv. system is normally sectionalized as shown by Fig. 2 to limit short-circuit duty. This diagram shows the positive sequence reactance of the principal lines and equipment as well as the equivalent reactance of the synchronous equipment connected to the various parts of the system for the maximum setup as of 1931. The set-up for previous years[1] was similar except that the Plymouth Meeting-Roseland line was not in service; there were only two step-down transformer banks at Plymouth Meeting; only two Plymouth Meeting-Westmoreland 66-kv. lines; and the 66-kv. system was not sectionalized at Plymouth Meeting or Westmoreland.

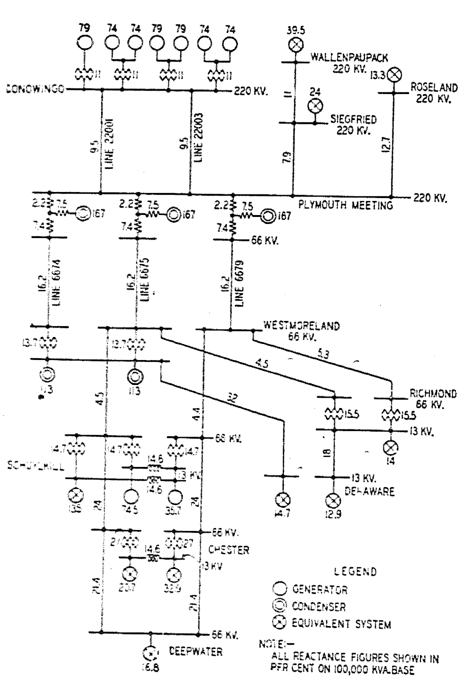

FIG. 2—SYSTEM REACTANCE DIAGRAM

PRELIMINARY STABILITY STUDY

Extensive stability calculations and studies were made before the 220-kv. system was built to check stability in case of line faults. The faults were assumed to involve only one phase since information then available indicated this was the general experience on 220-kv. systems. These studies were based on the ultimate system development which involves extension of the 220-kv. system as shown by the dashed lines in Fig. 1. Since the system is rather complicated, one of the studies was made with a mechanical model as described in the paper *Quantitative Mechanical Analysis of Power System Transient Disturbances* by R. C. Bergvall and P. H. Robinson.[2]

The results of the stability studies in general showed that synchronism could be maintained for practically all expected fault conditions, although considerable swinging was indicated for some conditions. On account of this swinging condition together with uncertainty regarding some of the factors used in the study such as clearing time and fault resistance, it was considered advisable to incorporate in the design several special features for increasing stability and reliability.

FEATURES OF INITIAL INSTALLATION HAVING SPECIAL INFLUENCE ON STABILITY

The measures for increasing stability and reliability which were provided with the initial design may be divided into two general classes: preventive and remedial. The first class was aimed at the prevention of faults by means of heavy insulation and protection against lightning with double ground wires for each line, shielding for substation structures and installation of lightning arresters at the substation as described in previous papers.[3,4]

The other class included steps to clear faults quickly should they occur and measures to stiffen the system in case of transient disturbances. A special relay system was installed, designed to clear all faults with minimum delay. A detailed description of the original relay system is given in one of the references.[4] The oil circuit breakers which were the fastest available for such high-voltage service at the time of installation were designed to interrupt fault current in less than one-half second. The generators were built with as low transient reactance as possible consistent with reasonable cost and efficiency. It was rather difficult to obtain low reactance for these machines on account of their slow speed (81.8 r.p.m.), but it was possible to reduce the transient reactance to about 29 per cent and still have a fairly well-balanced design. The transformer reactance was kept as low as possible with safety and this resulted in a value of about 9 per cent. The three windings of the Plymouth Meeting transformers were so arranged that the synchronous condensers connected to the tertiary would be very closely coupled to the 220-kv. system, so as to secure the maximum benefit from them in maintaining stability.

One 30,000-kva. condenser is provided for each of the three transformers at Plymouth Meeting, and these condensers have rather special excitation as described by L. W. Thompson and P. J. Walton.[5] In case of a system disturbance, the exciter voltage is automatically increased at a rate of over 6,000 volts per second, and the load on each condenser increases from a normal value of

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[1] For references see Bibliography.

10,000 kva. to about 58,000 kva. in one-half second. Two additional condensers with automatic high-speed excitation are installed at Westmoreland substation and the 60-cycle motors of a considerable number of frequency converters on other parts of the Philadelphia Electric Company system are used as condensers. These frequency converters and all generators on the Philadelphia electric system have automatic voltage regulators.

The Conowingo generators have special high-speed excitation designed to prevent a drop of more than 2 per cent in the induced voltage of the generators when full-load lagging reactive kva. is suddenly applied to the generator while carrying full rated load at unity power factor.

INSTABILITY EXPERIENCED WITH INITIAL OPERATION

During the first year, 1928, three faults occurred on the Plymouth Meeting-Conowingo lines, each fault involving two phases instead of only one as expected and used as the basis for design. Conowingo became unstable for each of these faults and also on one other occasion when a severe disturbance occurred on the Philadelphia Electric Company 66-kv. system.

The first case of instability was caused by a lightning flashover of two phases of the Plymouth Meeting-Conowingo line about two miles from Plymouth Meeting, tripping the faulty line at Plymouth Meeting and both lines at Conowingo. Five generators were in operation at Conowingo carrying 168,000 kw. and all five machines tripped on over-speed or over-voltage. The Philadelphia Electric System load was 202,000 kw. none of which was lost. The Plymouth Meeting-Siegfried line stayed in and helped materially in supporting the Philadelphia Electric System load after Conowingo was lost.

The second case of instability was caused by an oil fire and explosion near the Chester-Schuylkill two-circuit, 66-kv. line, which enveloped the line in flame causing a two- or three-phase short circuit. This burned down some of the conductors, tripping both circuits. Chester was cut away from the system with a load of about 40,000 kw. One Plymouth Meeting-Westmoreland 66-kv. line tripped at Westmoreland and four out of six machines at Conowingo tripped on over-speed or over-voltage. The lights in Philadelphia were noticed to flicker for several seconds. The Philadelphia Electric System load previous to the trouble was 297,000 kw. of which about 30,000 kw. was lost as a result of the disturbance. The generation at Conowingo dropped from 218,000 kw. to 26,000 kw. The Philadelphia Electric System steam generating capacity in operation at the time was 270,000 kw., the large excess being due to the noon hour drop in load.

The third case of instability was on the 4th of July, 1928, when the system load was small on account of the holiday. This case was caused by a lightning flashover of two phases of the Conowingo-Plymouth Meeting line about five miles from Conowingo while five generators were in operation carrying 112,000 kw. Switches tripped on the faulty line at Plymouth Meeting and on both lines at Conowingo. One Conowingo generator tripped on over-speed. A Hall high-speed recorder which had just been installed at Plymouth Meeting indicated the fault duration as approximately one second and showed that Conowingo became unstable during the fault and remained out of step with the system for about six seconds until separated apparently by opening of the unfaulted Plymouth Meeting-Conowingo line. The remaining generators on the system picked up the load without apparent difficulty.

The final case of instability experienced during the first year's operation was caused by a lightning flashover of two phases of the Plymouth Meeting-Conowingo line about four miles from Plymouth Meeting, while four generators were in operation at Conowingo carrying 140,000 kw. The faulty line tripped at both ends with an estimated clearing time of 0.3 seconds at Plymouth Meeting and 1.7 seconds at Conowingo. All four generators operating at Conowingo tripped on over-speed and the record of an automatic oscillograph which had been installed on the system by this time showed loss of synchronism in less than one-half second after the beginning of the fault. The Philadelphia electric system load at the time of the disturbance was 360,000 kw. and no load was lost. This oscillograph record is shown in Fig. 3.

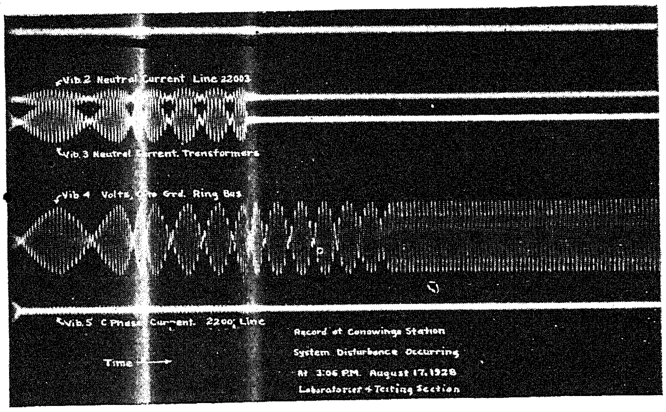

FIG. 3—OSCILLOGRAM OF SYSTEM DISTURBANCE AUGUST 17, 1928

Instability caused by two-phase fault on Conowingo line

Another disturbance experienced during the first year's operation which resulted in the loss of Conowingo was caused by a two-phase fault near the middle of one of the two 66-kv. Plymouth Meeting-Westmoreland lines while Conowingo was carrying 180,000 kw. Both Plymouth Meeting-Westmoreland lines tripped separating Conowingo from the system and all five generators in operation at Conowingo tripped on over-speed or over-voltage. It is believed this interruption was due to too low setting of the relays on the Plymouth Meeting-

Westmoreland lines which tripped the unfaulted line as the result of overload rather than through instability.

This experience during the first year when every fault on the Plymouth Meeting-Conowingo line caused instability was naturally very alarming, and extensive study was given the matter to determine the cause of the trouble and to find out what could be done to correct the condition. The study was handicapped at first by the lack of definite records of the time and sequence of events. Some doubt existed regarding the performance of relays, switch time and operation of the governors. All four cases of instability occurred before the installation of synchronous condensers was complete and before the high-speed excitation system was working satisfactorily.

Temporary Means of Improvement

While there was practically no interruption of service to customers, it was realized that something must be done immediately to increase the reliability of Conowingo. Therefore a number of temporary measures was adopted pending the outcome of further study and experience.

It was evident that most trouble was likely to occur during lightning storms and that service to the customer could be maintained by providing sufficient steam reserve. Arrangements were therefore made with the station attendants and line patrolmen to watch for electric storms and to report their appearance to the system load dispatcher so that the load on Conowingo could be reduced and the steam reserve built up whenever an electric storm was on the system. A practise was also adopted of operating the 220-kv. system split during storms if four or more generators were in operation at Conowingo so that the station would operate in two separate sections. While this meant the certain loss of half the station with a fault on either line, it greatly reduced the chance of losing the entire station. The use of this storm set-up was discontinued in 1931 when changes had been made in some of the equipment so that it was no longer required.

Special Tests

While relief was obtained by the temporary measures, a series of tests was made to check practically all uncertain factors affecting the performance of the system.

The most comprehensive of the tests was a series of single-phase and two-phase faults made at various locations on the Plymouth Meeting-Conowingo lines while records were obtained by means of observers and special recording equipment on all parts of the system. This test was made with most of the synchronous equipment and lines in service but without load because of danger of disturbing the customers. Twenty-nine oscillograph elements were used in the test and the connections to these elements were changed around to measure various quantities for the different faults. High-speed moving pictures were taken of all the faults and of the governors on several of the generators.

The main conclusions from this test were as follows:

1. Relay operations were proper and all faults were cleared with more than sufficient margin to prevent the disconnection of any equipment other than the faulted line.
2. The overall clearing time for faults including switch and relay operation was somewhat longer than expected, averaging a little over one-half second for the first switch and one second for the second switch.
3. There was no serious swinging of one generator against another at Conowingo.
4. There was no hunting of governors.
5. The faults were apparently of low impedance and did not blow from one conductor to another.
6. The high-speed excitation system was not performing as well as expected.
7. The calculated short-circuit current was in reasonably close agreement with the actual test values.

Another test was made by opening switches on the Conowingo generators while the machines were carrying load to show the speed characteristics and governor response as well as to furnish a service check of the overspeed and over-voltage protection. It showed that the speed increased so rapidly when full load was dropped that some of the units advanced 180 deg. ahead of the system in less than 0.3 seconds. This naturally explains how the plant can get out of synchronism so quickly when the electrical output is seriously reduced by the reduction in voltage as a result of a severe system fault.

Special tests were made of the synchronous condensers under load conditions in order to check the operation of the high-speed excitation system and determine the best adjustments. After some changes had been made, the performance was brought up to expectations. The test of the Plymouth Meeting condensers is described in another paper previously mentioned.[5]

Another test was the measurement of zero sequence reactance of the Conowingo lines. This was one of the uncertain factors used in the calculations and the test showed that the measured value was very close to that used in the calculations.

Collection of Records of System Disturbances

Complete records and reports of each system disturbance are obtained by the following means:

1. *Automatic Oscillographs.* As soon as obtainable, automatic oscillographs were permanently installed at Conowingo and at Plymouth Meeting. The installation includes a six-element oscillograph at Conowingo and a seven-element oscillograph at Plymouth Meeting, while most of the time a second oscillograph with six elements has been used at Plymouth Meeting. A four-element Hall recorder is installed at Westmoreland. Through cooperation with the manufacturers, and communication companies, additional oscillographs and recorders have been temporarily installed from time to time to obtain field experience and special information. These oscillographs record residual cur-

rent in all lines, power in each of the Plymouth Meeting-Conowingo lines, positive sequence voltage and selected line currents and voltages. They are started automatically with station residual current. Arrangements are being made also to start the oscillographs with line current since three-phase faults recently experienced have had insufficient residual current to start the oscillograph.

2. Reports from Station Operators. Most of the relays are equipped with operation indicators and the more important relays are connected to a drop signal system so that it is possible to see which relays operate for each disturbance. From this, it is usually possible to determine the phases which are faulted and the general location of the fault. The operators are instructed to note the time of each disturbance and to make a record of all relay operations. They also report the reading of ammeters in the neutral ground connection of the high-voltage transformer windings. These ammeters are ratcheted so that the pointer remains at the maximum reading until released by the operator. Graphic recording meters make a continuous record of power, voltage and frequency. Of course, station records are kept which show the system set-up and machines in operation at all times.

3. Line Reports. Thorough inspection is made of the line immediately after each fault to determine the damage, location and number of phases involved. If necessary, the patrolmen climb each tower to locate the fault.

All available information is collected for each important disturbance and these records are analyzed to determine the cause, number of phases involved, location of fault, fault current, switching sequence, time of clearing, power swings, etc. The records of all severe disturbances have been referred to the manufacturers of the major equipment to secure their recommendations regarding means of improvement.

Analytical Study of Two-Phase Faults

In view of the experience with two-phase faults during the first year, further stability calculations were made based on this type of fault. These calculations showed that stability could not be expected with solid 220-kv. bus operation and heavy load on the plant with a two-phase fault near the end of the line cleared in approximately one-half second, but if the fault was cleared in about 0.15 second, the system would probably remain in synchronism. A similar study was made for two-phase faults on the Plymouth Meeting-Westmoreland lines and this also showed the need of high-speed switching.

Development of High-Speed Circuit Breakers

The manufacturers thought it would be possible to increase greatly the speed of clearing faults by means of new design circuit breakers and relays, and with the need for higher speed operation apparent, they immediately undertook the development of suitable equipment. Some experimental models of new relays and oil circuit breakers were tested at Plymouth Meeting by means of short circuits on the 220-kv. system. These tests were very successful showing that interrupting time of the order of 0.14 second was possible. A description of some of the tests was given in the technical press.[6,7]

Changes In Equipment to Increase Stability and Reliability

As a result of the various studies and tests combined with normal development of the system, it was decided to make the following permanent improvements:

1. New high-speed circuit breakers were purchased and rearrangements made at Plymouth Meeting and Westmoreland so that high-speed breakers were provided for normal operation on all 220-kv. and 66-kv. aerial lines.
2. The 220-kv. line breakers at Conowingo were rebuilt so that high-speed operation was obtained.
3. The relay system was revised and new high-speed relays installed on all lines to reduce the operating time to about one cycle.
4. A third step-down transformer was installed at Plymouth Meeting.
5. A third 66-kv. line was added between Plymouth Meeting and Westmoreland.
6. The 66-kv. system was split into three separate sections at Plymouth Meeting and Westmoreland.
7. The 220-kv. line insulation was increased from 14 to 16 units for suspension insulators and from 16 to 18 units for strain insulators.

Operating Experience

The general experience with Conowingo since the first year's operation has been very satisfactory. Many disturbances have occurred due to faults on practically all parts of the system, but since the first year when there were four cases of instability, various steps have been taken to improve operation and there has been only one case where Conowingo became unstable.

The disturbances have been classified according to the location of the fault on the system and will be discussed under the following general headings:

Plymouth Meeting-Conowingo Line Faults

The principal data for each fault experienced on the Plymouth Meeting-Conowingo lines are shown in Table I. This gives the best known information regarding the type and location of the fault, the clearing time, load conditions and the general effect on the system. Some of the information is only approximate but has been included wherever possible for the sake of completeness.

The residual current is shown in amperes as measured in the faulted line. The positive sequence voltage is expressed in kilovolts equivalent line to neutral on the 220-kv. system at Plymouth Meeting measured during the fault. Comparison of the positive sequence voltage

TABLE I—PLYMOUTH MEETING-CONOWINGO LINE FAULTS

Year	1928			1929				1930				1931			
Date	6/9	7/4	8/17	6/14	6/15	7/29	8/21	1/14	6/26	7/6	7/21	5/10	7/9	7/9	7/24
Time	5:27 p.m.	4:12 p.m.	3:06 p.m.	11:48 p.m.	1:21 a.m.	4:12 p.m.	5:06 a.m.	12:31 p.m.	7:48 p.m.	7:19 p.m.	7:14 p.m.	8:52 p.m.	7:41 p.m.	9:01 p.m.	1:47 a.m.
Line No.	22,003	22,003	22,001	22,001	22,001	22,003	22,003	22,003	22,003	22,001	22,003	22,003	22,001	22,001	22,003
Cause	Ltng.	Ltng.	Ltng.	Ltng.	Ltng.	Ltng.	—	Ltng.	Ltng.	Ltng.	Switch failure	Ltng.	Ltng.	Ltng.	Ltng.
No. phases	2	2	2	1	1	2	1	1	1	1	1	2	1	3	1
Miles from Ply. Mtg.	2	53	4	—	—	—	—	17	28	—	0	18	5	22	33
Residual current Con.	—	—	—	Open	Open	—	Open	915	500	420	450	1,370	940	—	Open
Residual current Ply.	—	—	—	1,300	1,350	—	—	—	700	3,000	—	2,600	4,500	—	2,600
Pos. sequence voltage Ply.	—	—	—	—	—	—	—	—	115	87	98	76	95	—	115
Clearing time (60-cycle) Con.	—	10	102	Open	Open	69	Open	45	107	71	37	11	12	Est. 40-50	Open
Clearing time (60-cycle) Ply.	—	62	20	18	16	20	35	31	35	33	55	8	15	"	8
Load Conowingo mw.	168	112	140	30	0	66	0	263	170	92	150	235	145	165	35
Load P. E. steam gen. mw.	34	46	220	225	211	349	226	159	258	145	323	89	248	241	163
Load P. E. system mw.	202	158	360	252	207	423	250	392	390	223	473	319	345	396	207
220-system set-up	Solid	Solid	Solid	Solid	Solid	Solid	Solid	Solid	Split	Solid	Solid	Solid	Solid	Solid	Solid
Conowingo stability	Unstable	Unstable	Unstable	OK	Off	OK	Off	OK	OK	OK	OK	OK	OK	Unstable	OK
Conowingo power swings	—	—	—	—	—	—	—	Est. 200–325	75–100	70–110	130–155	164–292	114–185	+313–313	31–38

with the normal value of approximately 127 furnishes a general indication of the shock to the system. The clearing time is given in cycles based on the operating frequency of 60 cycles per second. The figures for Conowingo power swings are the first maximum and minimum values of average power output of Conowingo for the surges immediately following the clearing of the fault.

There has been a total of 15 faults on the Conowingo lines including nine single-phase, five two-phase, and one three-phase. Instability resulted from the three-phase fault and from three of the two-phase faults which occurred before the high-speed circuit breakers were in service.

The faults during the first year are discussed in the beginning of the paper. In the second year there were no faults while much load was on the station, so there was no real test of stability. At the beginning of the third year, there was a rather unusual experience of an electric storm in the winter. This caused a lightning flashover of one phase on the Conowingo line while the station was operating with solid 220-kv. bus and loaded to maximum capacity. Very heavy swinging occurred but stability was maintained. Fig. 4 is an oscillograph record of this disturbance as obtained at Conowingo. The fault occurred on *B*-phase of line 22,003. The clearing time at Plymouth Meeting is shown by the length of the first step in the residual current record and the clearing time at Conowingo by the total length of the residual current record. Allowance must be made for the starting time of the oscillograph which in this case is about 8 cycles. The record of voltage on line 22,003 is interrupted sometime before the fault is cleared, due to the operation of an auxiliary switch on the breaker mechanism. The record of *A*-phase current in the unfaulted line (No. 2 vibrator) shows the surging after the fault was cleared.

Another interesting disturbance in 1930 was a fault while using the storm set-up. The 220-kv. system was operating split with two generators carrying 70,000-kw. load connected to the line on which the fault occurred while four generators with about 100,000 kw. were on the other line. Of course, the two generators carrying 70,000-kw. load were lost but the remaining section of the system was disturbed very little.

The last fault of 1930 is rather unusual since it was the equivalent of a 220-kv. bus fault at Plymouth Meeting. An insulation failure occurred inside of one of the Conowingo line circuit breakers which opened but did not clear the fault since it involved the switch terminal connected to the bus. The fault burned clear after a little while and system operation was not very greatly disturbed.

The new equipment for high-speed clearing of faults was in service in 1931 and showed its value for the first fault which was a flashover of two phases while Conowingo was almost fully loaded. The fault was cleared from Plymouth Meeting in about eight cycles or 0.133

seconds and from Conowingo in 11 cycles as shown by the oscillograph record from Conowingo, Fig. 5. The time lost between the beginning of the fault and the first visible record is approximately three cycles. The reduction in oscillograph starting time as compared with previous records such as Fig. 4 was necessary on account of the high-speed switch operation. The improvement in starting time is secured through the use of a new type lamp. Rather heavy surging occurred after the fault but stability was maintained.

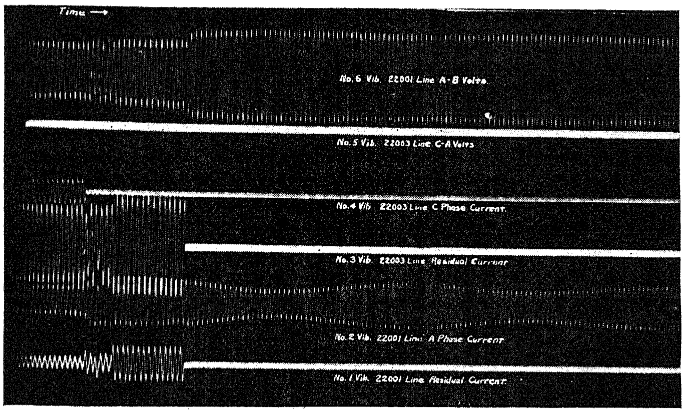

Fig. 4—Oscillogram of System Disturbance January 14, 1930

Stability maintained through single-phase fault on Conowingo line with full load on station

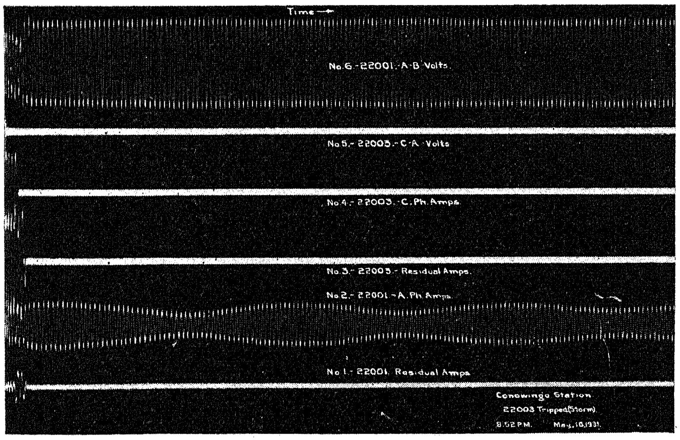

Fig. 5—Oscillogram of System Disturbance May 10, 1931, Showing Operation of High-speed Circuit Breakers

Stability maintained through a two-phase fault on the Conowingo line

The worst disturbance and the only one which has caused instability since the first year's operation was the three-phase lightning flashover in 1931. The plant was carrying 165,000 kw. and the fault was not cleared for almost a second because there was practically no ground current, and the only high-speed relays in service were ground relays. Since this experience high-speed balanced protection has been added to take care of such conditions. Due to the relatively small ground current the automatic oscillographs did not start until the circuit breaker had interrupted the fault and no definite record was obtained of the clearing time. The record which was obtained, Fig. 6, is of interest, however, since it shows the power swings of the station out of synchronism with the system. The unfaulted line No. 22,001, finally tripped as shown by cessation of power swings. The crest of the power swing was used as a check on stability calculations by comparison with the crest of the power angle diagram calculated for this condition. These check within a few per cent which seems to indicate that the assumptions used in the stability calculations are reasonably satisfactory.

Interconnection Line Faults

The faults experienced on the 220-kv. interconnection lines running from Plymouth Meeting to Siegfried and Roseland are listed in Table II. So far as known, all eight faults were single-phase and while most of these were of relatively long duration, none seriously affected stability. The load on these lines is usually rather small, seldom exceeding 50,000 kw. The Roseland line was not placed in service until near the end of 1930 so that comparatively little experience is available on this line. It was equipped initially with high-speed circuit breakers, but the first fault was slow in clearing because it was so near the Roseland end of the line that it was outside the zone of operation of the high-speed relays. The next fault which was probably much closer to Plymouth Meeting was cleared in eight cycles. The last disturbance on the Siegfried line was rather unusual, being caused by the failure of the circuit breaker at Plymouth Meeting when the line was being opened. Some material jammed the oil valve connecting one of the switch tanks to the oil system and most of the oil drained out without being noticed before the breaker was opened. Naturally, the circuit breaker could not open the circuit properly under this condition and flashover to ground occurred inside the tank. This was equivalent to a 220-kv. bus fault at Plymouth Meeting and it was cleared as expected by trip-out of all 220-kv. feeds into Plymouth Meeting. This involved considerable time delay but the system was not seriously dis-

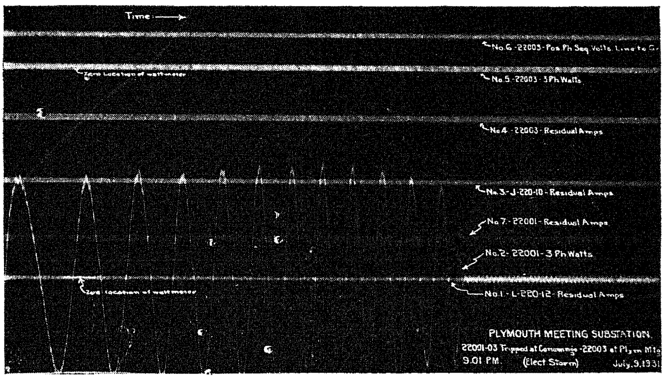

Fig. 6—Oscillogram of System Disturbance July 9, 1931

Instability resulting from three-phase fault on Conowingo line

2. *Quantitative Mechanical Analysis of Power System Transient Disturbances*, by R. C. Bergvall and P. H. Robinson. Trans. A.I.E.E., July 1928, p. 915.

3. *220-Kv. Transmission Line for the Conowingo Development*, by P. H. Chase. Trans. A.I.E.E., July 1928, p. 900.

4. *Electrical Features of the Conowingo Generating Station and Receiving Substations at Philadelphia*, by R. A. Hentz. Trans. A.I.E.E., July 1928, p. 890.

5. *Automatic Regulation of Synchronous Condensers Equipped with Superspeed Excitation*, by L. W. Thompson and P. J. Walton. Trans. A.I.E.E., January 1930, p. 315.

6. "Need for High Speed Breakers," by E. K. Lewison and "Results Obtained," by R. Paxton. *Electrical World*, October 12, 1929, Vol. 94, p. 735.

7. "220-Kv. Tests on Deion Grid Breakers," by L. W. Dyer, *Electrical World*, April 26, 1930, Vol. 95, p. 844.

8. *Lightning Investigation on 220-Kv. System of P. P. & L. Company 1928 and 1929*, by N. N. Smeloff and A. L. Price. Trans. A.I.E.E., July 1930, p. 895.

9. *Lightning Investigation on the 220-Kv. System of the P. P. & L. Co. (1930)*, by Edgar Bell and A. L. Price, Trans. A.I.E.E., September 1931, p. 1101.

Discussion

J. B. MacNeill: Looking to the future, the more stable operation of large systems under fault conditions will result in large economies in the construction of such systems. At the present time the load that can be carried through a fault is determined by the speed of the protective devices. It has been estimated that in some systems 100 per cent additional load can be carried during storms by the use of sufficiently high-speed circuit breakers and relays over what was possible with corresponding apparatus a few years ago.

On the Conowingo system, I am informed 8-cycle operation of the switches will hold the system together on the majority of faults. This speed is obtainable even at 220,000 volts with the most modern apparatus at all fault currents which affect stability.

When sufficient data become available on high-speed operation of circuit breakers and relays, and enough is known regarding the nature of faults and the speed of deionization thereof, it is not too much to predict that automatic reclosing even of 220,000-volt circuit breakers may be possible. A study of the phenomena occurring both at the fault and in the circuit breaker will be necessary, and considerable operating experience over a period of time will be advisable before speedy reclosure can be considered. In time, however, the necessary data will be secured, and we can have the economies in system operation which automatic reclosing to main power breakers will secure.

An interesting feature of the development work done by Mr. Hentz's organization, was the use of 220,000 volts as a single-phase voltage for testing of oil circuit breakers. Some of the results of these tests have been presented to the Institute in the past. Of especial interest is the light these tests throw on the probable switching speeds for 330,000-volt systems, since 220,000 volts is considerably greater than the line-to-neutral voltage of 190,000 volts, of a 330,000-volt system. With present forms of construction, switching times for 330,000 volts may be approximately 12 cycles, as compared to 8 cycles for 220,000 volts. This will have a considerable effect on the stability of 330,000-volt systems. It is felt, however, that an extension of ideas now available for circuit breaker improvement may result in switching speeds for 330,000 volts comparable with those obtainable on 220,000 volts.

R. M. Spurck: This paper indicates that the stability problems of a large system have been solved. It is hoped that other operating companies who may have stability problems will study this paper and plan on obtaining similar operating data.

It seems worth while to bring out that curing stability problems by high-speed circuit clearing also brings with it many other benefits, such as reduced area of disturbance on the system during faults, reduction in voltage dips, and apparatus that requires less maintenance.

R. A. Hentz: One of the most significant matters in connection with the instability which we experienced is the large percentage (40 per cent) of faults on the Conowingo lines involving more than one wire. From all the information we could glean at the time the system was designed, single wire to ground were to be practically the only faults experienced and that other faults would be extremely rare, but it is evident that, in the region transversed by these lines and with the type of tower construction used, faults involving two wires to ground and even three wires to ground will occur and that this fact should be taken carefully into account in the design of any system.

It is proposed to continue the collection of the data incorporated in the paper and with the passing of years the accumulated experience so obtained will be of further help to those interested in stability problems.

The development of high-speed oil circuit breakers and relays referred to by Mr. MacNeill was the principal factor in eliminating the difficulties which we experienced during the first summer of operation. The fact that this can be accomplished in such very large breakers as those installed on the Conowingo lines, (rated 230 kv. and with 2,500,000 kva. interrupting capacity) gives promise that interruption of faults with rapidity enough to eliminate or at least minimize instability is possible for most other systems.

Radio Interference from Insulator Corona

BY F. O. McMILLAN*

Member, A.I.E.E.

***Synopsis.**—Extended laboratory tests reveal that the corona formation voltage and the initial radio interference voltage of clean, dry insulators are identical. Oscillograms show many interesting characteristics of insulator corona current and of the radio interference caused by it. Means for reducing the interference are suggested.*

RADIO interference originates on high voltage insulators from two principal causes: (1) from very small ionization or corona discharges due to overstressed air in the dielectric field; and (2) from intermittent leakage currents flowing through conducting deposits of various kinds on the insulator surfaces. Both of these causes of interference are of sufficient importance to warrant investigation. However, because the corona discharge type of interference is universally troublesome, and serious surface leakage is more or less localized in certain geographical areas which are subject to salt fogs or special industrial conditions, this investigation has been devoted to a study of interference due to corona.

The voltage gradient or stress on a dielectric transmitting dielectric flux is directly proportional to the flux density, and inversely proportional to the ease with which the insulating material transmits flux. Therefore when two dielectrics are transmitting the same dielectric flux in series, the dielectric having the lowest permittivity obviously will be subjected to the highest voltage gradient per unit thickness. The insulation of high voltage power lines consists of two dielectrics, air and some other solid insulating material such as porcelain or glass. In their practical application, these gaseous and solid materials are combined in series and parallel arrangements in the dielectric circuit.

Air and solid materials have widely different dielectric characteristics. High voltage insulator porcelain transmits dielectric flux approximately six times as readily as air and has a dielectric strength several times greater. Glass used in modern high voltage insulators has a permittivity four and one-half times that of air, and, like porcelain, also has a dielectric strength several times that of air. Because of the widely different permittivities, exceedingly careful design is necessary to use air and porcelain or air and glass in series combinations with high impressed voltages. Thin air-gaps in series with thick sections of porcelain or glass must be avoided in regions of dense dielectric flux, or failure of the air by ionization will result. This undesirable condition is found in three different places on many of the standard multi-shell pin type insulators as now designed, as follows:

1. At points where the conductor and tie wire are not in intimate contact with insulator head.
2. At the contact between the pin or thimble and the insulating material.
3. Between shells of multi-part insulators.

When insulators so designed are used on high voltage lines, the air in these regions of high voltage gradient fails due to the ionization-by-collision phenomenon. This air failure occurs very suddenly at the time the critical ionizing gradient is reached each half cycle of the alternating voltage impressed on the insulator. Because of the changed energy storage in the capacitance and inductance of the electric circuit the sudden flow of ionizing current resulting from the corona formation sets up oscillations in the electric circuit.

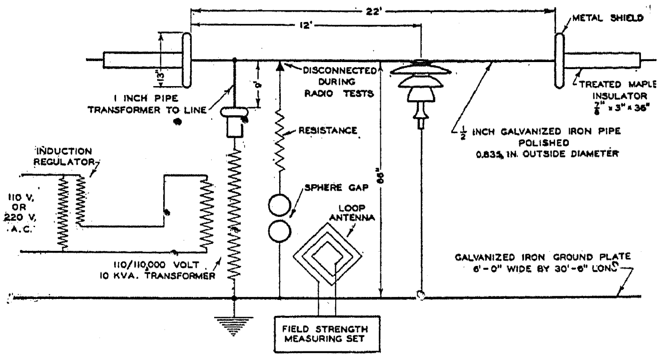
22'
12'
METAL SHIELD
13"
6"
DISCONNECTED DURING RADIO TESTS
1 INCH PIPE TRANSFORMER TO LINE
TREATED MAPLE INSULATOR 7/8" × 3" × 36"
RESISTANCE
1/2 INCH GALVANIZED IRON PIPE POLISHED 0.835 IN. OUTSIDE DIAMETER
INDUCTION REGULATOR
110 V. OR 220 V. A.C.
SPHERE GAP
66"
LOOP ANTENNA
GALVANIZED IRON GROUND PLATE 6'-0" WIDE BY 30'-6" LONG
110/110,000 VOLT 10 KVA. TRANSFORMER
FIELD STRENGTH MEASURING SET

FIG. 1—SCHEMATIC DIAGRAM OF APPARATUS FOR LABORATORY INVESTIGATION OF RADIO INTERFERENCE AND CORONA FORMATION VOLTAGES ON PIN INSULATORS

The oscillations thus generated cover a wide band of frequencies from the lower range of audibility through the higher radio frequencies. These higher frequency radiations cause radio interference.

Laboratory tests show conclusively that radio interference from clean, high-voltage insulators is caused by corona discharges occurring at locations on the insulator where the air is ionized by excessive dielectric flux concentration. Large numbers of tests conducted in a darkened room showed that corona formation and radio interference always are coincident. To obtain this result, great care must be used in performing the tests. The radio receiving set must have high amplification, the loop antenna must be closely coupled with the circuit radiating the interference, and the corona observations made very carefully in a dark room. Obviously

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when the corona formation first occurs under a conductor, behind a tie wire, or in the pin-hole of an insulator, it cannot be observed. Radio noise will occur coincident with this hidden corona and therefore precedes the first observable appearance of corona. The fact that corona formation and radio interference are coincident was determined by a great many tests using insulators so designed and treated that the first corona occurs in locations easily observed in a darkened room. Some of these test data are shown in Table I.

RADIO INTERFERENCE AND VISUAL CORONA FORMATION VOLTAGES

At the time the investigation of radio interference was started, it was known that in general the interference from pin type insulators is more pronounced than that from the suspension type; also, that more interference was radiated from high voltage than from low voltage pin type insulators. For these reasons, practically all tests have been confined to pin and pedestal insulators rated at from 66 to 73 kv. The radio interference and visual corona formation voltage tests were made with apparatus connected as shown in Fig. 1. Great care was exercised to avoid extraneous interference from the testing equipment. To prevent interference from minute discharges between metallic parts in close proximity to each other, all pieces of hardware associated with the experimental apparatus were bonded carefully either to the high voltage conductor or to the ground. A polished ½-in. galvanized iron pipe (0.83 in. outside diameter) shielded at each end, was used as the conductor; this pipe was free from corona for all voltages up to 100 kv.

TABLE I—RADIO INTERFERENCE AND CORONA FORMATION VOLTAGE DATA FOR PIN INSULATORS

INSULATOR NUMBER	NOMINAL RATING KV	INITIAL RADIO INTERFERENCE DATA		INSULATOR VISIBLE CORONA DATA					REMARKS
		INSULATOR POTENTIAL KV	LOCATION ON INSULATOR	HEAD KV	PIN KV	SHELLS*			MEAN ATMOSPHERIC CONDITION BAROMETRIC PRESSURE 757.2 MM. OF HG. TEMPERATURE 21.3 DEGREES CENTIGRADE WET BULB TEMPERATURE 55.3 DEGREES FAHRENHEIT DRY BULB TEMPERATURE 70.4 DEGREES FAHRENHEIT RELATIVE HUMIDITY 36 PER CENT
						1-2 KV	2-3 KV	3-4 KV	
A-4	66	20.4	PIN	20.4	18.4	30.5	25.7	–	Solder coating in conductor and tie wire grooves.
B-5	66	23.0	HEAD	23.0	32.7	62.2	86.6	–	Silver and Bismuth coating on head. Zinc thimble in pin hole.
B-6	66	23.5	HEAD & PIN	23.5	23.5	67.6	75.0	–	Silver and Bismuth coating on head and in pin hole.
D-4	66	8.1	HEAD	8.5	52.0	53.6	42.9	–	Standard porcelain insulator with ¾ in. malleable thimble.
D-5	66	29.3	CAP TO PORC.	30.2	37.1	50.3	NOT TAKEN	–	Malleable cap on head ¾ in. malleable thimble, discharge cap to porcelain.
D-6	66	32.1	CAP TO PORC.	32.1	40.4	54.4	58.9	–	Malleable cap on head ¾ in. malleable thimble, discharge cap to porcelain.
D-6a	66	42.4	BETWEEN SHELLS 2&3	47.6	45.7	43.4	42.4	–	Crevices head and thimble filled with Portland Cement.
D-6b	66	17.9	HEAD	17.9	71.5	97.1+	70.7	–	Crevices head and thimble filled with Portland Cement, between shells with Pothead Compound.
D-7	66	25.2	PIN	27.0	25.2	46.5	51.9	–	Malleable cap on head ¾ in. malleable thimble, discharge cap to porcelain.
D-7a	66	60.7	HEAD	60.7	86.7	53.8	56.8	–	Crevices head and thimble filled with Pothead. Comp'd. Discharge at head.
D-7b	66	38.7	HEAD	38.7	64.1	91.6+	91.6+	–	Crevices at head, shells, and thimble filled with Pothead Compound. Discharge over compound at head 45.3 KV.
F-4	70	25.2	HEAD	25.5	38.3	44.7	41.1	37.4	Sanded head coated with electrolytic copper ¾ in. malleable thimble.
F-5	70	9.5	HEAD	9.5	← NOT TAKEN →				Same as F-4. Discharge in fault in metal coating.
G-1	70	12.3	HEAD	12.3	38.5	81.1+	81.1+	–	Standard Insulator. Corona on head at tie and in pin hole around metal.
G-2	70	50.3	HEAD	50.3	81.1+	81.1+	81.1+	–	"Nostatic" coating on head and in pin hole.
G-2a	70	40.3	HEAD	40.3	← NOT TAKEN →			–	After 10 A.I.E.E. Std thermal test cycles, 35 impulse flashovers at 450 Kv. maximum, and five hours heavy conductor and tie wire vibration, corona first formed where impulse arcs destroyed the conducting coating on the head.
G-3	70	59	HEAD	59.0	80.0+	80.0+	80.0+	–	"Nostatic" coating on head and in pin hole.

*Shells numbered from head to pin

TESTS ON PIN INSULATORS

A few representative determinations of the radio interference and corona formation voltages for pin insulators are given in Table I. The tabulated values show that radio interference and visible corona occur at the same value of insulator voltage except when the corona starts in locations not in view.

The unshielded insulators started radiating interference and showed visible corona at the surprisingly low voltages of from 8.0 to 12.2 kv. to neutral or approximately 25 per cent of the normal operating voltage. However, all of the shielded insulators except F-5 showed a marked improvement in the radio interference formation point, and only one insulator G-2 was free from radio interference at normal operating voltage.

The lower case subscript letters (Table I) indicate insulators treated or modified as shown under the remarks in the table.

It may be observed that improvements were effected by sealing all crevices with cement or compound; the radio interference voltage was increased thereby from 32 to 123 per cent.

FUNDAMENTALS OF PREVENTING INSULATOR RADIO INTERFERENCE

Because of the nature of the disturbing electrical discharges on insulators there are at least three methods of attack to eliminate them. These are to:

1. Design the insulator in such a manner that no regions of overstressed air exist under normal operating conditions.
2. Provide conducting coatings to act as dielectric

flux distributors at the head, in the pin-hole, and on other necessary parts.

3. Displace the overstressed air with an insulating compound dielectrically strong enough to withstand the potential.

From at least one standpoint, method 1 is the most desirable of the three because it employs no auxiliary material which may be subject to deterioration in service. This method, however, is the most radical departure from present practise and therefore probably will require a longer development period if found practicable at all. If method 2 employing a dielectric flux distributor is used, the conducting material must be in such intimate contact with the solid dielectric that all air between the conducting coating and the porcelain or glass is excluded. The use of method 3 requires that all air in regions subject to being overstressed to the ionizing point be displaced by an insulating compound.

Some difficulties are encountered in employing any of these three methods. Conducting coatings must remain at all times in intimate contact with the insulator material, and must withstand weathering, electrolysis due to leakage currents, temperature changes, and abrasion due to conductor and tie wire motion. Further such coatings must not terminate in positions that will overstress the air in other parts of the dielectric circuit, and the flashover value of the insulator must not be reduced below the required factor of safety. If insulating compound is used, it must withstand weathering, stay in place through the extremes of temperature to which it may be subjected, and not be displaced by the motion of the conductor and tie wire to the extent of impairing its effectiveness.

Although the design of a satisfactory insulator of practical proportions and which will be suitable for mass production apparently is not a simple problem, it is believed to be entirely possible to meet the requirements outlined and build an insulator entirely free from radio interference at a safe margin above its normal operating voltage.

INSULATOR CORONA CURRENT INVESTIGATION

The magnitude and character of the corona current of high voltage insulators is of great interest and importance because these characteristics are direct indications of the radio interference radiation produced. Corona current is not easily studied because (1) it is quite small, attaining maximum values of approximately 40 microamperes for normal voltage on a standard 66-kv. insulator; (2) it is obscured by the insulator charging current which is approximately 0.4 milliampere maximum or 10 times the corona current for an insulator of this size; and (3) it has superimposed oscillations of very high frequency.

An instrument suitable for the investigation of currents of these magnitudes and characteristics must have a high sensitivity and be capable of responding faithfully to high frequencies. The device that meets these requirements most satisfactorily is the low voltage cathode ray oscillograph. Because of the low anode-cathode voltage and resulting low velocity of the electron stream, this instrument has the highest sensitivity of all of the cathode ray oscillographs; and the cathode beam diffusion due to the relative immobility of the positive focusing ions is not excessive for beam speeds of 3x10^6 cm. per sec. on the fluorescent screen.

Although the cathode ray oscillograph is well suited to the study of high frequency discharges, it will not of itself separate the corona current from the charging current. To overcome this difficulty the circuit shown in Fig. 2 was devised to balance out the charging current leaving only the corona current to produce deflection. With this circuit arrangement, it is possible by adjusting R_1 and C_3 to make the vertical deflecting potential on the oscillograph zero for all insulator voltages up to the critical corona formation voltage. At this critical voltage and for higher voltages, the oscillograph vertical deflecting voltage is proportional to the instantaneous value of the insulator corona current in R_2. This condition obtains because C_1 is free from corona. A glass insulator with dielectric flux distributing coatings on the head and in the pin-hole, and found by test to be free from corona up to 50 kv., was used for C_1 when the accompanying cathode ray oscillograms were taken.

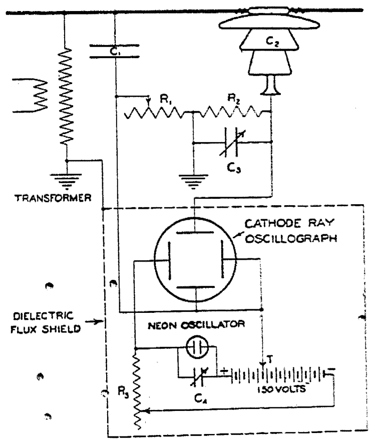

FIG. 2—CIRCUIT FOR OBTAINING CATHODE RAY OSCILLOGRAMS OF INSULATOR CORONA CURRENT. NOTE THAT THE INSULATOR CHARGING CURRENT IS BALANCED OUT

The horizontal deflecting or sweeping voltage for the oscillograph was obtained from a neon oscillator connected as shown in Fig. 2. This type of oscillator produces practically a straight line time axis that can be operated over a wide range of sweeping frequencies by the adjustment of R_3 and C_4. This sweeping oscillator can be synchronized readily with the frequency producing the phenomena under observation so that a stationary image either for observation or recording photographically is obtained. To eliminate stray dielectric fields a grounded wire cage 5x5x6 ft. high was used to shield all apparatus associated with the oscillograph and neon oscillator.

Voltage was applied to the laboratory line and increased to a value just below the critical corona voltage for the insulator under test. At this voltage, resistance R_1 and capacitance C_3 were adjusted to make the vertical deflecting voltage zero. These adjustments were made while the neon oscillator was sweeping the cathode beam over the time axis, conditions under which any slight inaccuracies in balance were detected readily by the deviation of the beam from a straight line. When this balance was established accurately, a series of corona current oscillograms was taken for the insulator at various conductor to pin voltages from 10 to 40 kv., r.m.s., in increments of 5 kv.

In Fig. 4 is shown a series of corona current oscillograms taken on a standard three-shell 66-kv. porcelain insulator with a ¾-in. malleable thimble cemented in the pin-hole. Oscillogram *G* (40 kv.) shows the normal corona current for this insulator operating on a three-phase transmission line at a line potential of 69.3 kv. Atmospheric conditions were representative and normal, air density being 1.006 and the absolute humidity 9.4 g. per cubic meter.

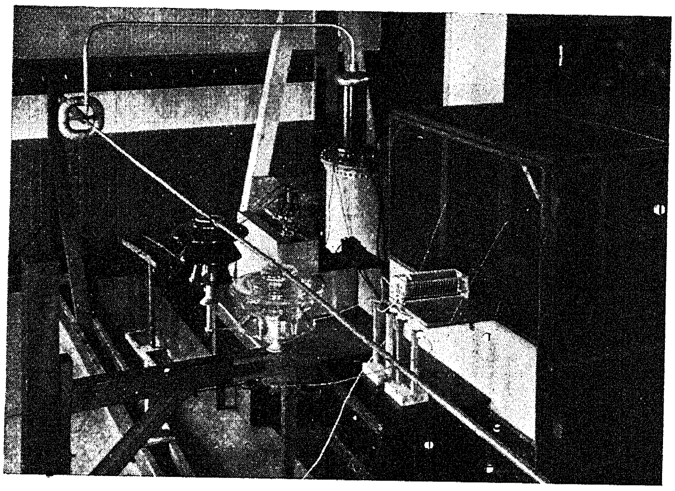

Fig. 3—Apparatus for Obtaining Cathode Ray Oscillograms of Insulator Corona Current

Oscillograms reproduced in Fig. 4 show that when the ionization gradient is reached, the current rises to a maximum value quite abruptly during the positive half cycle and decays rapidly with considerable oscillation, reaching zero practically coincident with the occurrence of maximum voltage on the insulator. During the negative half cycle the corona current does not reach its maximum value in so short a time interval as in the positive half cycle, the oscillations appear to be much more violent, and the corona current is sustained at a high value for a longer time. The higher sustained value of negative current is shown clearly by the fact that for the same line voltage the area under the negative wave is greater than under the positive wave.

Unfortunately the oscillograms do not show the oscillations clearly. This is because that in order to form an image on photographic film it is necessary to trace several successive cycles of corona current on the fluorescent screen. On account of the chance character of corona formation, successive cycles of corona current are not alike, and where the oscillations are the greatest, the variation in successive figures is largest. This variation in the registration of successive cycles is shown on the oscillogram by variations in intensity of exposure and sharpness of the image.

Corona current oscillograms were taken also on a special three-shell 66-kv. porcelain insulator with a silver and bismuth coating on the head and in the pin-hole. This insulator (No. B-6 of Table I) was practically identical with the previous one except for the addition of the metallic coatings. In this case no corona current was observed below 30 kv. The difference in the character of positive and negative corona currents on this insulator was greater than for the standard insulators. The positive current shows much less oscillation than the negative, and the duration of the positive current was observed to be much shorter than that of the negative.

Metallic dielectric flux distributors on this special insulator greatly improved the characteristics from the standpoint of radio interference. The interference starting voltage was increased from slightly above 10 kv. to 23.5 kv., and the maximum corona current at 40 kv. was reduced from 40 to 11 microamperes. However, the voltage gradient was noted to be excessive at the edges of the metallic coatings on the head and in the pin-hole at 23.5 kv., which is only 61.6 per cent of normal operating voltage.

Similar oscillograms were taken for a standard 70-kv. one-piece glass insulator without any treatment to prevent corona formation. These oscillograms show the same general corona current characteristics obtained with the standard porcelain insulator. Cathode ray oscillograms for a similar glass insulator with high resistance iridescent dielectric flux distributors on the head and in the pin-hole revealed no corona current for voltages up to 40 kv. when these coatings were in good condition.

Oscillographic Study of Radio Receiver Output

In order to determine the value of the cathode ray oscillograms of insulator corona currents for indicating the interfering noise that will be produced in a radio receiver, a large number of both Duddell and cathode ray oscillograms was taken of the audio frequency output from a superheterodyne receiver. These were taken with the receiver tuned for a frequency of 1,000 kc. and receiving only the radiation from the laboratory line equipped with one insulator. The circuit conditions and voltages used were the same as those employed when the cathode ray oscillograms of corona current discussed previously were taken.

A typical set of oscillograms for this portion of the investigation may be seen in Fig. 5. In these the polarity and magnitude of the insulator voltage as well as the audio frequency current output of the radio receiver are shown for the standard, three-shell, 66-kv. porcelain insulator for which corona current oscillograms were shown in Fig. 4. It should be observed that the oscillations in the audio output have frequencies reaching values of 5,000 cycles per sec., which is the natural period of oscillation for the oscillograph vibrator element. Therefore the amplitude indications are not very reliable. Notwithstanding this fact, the oscillograms show accurately the beginning of the corona discharge and quite accurately the end of the corona discharge with reference to each half cycle of the insulator voltage. At 14.1 kv. maximum, there is just the slightest trace of interference, occurring at practically the crests of the voltage waves. At 21.2 kv. maximum, the corona starts at a very small electrical angle and at 28.3 kv. maximum, it appears at the zero point of the voltage wave. For higher voltages the corona starts before the voltage wave reverses polarity. For all values of voltage the corona discharge stops at the crest of the voltage wave. This is shown by the fact that at that point the oscillations stop. At the higher voltages the smooth decay of current following the oscillations is due to the recovery of the receiving set from overloading.

The phenomenon of obtaining critical ionizing voltage gradients on the insulator at zero, and even with opposing conductor potentials, is due to the large space charge established in the space and on the insulator surface surrounding the conducting materials where ionization has taken place during the previous half cycle. This is brought out in greater detail in the following discussion.

A THEORY OF THE FORMATION OF INSULATOR CORONA

A theory of the phenomena associated with corona formation on an insulator can be developed by assuming the initial half cycle of conductor voltage to be either positive or negative. For the following discussion the initial polarity is assumed to be negative and the ionization of the air surrounding the conducting parts and in zones of high voltage stress is assumed to be only the normal atmospheric value at the instant the voltage first is applied.

During the initial negative half cycle, the dielectric field of the negatively charged conductor repels the free electrons and negative ions in the regions of high voltage gradient; when the critical ionizing voltage gradient is reached, they bombard the air molecules with such violence that ionization by collision results. The additional electrons and negative ions produced by collision are repelled from the conducting material by the electric field, and form the major part of an outside negative space charge. The heavy immobile positive ions move relatively slowly toward the conductor in the regions of ionization. In this manner corona increases and the positive ion space charge that accumulates due to the relative immobility of the positive ions subtracts from the negative field of the conductor until the crest of the voltage wave is reached. At this point ionization stops because the space occupied by the corona is completely ionized; the voltage gradient at the outside boundary of the corona discharge is just equal to the critical value and must decrease below this value as the conductor voltage decreases. For the negative conductor condition under

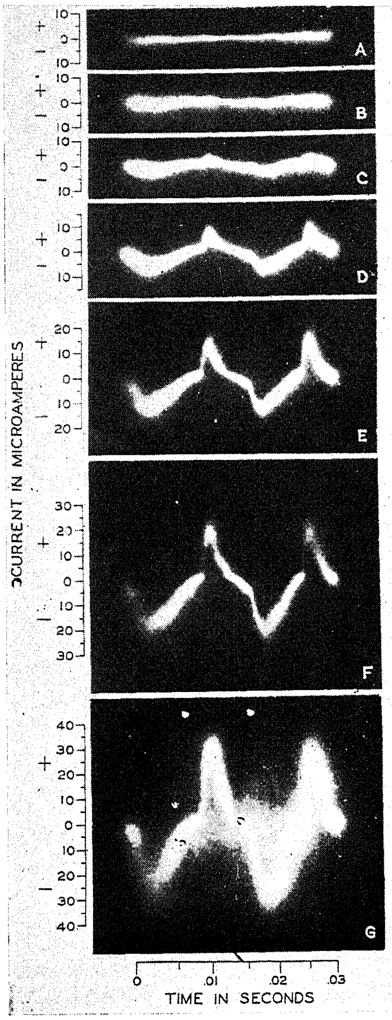

FIG. 4—CATHODE RAY OSCILLOGRAMS OF CORONA CURRENT FOR A STANDARD 66-KV. PORCELAIN PIN TYPE INSULATOR

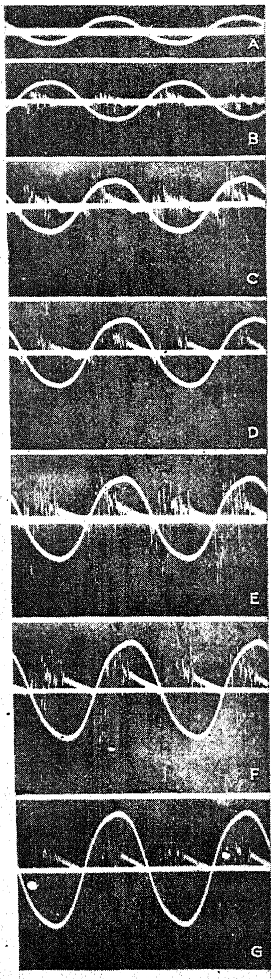

FIG. 5—INSULATOR VOLTAGE (60-CYCLE) AND OUTPUT OF RADIO RECEIVER SHOWING NOISE FROM CORONA ON INSULATOR OF FIG. 4

	Insulator kv. conductor to pin	
	r.m.s.	maximum
A—	10	14.1
B—	15	21.5
C—	20	28.3
D—	25	35.4
E—	30	42.5
F—	35	49.5
G—	40	56.6

Test conditions (Figs. 4 and 5)	
Barometric pressure	756.0 mm.
Temperature, dry bulb	21.6 deg. cent.
wet bulb	15.3 deg. cent.
Relative humidity	50.0 per cent
Absolute humidity	94.0 g. per cu.m.

consideration the positive space charge limits the corona to a comparatively small discharge distance. Many of the free electrons are swept out of the ionization zone and the loss of positive ions by recombination adjacent to the conductor thereby is reduced to a relatively small number. Large numbers of positive ions always are present therefore adjacent to the conductor; this is such a favorable position that a comparatively small positive space charge will reduce greatly the field of a large negative conductor voltage at the outside boundary of the corona discharge.

After the negative voltage wave passes the crest and starts to decrease, the negative space charge at the outer boundary of the corona no longer is replenished by new ionization; hence it begins to diffuse and be lost by recombination. This action releases the positive space charge and allows it to move more rapidly back toward the conductor, which movement tends to increase the gradient in the space between the positive space charge and the conductor. Because of the decrease in conductor potential, ionizing voltage gradients prior to actual reversal of conductor potential are reached only when the space charge is very large. The latter phenomenon does occur, however, for all maximum applied voltages in excess of approximately twice the maximum visual critical corona voltage.

When the conductor voltage reverses and becomes positive after the described phenomena have taken place, the dielectric flux of the residual positive space charge adds to the field of the positive potential on the conductor. The addition of those fluxes causes the critical ionizing voltage gradient to occur at lower and lower values of instantaneous positive conductor voltages as the maximum conductor voltage is increased above the visual critical corona value. The resultant positive field accelerates the electrons in the surrounding space, and when high velocity collisions occur with molecules of air, ionization is produced. Large numbers of the electrons liberated are swept from the field and removed by the conductor, leaving more immobile positive ions to add to the positive space charge. This increase in space charge increases the dielectric flux density and extends the ionizing voltage gradient to a far greater distance from the conductor than would be possible with the conductor voltage acting alone. When the crest of the positive voltage wave is reached, the voltage gradient is at maximum and the corona discharge stops because the space occupied by the corona is completely ionized; the decrease in the conductor voltage makes ionization at a greater distance from the conductor impossible. As the conductor potential decreases it drops below space charge potential and a field is established between space charge and conductor. The field produces ionizing voltage gradients before the conductor voltage reverses, when the space charge potentials are very high. This phenomenon occurs for maximum applied voltages in excess of approximately twice the maximum visual critical corona voltage.

During the rise in voltage on the positive half cycle a much larger positive space charge is established than that produced during the same period of the negative half cycle. This is because when the conductor is positive, the positive space charge flux adds to the conductor field and extends the ionization gradient, when it is negative, the positive space charge flux subtracts from the negative conductor field at the outside corona boundary.

When the conductor voltage reverses and becomes negative following the establishment of a large positive space charge in the manner just described, a very high voltage gradient is produced between the negative conductor and the residual positive space charge because of the very short distance between them. When this voltage gradient reaches the critical ionizing value (which may occur before the voltage actually becomes negative) breakdown occurs in the form of minute arcs which neutralize the space charge. These corona arcs produce sudden changes in the electric circuit conditions, thereby causing high frequency oscillations of a violent character.

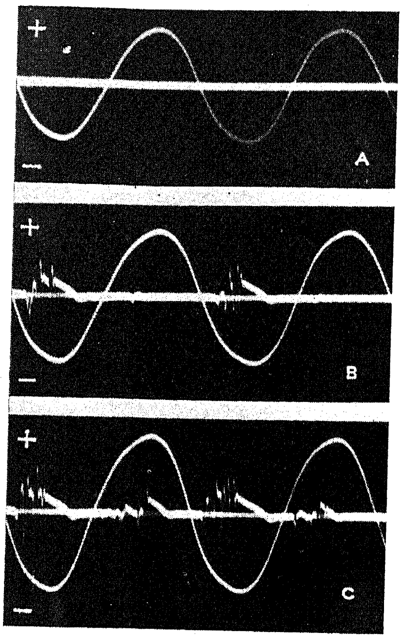

FIG. 6—INSULATOR VOLTAGE (60 CYCLE) AND OUTPUT OF RADIO RECEIVER SHOWING MUCH MORE INTERFERENCE DURING THE NEGATIVE THAN DURING THE POSITIVE HALF CYCLES. PORCELAIN INSULATOR (66 KV.) WITH SILVER AND BISMUTH COATING ON HEAD AND IN THE PIN-HOLE

	Insulator kv. conductor to pin	
	r.m.s.	maximum
A—	30	42.5
B—	35	49.5
C—	40	56.6

Barometric pressure	760.0 mm.
Temperature dry bulb	21.5 deg. cent.
wet bulb	16.0 deg. cent.
Relative humidity	56.0 per cent
Absolute humidity	10.6 g. per cu.m.

Radio Interference from Positive and Negative Corona

Near the critical corona formation voltage the electric circuit oscillations and the electromagnetic radiation resulting from them are inherently much worse for 60-cycle a-c. circuits when the conductor is negative than when it is positive. Theoretical considerations outlined in the preceding paragraphs indicate that this condition is to be expected and experimental investigations show a very decided polarity effect under certain conditions. For the special 66-kv. porcelain insulator with silver and bismuth coatings on the head and in the pin-hole, oscillograms taken show a very striking polarity effect in the interference radiation. (See Fig. 6.)

From the experimental investigations it appears that the marked difference in radio interference when the line is positive and when it is negative, respectively, can be observed only when the insulator has a definite critical corona formation voltage. When the voltage is increased far above this critical value, the polarity effect becomes less marked and at still higher voltages probably would disappear almost entirely. This phenomenon accounts for the fact that practically no interference polarity effect was observable in the tests on standard insulators. (See Fig. 5.) On such insulators there is a heterogeneous group of visual critical voltage gradients that cause corona to form in small areas at various locations for every different value of insulator voltages from approximately 25 per cent normal operating voltage up to the flashover value. There the polarity effect is obscured because when sufficient voltage is applied to produce appreciable corona on the insulator, the regions that go into corona at low voltage are subjected to an excessive voltage gradient and therefore produce practically the same interference on both positive and negative half cycles.

Summary

The findings of this study may be summarized as follows:

1. Standard multi-shell pin type insulators of conventional design are susceptible to corona formation at three different points.

2. Radio interference and visible corona formation for clean insulators start at the same voltage.

3. Radio interference from insulator corona may be reduced by (a) designing the insulator so as to eliminate regions of overstressed air; (b) providing conducting coatings on different parts to act as dielectric flux distributors; and (c) displacing the overstressed air with an insulating compound.

4. Insulator corona current can be separated from the charging current and studied independently by means of the cathode ray oscillograph.

5. Duddell oscillograms of the line voltage, and the audio frequency output of a radio set receiving the radiation from an insulator or other source of corona, provide an excellent means for studying certain corona characteristics.

6. The radio interference radiation from an insulator having a definite critical corona voltage shows a decided polarity effect for all corona voltages up to approximately twice the critical disruptive value.

Acknowledgments

The writer is indebted to his associates in this work, Prof. E. C. Starr and Mr. L. T. Fisher, for their suggestions, contributions, and active participation in the investigation. The work has been made possible through the cooperation of the engineering experiment station (Oregon State College) under the direction of Dean H. S. Rogers and the foreign systems coordination committee of the National Electric Light Association, represented by Mr. H. H. Schoolfield.

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Predetermination of the A-C. Characteristics of Dielectrics

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Synopsis.—*From the charge and discharge currents of any type of dielectric under continuous potential it is possible to predict accurately the loss, power factor and capacity at 60 cycles. This is done by an empirical determination of the equation for the relaxation function of the dielectric at a given temperature, followed by the application of von Schweidler's method. The method developed is available at any frequency, provided that the continuous potential charge and discharge currents may be measured over initial time intervals comparable with the alternating period.*

It is shown that agreement of the relaxation function with the t^{-n} *expression is not general, but on the contrary, it is confined to dielectrics showing negligible or no irreversible conduction. The inadequacy of the* t^{-n} *expression is proved on experimental and analytical reasons. Chief among them is the failure of the* t^{-n} *expression to predict accurately the variation of dielectric loss with frequency.*

A convenient and sufficient expression for the relaxation function is shown to be a sum of three exponentials. Further, experiment and analysis prove that the method of three exponentials predicts accurately the a-c. behavior of a dielectric at 60 cycles.

The usual forms of irreversible conduction encountered are defined and classified. The case where the initial constant current does not obey Ohm's law has been considered analytically in its contribution to the a-c. behavior.

The manner in which the complete a-c. behavior of a dielectric is accurately predicted from suitable d-c. measurements is illustrated by three typical sets of experimental data; paper, oil, and impregnated paper.

* * * * *

INTRODUCTION

THE purpose of this paper is to give the experimental proof that the a-c. behavior of a dielectric at 60 cycles may be accurately predicted from suitable d-c. measurements. The fact that dielectric loss may be explained in terms of reversible absorption, or residual charge, and anomalous conduction, as observed under continuous potential, has led to several well known extensions to the alternating case of various theories of dielectric absorption. Notable among these are Wagner's[1] extension of Maxwell's stratified dielectric and von Schweidler's[2] extension of Pellat's theory. von Schweidler has deduced the expression for the dielectric loss in an alternating field from the equation of the empirical absorption current-time relation as observed under continuous potential. The work of F. Tank,[3] 1915, apparently represents the first effort made using von Schweidler's method for a direct check between measured and computed losses. Since then other checks on this method have been made by Whitehead,[4] Benedict,[5] and others. This paper discusses further the possibilities and limitations.

The continuous potential measurements, using the amplifier-oscillograph, give records on photographic films of the charge and discharge current-time relations for the interval of time 0.001 to 0.040 sec. after the application of voltage or of short-circuit. The geometric transient, both on charge and discharge, confined by choice of circuit constants to extremely short intervals of time is, therefore, absent from such records. This permits the accurate determination of the reversible absorption current directly from the discharge records and of the irreversible conduction current as the difference between the corresponding ordinates of charge and discharge.[6] The usual measurements of charge and discharge current taken with a galvanometer and beginning about a minute after the application of voltage or of short-circuit, give values which in most cases have no bearing on the a-c. behavior at commercial frequencies. The condition for suitable d-c. measurements for the accurate prediction of the a-c. behavior is that the range of time covered by the d-c. measurements must be of the same order of magnitude as the period of the alternating wave.

VON SCHWEIDLER'S ANALYSIS

The several components of alternating current in a dielectric at a given frequency may conveniently be represented in a vector diagram. Fig. 1 shows the familiar Wagner diagram for a capacitor exhibiting the anomalous properties of reversible absorption and irreversible conduction in addition to the normal property of specific inductive capacity. von Schweidler's method permits the computation from the curves of d-c. charge and discharge currents of each of the separate components, thereby giving dielectric loss, phase difference, and the increase in capacity due to absorption. Under continuous potential the irreversible conduction current is usually characterized by a constant initial conductivity which is independent of the applied stress for low stresses and which is almost invariably larger than the final or normal conductivity. The contribution of the initial irreversible conduction to the alternating case is discussed in a later section. Reversible absorption under continuous potential is characterized by a decaying current function of time

$$i_r(t) = EC_0\phi(t) \qquad (1)$$

where E is the applied voltage, C_0 the geometric capacitance, and $\phi(t)$ the relaxation function corresponding to the dielectric at a given temperature; it is measured directly in the d-c. discharge curve. Equation (**1**) embodies Curie's[8] well known laws; namely, that the ordinates of the absorption current-time relation be directly proportional to the applied voltage and to the dimensions of the capacitor. If these laws are obeyed then the Hopkinson[7]-Curie[8] principle of superposition is applicable to the determination of the current under a varying voltage. von Schweidler's fundamental equation for the current, as dependent on any e.m.f. variation $E(\tau)$ in the past, and as based on the principle of superposition, is

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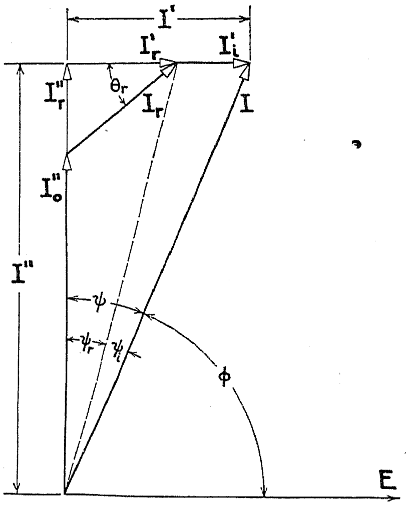

FIG. 1—VECTOR DIAGRAM OF A CAPACITOR SHOWING REVERSIBLE ABSORPTION AND IRREVERSIBLE CONDUCTION IN ADDITION TO SPECIFIC INDUCTIVE CAPACITY

Current Vector	Contribution of
I''_o	Geometric Capacitance
I_r	Reversible Absorption
I'_i	Irreversible Conduction

$$i(t) = E(o)\phi(t) + \int_o^t \frac{d}{d\tau} E(\tau)\phi(t-\tau)\,d\tau \qquad (2)$$

which he applies to determine the steady state for the case of the alternating sinusoidal applied potential. The additional admittance contributed to a capacitor by reversible absorption, in complex notation, is shown to be:

$$Y_r = \omega C_0(B + iA) \qquad (3)$$

where $\omega = 2\,\pi \times$ frequency and

$$\left.\begin{aligned} A &= \int_o^\infty \cos \omega t \,.\, \phi(t) \,.\, dt \\ B &= \int_o^\infty \sin \omega t \,.\, \phi(t) \,.\, dt \end{aligned}\right\} \qquad (4)$$

THE RELAXATION FUNCTION

Hopkinson, Curie, Tank, and others have reported experimental agreement of the relaxation function with the expression:

$$\phi(t) = \beta t^{-n} \qquad (5)$$

von Schweidler gives in this case the values of the infinite integrals of (**4**), as:

$$\left.\begin{aligned} A &= \omega^{n-1}\beta\Gamma(1-n)\cos\frac{(1-n)\pi}{2} \\ &\text{converges for } o<n<1 \\ B &= \omega^{n-1}\beta\Gamma\,(1-n)\sin\frac{(1-n)\pi}{2} \\ &\text{converges for } o<n<2 \end{aligned}\right\} \qquad (6)$$

In all the cases mentioned above the values of n given lie between 0 and 1. Benedict[5] shows from experiments with solid dielectrics the same type of agreement up to 100 milliseconds. Whitehead,[4] however, has reported several cases of values of $n>1$, which introduces a serious objection to the use of the type of relaxation function given in equation (**5**). As an example, in tests on commercial abietic acid over a range of temperature, Whitehead[9] has found that the discharge current-time relation is of the t^{-n} type with values of n ranging from 0.1 to 2.5.

In spite of the wide spread experimental agreement with the t^{-n} expression it is found to be inadequate for the accurate prediction of the a-c. behavior for the following reasons:

1. Analytically, when $n>1$, the value of the A integral in (**6**) above does not converge and it becomes impossible to compute the increase in capacity due to absorption.

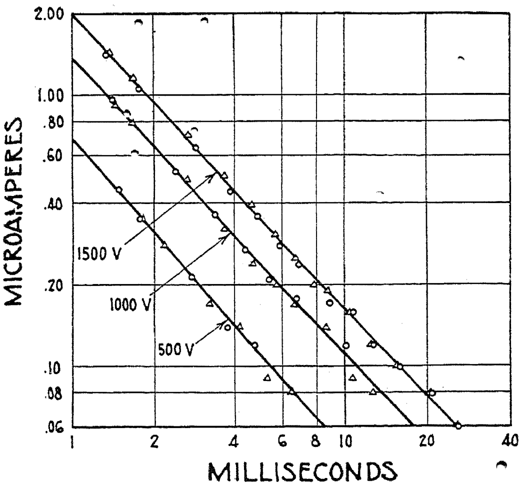

FIG. 2—VARIATION WITH VOLTAGE OF CHARGE AND DISCHARGE CURRENT-TIME RELATIONS AT 60°C. DRY PAPER SPECIMEN A7. ILLUSTRATES AGREEMENT WITH t^{-n} EXPRESSION FOR VALUES OF $n > 1$.

2. Examination of the expression for the B integral discloses a mode of variation of power factor with frequency which is contrary to both theory and experiment. The computed power factor will decrease, remain constant, or increase with frequency depending on whether the value of n is less than one, equal to one, or greater than one, but in no case will it show a power

factor maximum as required by experience based on well accepted theories.

3. Experimentally, the negative power of time calling as it does for infinite current at zero time, gives rise to computed losses which are usually in excess of the measured losses.

4. Finally, the t^{-n} expression is by no means general, for noted departures are found in many cases.

Fig. 2 gives the current-time relations at three voltages for a specimen of dry, unimpregnated paper showing agreement with t^{-n} expression for $n > 1$. Dielectric loss in dry paper specimens is entirely due to reversible absorption as evinced by the equality of charge and discharge currents. The irreversible conduction current even at 60 deg. cent. is too small to be detected by the amplifier-oscillograph. Fig. 3 gives the corresponding discharge current-time relations for the same paper specimen after impregnation with an oil whose electrical properties are described later. Note in this case the decided departure of the absorption (discharge) current-time relation from the t^{-n} expression. We find in general, that if a capacitor exhibits charge and discharge curves which are identical, that is, if its dielectric loss is due entirely to reversible absorption, then the empirical current-time relation obeys the negative power law of equation (5). If, on the other hand, a capacitor shows measurable irreversible conduction, or if it is made to acquire it, either through elevation in temperature or as in the case of a paper specimen, through impregnation, then the observed discharge current-time relation will differ materially from the simple t^{-n} expression.

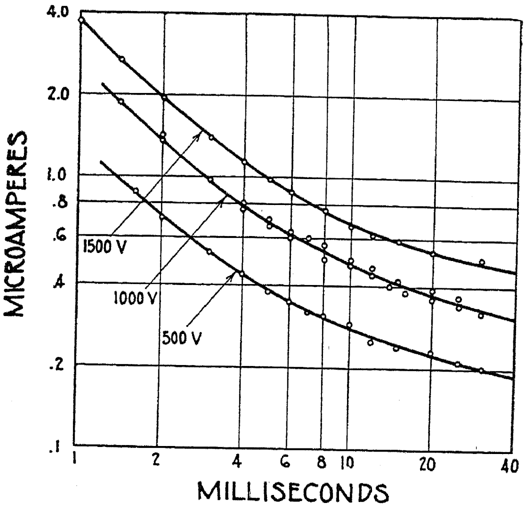

FIG. 3-VARIATION WITH VOLTAGE OF DISCHARGE CURRENT TIME RELATIONS AT 60°C. IMPREGNATED PAPER SPECIMEN C7. ILLUSTRATES DEPARTURE FROM t^{-n} EXPRESSION.

THE METHOD OF THREE EXPONENTIALS

A more general expression for the relaxation function which is not open to the analytical objections of the t^{-n} type is a series of exponentials of the form

$$\phi(t) = \beta_1 e^{-\alpha_1 t} + \beta_2 e^{-\alpha_2 t} + \ldots + \beta_n e^{-\alpha_n t} \tag{7}$$

for which the A and B integrals of (4) become

$$\left.\begin{aligned} A &= \frac{\alpha_1\beta_1}{\omega^2 + \alpha_1^2} + \frac{\alpha_2\beta_2}{\omega^2 + \alpha_2^2} + \ldots + \frac{\alpha_n\beta_n}{\omega^2 + \alpha_n^2} \\ B &= \frac{\omega\beta_1}{\omega^2 + \alpha_1^2} + \frac{\omega\beta_2}{\omega^2 + \alpha_2^2} + \ldots + \frac{\omega\beta_n}{\omega^2 + \alpha_n^2} \end{aligned}\right\} \tag{8}$$

Hopkinson, Steinmetz, Wagner, and others have found that three terms of equation (7) usually will suffice to express the empirical equation for the observed relaxation function.

The resolution of a given observed current-time relation into the sum of three exponentials is indeterminate and the two constants of each term may take wide ranges of values. Analytically this means that a set of six constants corresponding to the three exponentials is so chosen that the resultant equation will be satisfied by any six given points of the observed current-time relation. This implies six simultaneous equations involving transcendental functions, the solution of which can only be obtained by graphical means. To illustrate with a concrete example, consider the 1,500-volt curve of Fig. 2 and resolve it into three exponentials by the method of successive residuals as outlined by J. Lipka.[10]

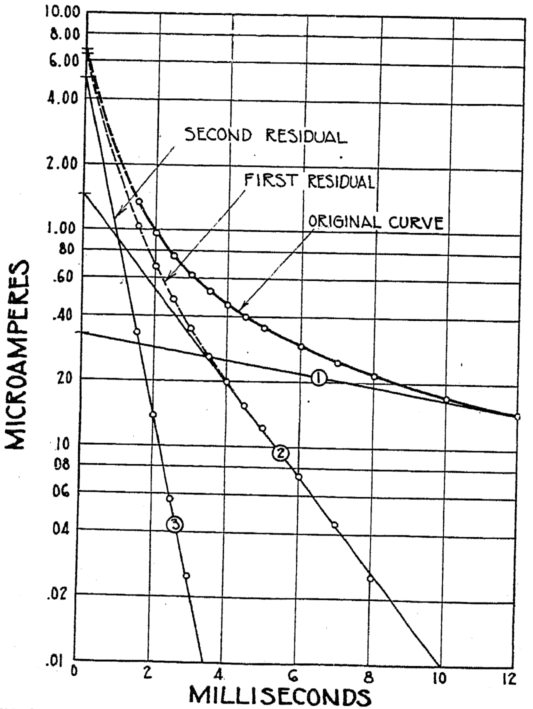

FIG.4-APPLICATION OF THE METHOD OF THREE EXPONENTIALS TO A CHARGE AND DISCHARGE CURRENT-TIME RELATION AT 1500 VOLTS. DRY PAPER SPECIMEN A7 AT 60°C

1. Redraw the curve in semilogarithmic coordinates; that is, the logarithm of the current against the time in seconds. This is shown by the original curve in Fig. 4.

2. At any arbitrarily chosen value of time, say 12 milliseconds in this case, draw a tangent to the curve terming this straight line the *first exponential.*

3. By subtracting the corresponding ordinates of the first exponential from the original curve obtain the *first residual* (see Fig. 4). Draw a tangent to this curve and note that its position is no longer arbitrary, but is defined by the later part of the curve with points lying in a straight line. This straight line is termed the *second exponential.*

4. By subtracting the corresponding ordinates of the second exponential from the first residual obtain the *second residual,* which this time is completely defined by a series of points lying in a straight line and constituting the last and *third exponential.* Obviously, if the second residual had not been expressible as a straight line it would have been necessary to continue the method to a fourth or even a fifth exponential. In practise, however, this is seldom the case.

5. For each exponential the intercept on the current axis is a measure of the constant multiplier β and its slope a measure of the inverse time constant α. This permits the computation of the empirical current-time relation and its direct comparison with the observed curve. Table I gives such a comparison showing remarkable agreement for the range of time investigated.

TABLE I—COMPARISON OF OBSERVED AND CALCULATED VALUES OF CURRENT. THE METHOD OF THREE EXPONENTIALS. CHARGE AND DISCHARGE CURRENT-TIME RELATION AT 1,500 VOLTS

Dry Paper Specimen A-7 at 60 deg. cent.

Exponential	α	$i\,(o)$
(1)	69	0.33×10^{-6}
(2)	500	1.45×10^{-6}
(3)	1,787	5.00×10^{-6}

(t) Time in seconds	Current: microamperes				
	Calculated				Observed
	$i_1(o)e^{-\alpha_1 t}$	$i_2\,(o)e^{-\alpha_2 t}$	$i_3(o)e^{-\alpha_3 t}$	$i(t)$	$i(t)$
0.002	0.2875	0.5335	0.1402	0.9612	0.960
0.004	0.2504	0.1962	0.0039	0.4505	0.450
0.006	0.2181	0.0722	0.0001	0.2904	0.290
0.008	0.1900	0.0266	—	0.2166	0.215
0.010	0.1655	0.0098	—	0.1753	0.171
0.012	0.1442	0.0036	—	0.1478	0.145

From equation (3) the components of current contributed by reversible absorption at a given frequency and voltage are:

$$\left.\begin{aligned} &\text{Quadrature component} \quad I_r'' = AI_0'' \\ &\text{In phase component} \quad I_r' = BI_0'' \end{aligned}\right\} \qquad (9)$$

where $I_0'' = \omega E C_0$, see Fig. 1, and where the A and B integrals are computed for a given frequency from the constants of the three exponentials by means of equations (8). The above values now permit the computation of the contribution of reversible absorption to the a-c. behavior, since A and B are simple functions of the α's and β's. Thus we have:

$$\left.\begin{aligned} &\text{Phase defect angle} \tan \Psi_r = \frac{B}{1 + A} \\ &\text{Dielectric loss} \qquad W_r = \omega E^2 C_0 B \\ &\text{Apparent capacitance } C' = C_0(1 + A) \end{aligned}\right\} \qquad (10)$$

Equations (10) show the dependence of the computed a-c. behavior on the values of the A and B integrals. These, however, for a given dielectric at a given temperature are functions of the constants α and β and also of the frequency. It therefore becomes important to examine how wide a range of choice of the constants α and β as related to the frequency is permissible for agreement of computed and measured values. Assume therefore for simplicity a current-time relation of a single exponential term and examine the theoretical variation with frequency of the A and B integrals which now become:

$$\left.\begin{aligned} A(\omega) &= \frac{\alpha\beta}{\omega^2 + \alpha^2} \\ B(\omega) &= \frac{\omega\beta}{\omega^2 + \alpha^2} \end{aligned}\right\} \qquad (11)$$

By change of variable

$$e^u = \frac{\omega}{\alpha} \qquad (12)$$

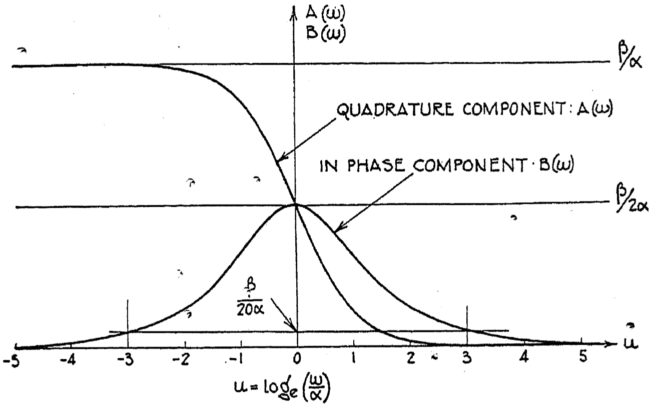

FIG. 5—VARIATION WITH FREQUENCY OF THE IN PHASE AND QUADRATURE COMPONENTS OF A SINGLE EXPONENTIAL.

obtain the relations

$$\left.\begin{aligned} A(u) &= \frac{\beta}{2\,\alpha}\, e^{-u} \operatorname{sech} u \\ B(u) &= \frac{\beta}{2\,\alpha} \operatorname{sech} u \end{aligned}\right\} \qquad (13)$$

which when plotted as shown in Fig. 5 are useful in showing the symmetry of the functions. The contribution to the in-phase component attains a maximum equal to $\beta/2\alpha$ at a value of frequency for which $\omega = \alpha$. The value of $B(\omega)$ falls off rapidly as the ratio ω/α departs either side from unity. For example, an exponential giving its maximum contribution to the in-phase component at $\omega = 100$ would contribute only one-tenth of the maximum value at either $\omega = 5$ or $\omega = 2{,}000$. This is readily deduced from Fig. 5. The variation with frequency of the in-phase component for a relaxation function expressible as the sum of several exponentials is given by the summation of a number of curves of the type $B(\omega)$ in Fig. 5, corresponding to the several exponentials. We have found that the maximum of a fourth exponential that might be introduced occurs at a frequency far from the frequency being studied, and that its contribution to the total in-phase component is negligible. The maximum of each contribution occurs at a value of ω equal to the inverse time constant α of that exponential. Therefore only the exponentials with values of α in the neighborhood of ω at the frequency at which correlation is intended contribute an appreciable amount to the loss component $B(\omega)$. Thus for agreement between measured and computed losses at 60 cycles, where $\omega = 377$, the values of α range approximately from 50 to 2,000. The same reasoning may be applied to examine the time range within which the d-c. measurements will give accurate prediction of the a-c. behavior. It shows that the values of the α's obtained for the three exponentials must range in the neighborhood of the value of ω corresponding to the frequency at which correlation is intended. This means that the range of time covered by the d-c. measurements must be of the same order of magnitude as the period of the alternating wave.

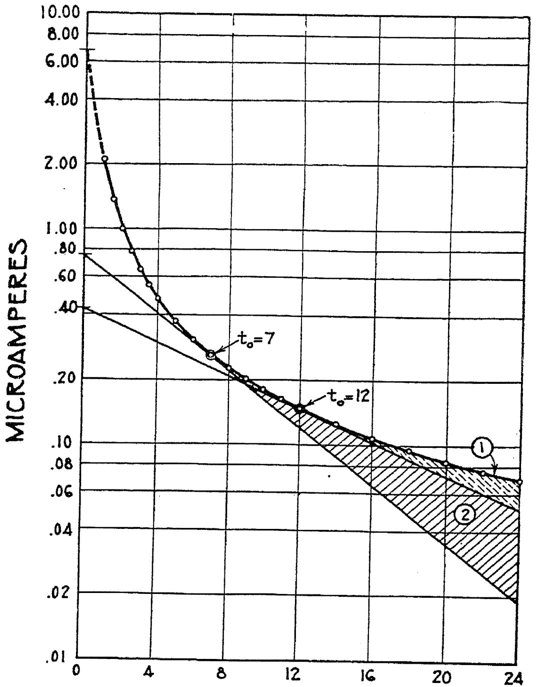

FIG. 6—CURRENT-TIME RELATION SHOWING THE FIRST TANGENT DRAWN AT TWO DIFFERENT VALUES OF "t_0" SHADED AREAS SEGREGATE IN EACH CASE THE NEGLECTED PORTION OF THE CURVE

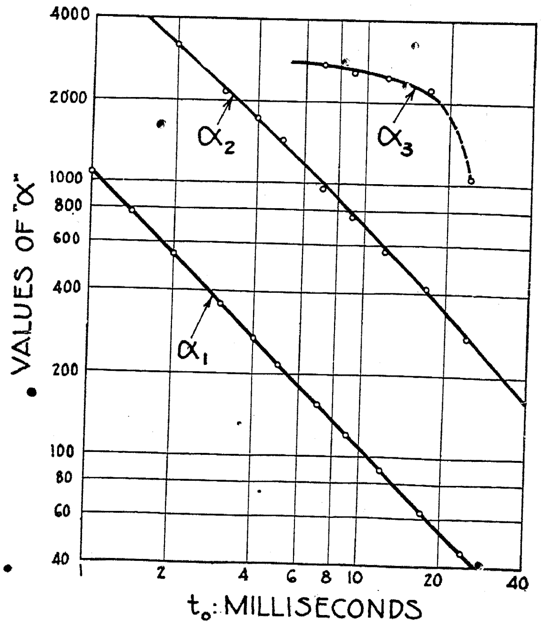

FIG. 7—VARIATION WITH "t_0" OF THE INVERSE TIME CONSTANT "α" OF EACH EXPONENTIAL

ACCURACY OF THE METHOD

It has been stated that the resolution of a current-time relation into three exponentials is quite indeterminate. The constants of the three exponentials depend entirely on the value of time t_0, of the empirical discharge current-time relation, at which the first tangent is drawn. Fig. 6 illustrates the point for the current-time relation of Fig. 4 showing the first tangent drawn at two different values of t_0 with the shaded areas indicating in each case the neglected portions of the curve.

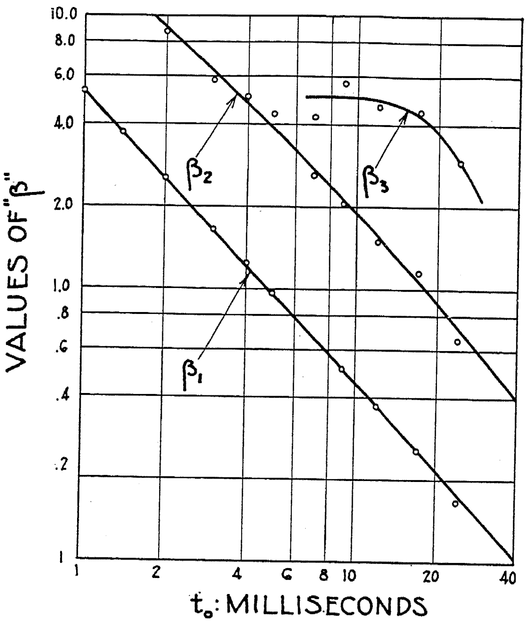

FIG. 8—VARIATION WITH "t_0" OF THE CONSTANT MULTIPLIER "β" OF EACH EXPONENTIAL.

Figs. 7, 8 and 9 give respectively for each exponential the variation with t_0 of the inverse time constant α, the constant multiplier β, and their ratio β/α. This ratio is shown in Fig. 5 to be a measure of the maximum contributions to the in-phase and quadrature components with varying frequency. Figs. 7 and 8 are significant in pointing out the definite dependence of the α and β constants on the chosen value of t_0, while their ratio, as shown by Fig. 9, is found to be essentially independent of t_0 at least for the first and second exponentials. The upper curve of this figure gives the summation of the three ratios, which is seen to be independent of t_0. From Fig. 5 and the equation (10) for the apparent capacitance it follows that this summation gives a measure of the dielectric constant at zero frequency, ϵ_1, in terms of the geometric dielectric constant, ϵ_0, at infinite frequency. Specifically, the relation is

$$\frac{\epsilon_1 - \epsilon_0}{\epsilon_0} = \frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2} + \frac{\beta_3}{\alpha_3} \qquad (14)$$

That this summation should prove independent of t_0 further corroborates the sufficiency of three exponentials.

While the constants of the three exponentials are definitely functions of t_0, the total contribution to the a-c. behavior, as given by the in-phase and quadrature components of current at a given voltage and frequency, is essentially independent of t_0 for a limited range of values. This is illustrated in Figs. 10 and 11 which show that for t_0 beyond six milliseconds the in-phase and quadrature components are essentially constant even though the separate contributions of each exponential may vary widely. This is particularly true of the in-phase component, Fig. 10, which is a measure of the power factor. and therefore the one most readily checked by experiment. No physical significance is to be attached to the actual numerical values of the constants of the three exponentials. They serve merely as an empirical means of computing the a-c. behavior. It is apparent from Fig. 10 that the optimum value of t_0 lies in the neighborhood of the period of the alternating wave thus corroborating from another point of view the criterion of what constitutes suitable d-c. measurements.

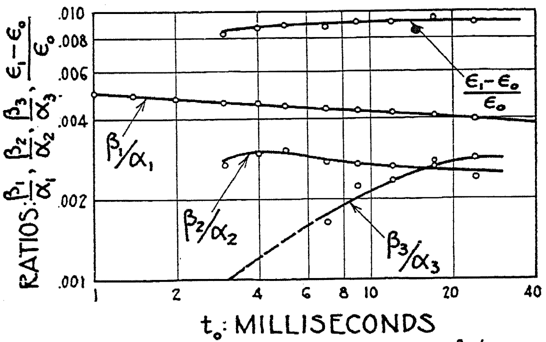

FIG. 9—VARIATION WITH "t_0" OF THE RATIOS β_1/α_1, β_2/α_2, β_3/α_3, AND THEIR SUMMATION, $(\epsilon_1-\epsilon_0)/\epsilon_0$

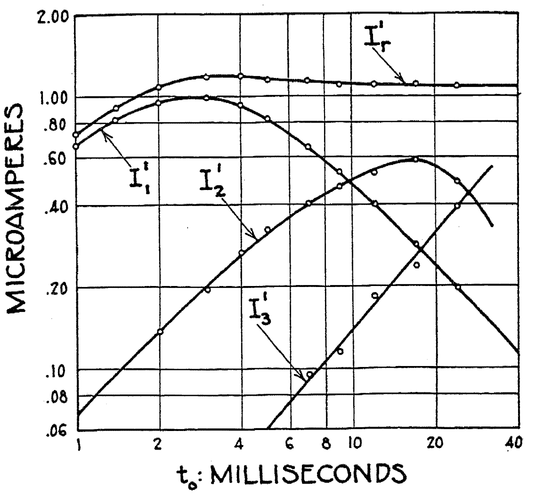

FIG. 10—CONTRIBUTION OF REVERSIBLE ABSORPTION AT 1500 VOLTS, 60 CYCLES. VARIATION WITH "t_0" OF THE IN PHASE COMPONENT OF CURRENT (I'_r) SHOWING THE CONTRIBUTIONS OF EACH EXPONENTIAL.

Irreversible Conduction

The irreversible conduction current as observed under continuous potential with the amplifier oscillograph for some of the cable oils tested is characterized by an initial current which remains constant for approximately one second and then decays with time until it merges into the final conductivity. This initial constant current, when it obeys Ohm's law, gives directly the value of the in-phase component of current at the same effective voltage. See Whitehead,[11] "The Conductivity of Insulating Oils, II" for a complete account of this initial conductivity.

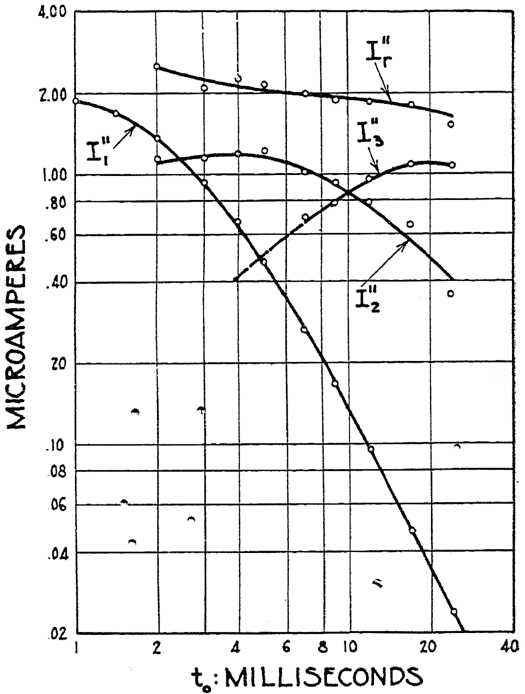

FIG. 11—CONTRIBUTION OF REVERSIBLE ABSORPTION AT 1500 VOLTS, 60 CYCLES. VARIATION WITH "t_0" OF THE QUADRATURE COMPONENT OF CURRENT (I''_r) SHOWING THE CONTRIBUTIONS OF EACH EXPONENTIAL.

In some of the impregnated paper specimens tested the irreversible conduction current does not obey Ohm's law, although it is still characterized by an initial constant conductivity. In such cases the irreversible constant current increases faster than the voltage as illustrated by the typical example shown in Fig. 12. For analytical purposes express the observed current-voltage relation by means of the power series:

$$i(e) = ae + be^3 + ce^5 + \ldots \qquad (15)$$

in which the coefficients may be directly determined from the observed current-voltage relation. To facilitate this, however, express the above relation thus:

$$i(e) = \gamma \sinh \mu e \qquad (16)$$

where the γ and μ constants are chosen to give the best agreement with the observed curve within the voltage range required. Obviously μ is a function of the temperature alone while γ is a function of the temperature and the dimensions of the capacitor. Expand equation (**16**) into a series

$$i(e) = \gamma \left[\mu e + \frac{(\mu e)^3}{3!} + \frac{(\mu e)^5}{5!} + \ldots \right] \quad (17)$$

in which higher powers than the fifth may be neglected.

To obtain the current-time relation for the alternating case substitute for the value of e in equation (**17**) its expression as a periodic sinusoidal function of time. This results in a power series of sin α, where $\alpha = (\omega t + \theta)$, in which all the higher powers of sinα may be broken up into harmonic components by means of Fourier's series. Thus, departure from Ohm's law causes distortion of the current wave in which the fundamental term alone is of interest, when power factor and dielectric loss measurements are made with a bridge network employing a tuned vibration galvanometer or an a-c. galvanometer whose field exhibits only the fundamental. Using effective values of current and voltage and calling I_i' (Fig. 1) the effective value of the *fundamental* component of current contributed by irreversible conduction obtain from the above analysis the expression:

$$I_i' = \frac{\gamma \mu E}{2} \left[1 + \cosh \mu E \right] \quad (18)$$

which is valid only when higher powers than the fifth may be neglected in equation (**17**).

It must be noted that the current contributed to the a-c. behavior by irreversible conduction is exactly in phase with the applied voltage and independent of the frequency. Letting I'' (Fig. 1) be the charging current as measured at a given frequency and voltage, the contribution of irreversible conduction to the a-c. behavior is as follows:

$$\left.\begin{array}{ll} \text{Phase defect angle } \tan \Psi_i = \dfrac{I_i'}{I''} \\ \text{Irreversible loss} \quad W_i = E I_i' \end{array}\right\} \quad (19)$$

where I_i' is given directly by the initial constant current under continuous potential when Ohm's law is obeyed or is computed by (**18**) when the non-linear current-voltage relation is determined according to equation (**17**). Table II gives the computed variation of power factor with voltage and temperature for the typical example shown in Fig. 12.

TABLE II—VARIATION WITH VOLTAGE AND TEMPERATURE OF POWER FACTOR DUE TO IRREVERSIBLE CONDUCTION. NON-LINEAR CURRENT-VOLTAGE CHARACTERISTIC

Impregnated Paper Specimen C-7

Temp. deg. cent.	γ Amp. $\times 10^{-8}$	μ	Power factor 500 volts	1,000 volts	1,500 volts
30	9.5	0.00082	0.00016	0.00017	0.00020
45	20.4	0.00076	0.00035	0.00039	0.00047
60	50.8	0.00072	0.00082	0.00090	0.00104

EXPERIMENTAL PROCEDURE

Continuous potential measurements and corresponding 60-cycle a-c. measurements were made on a number of samples of oil, paper, and impregnated paper at voltages of 500, 1,000, and 1,500, and at temperatures of 30, 45, and 60 deg. cent. For a complete description of the constant temperature, drying, impregnating, and measuring tanks and of the test specimens see Whitehead and Kouwenhoven,[12] "Fundamental Properties of Impregnated Paper."

The measurements under continuous voltage include the determination of the charge and discharge current-time relations with the aid of the amplifier-oscillograph. The amplifier increases the normal sensitivity of the electromagnetic oscillograph 10^5 times, giving a deflection on the photographic film of 1 mm. per 4×10^{-8} amperes. The further development of the amplifier oscillograph, as applied to this work, is described by S. K. Waldorf[13] in a separate paper.

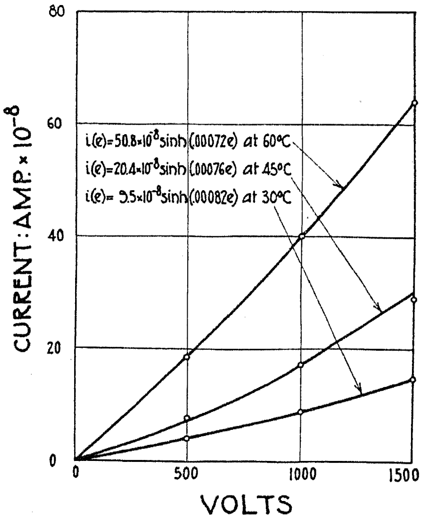

FIG. 12—VARIATION WITH TEMPERATURE OF CURRENT-VOLTAGE RELATIONS FOR THE INITIAL, CONSTANT, IRREVERSIBLE CONDUCTION CURRENT IMPREGNATED PAPER SPECIMEN C7.

The a-c. measurements include the determination of power factor and apparent capacitance using a modified Schering bridge whose power factor sensitivity is $\pm 5 \times 10^{-6}$. The development, theory, and special features of this bridge are described in a separate paper by Kouwenhoven and Baños.[14]

EXPERIMENTAL RESULTS

Paper Specimens. Dry paper specimens reveal a dielectric loss due entirely to reversible absorption; that is, the irreversible conduction current is not measurable as shown by the equality of charge and discharge currents in Fig. 2. Fig. 13 gives a typical pair of oscillograms at a given temperature and voltage. From such oscillograms are computed the current-time relations shown in Figs. 14 and 15 which illustrate the respective variations with voltage and temperature. The insert in Fig. 14 shows how closely Curie's law is obeyed, while the insert in Fig. 15 illustrates the variation of the current ordinates with temperature.

Table III gives the variation with temperature of the α and β constants of the absorption current-time relation for the dry paper specimen before and after impregnation with an oil whose electrical characteristics are described in the next section. Interesting changes of the constants are noted as result of impregnation. These differences are best appreciated by direct comparison of Figs. 2 and 3. Table IV gives the correlation of a-c. and d-c. measurements for the whole range of temperature and voltage explored. Note that in general the agreement between computed and measured losses is very close.

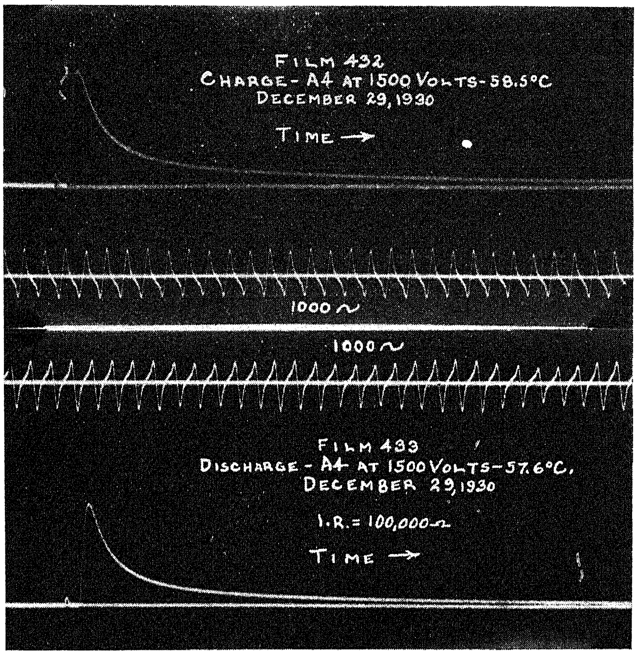

FIG. 13

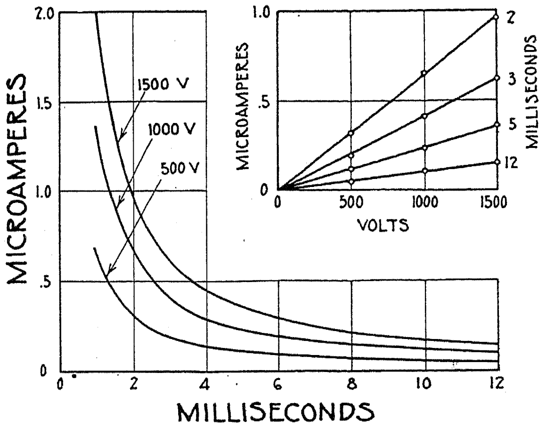

FIG. 14—VARIATION WITH VOLTAGE OF CHARGE AND DISCHARGE CURRENT-TIME RELATIONS AT 60°C DRY PAPER SPECIMEN A7.

TABLE III—VARIATION WITH TEMPERATURE OF THE AVERAGE "α" AND "β" CONSTANTS IN THE EMPIRICAL CURRENT-TIME RELATION

$$i(t) = EC_0\{\beta_1 e^{-\alpha_1 t} + \beta_2 e^{-\alpha_2 t} + \beta_3 e^{-\alpha_3 t}\}$$

Specimen	Temp. deg.cent.	C_0 $\mu\mu$f	β_1	β_2	β_3	α_1	α_2	α_3
Dry paper A-7.	30	741	0.52	2.52	10.32	112	638	2,160
	45	751	0.40	1.93	6.49	94	614	2,067
	60	760	0.31	1.37	4.09	77	553	1,797
Impregnated paper C-7	30	1,191	0.49	2.33	6.70	95	570	1,827
	45	1,200	0.44	1.92	6.34	59	561	2,030
	60	1,209	0.55	1.32	3.13	30	490	1,635

Oil Specimens. Fig. 16 gives a set of typical charge oscillograms at 1,500 volts and at 30, 45, and 60 deg. cent. for the cable oil used to impregnate the above paper specimen. It is noted that the charge current is characterized by a large initial constant current which accounts for the major part of the loss. Superimposed on this initial current is a smaller decaying current function of time which is reversible and, therefore, attributable to dielectric absorption in oil. Fig. 18 shows the variation with temperature of the charge and discharge current-time relations at 1,500 volts. The equations for discharge show that the small element of dielectric absorption is expressible by a single exponential thus corroborating the results published by Whitehead[11] in "The Conductivity of Insulating Oils, II." Fig. 17 gives the evidence that the initial constant current in the insulating oils tested obeys Ohm's law.

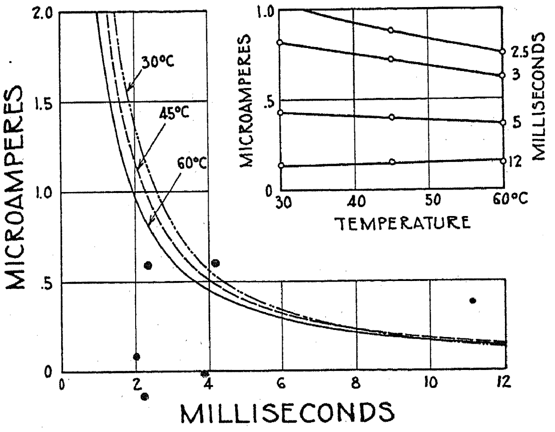

FIG. 15—VARIATION WITH TEMPERATURE OF CHARGE AND DISCHARGE CURRENT-TIME RELATIONS AT 1500 VOLTS. DRY PAPER SPECIMEN A7.

Table V gives the comparison between several measured and corresponding computed losses for one oil specimen B-7. The data are typical and the close agreement shown is further evidence of the soundness of the method used in computing the a-c. behavior.

Impregnated Paper Specimens. Some of the changes brought about on a dry paper specimen by impregnation with an insulating oil have already been pointed out in Fig. 3 and Table III. Fig. 3 shows the effect of the conductivity of the insulating oil in increasing the absorption and changing its character. Fig. 19 gives a typical pair of oscillograms in which the presence of the irreversible conduction current is apparent in the charge record. This fact is better appreciated from the equa-

TABLE IV—ANALYSIS OF POWER FACTOR AND DIELECTRIC LOSS AT 60 CYCLES BY COMPUTATION FROM D-C. CHARACTERISTICS

Paper Specimen A-7

Temp deg. cent.	Volts	Measured at 60 cycles Charging current amp. × 10^{-6}	Power factor	Dielectric loss watt × 10^{-6}	Computed from d-c. In-phase component of current amp. × 10^{-6}	Power factor	Dielectric loss watt × 10^{-6}
T	E	I''	tan Ψ	W	$I_r{}^1$	tan Ψ_r	W_r
60	500	144	0.00237	169	0.340	0.00236	170
	1,000	288	0.00237	676	0.684	0.00237	684
	1,500	432	0.00237	1,518	1.047	0.00242	1,560
	Average	—	0.00237	—	—	0.00238	—
45	500	142	0.00276	197	0.410	0.00288	205
	1,000	285	0.00276	786	0.830	0.00291	830
	1,500	427	0.00276	1,796	1.289	0.00302	1,930
	Average	—	0.00276	—	—	0.00293	—
30	500	141	0.00346	243	0.533	0.00378	266
	1,000	282	0.00346	974	1.024	0.00363	1,024
	1,500	422	0.00346	2,192	1.558	0.00368	2,335
	Average	—	0.00346	—	—	0.00369	—

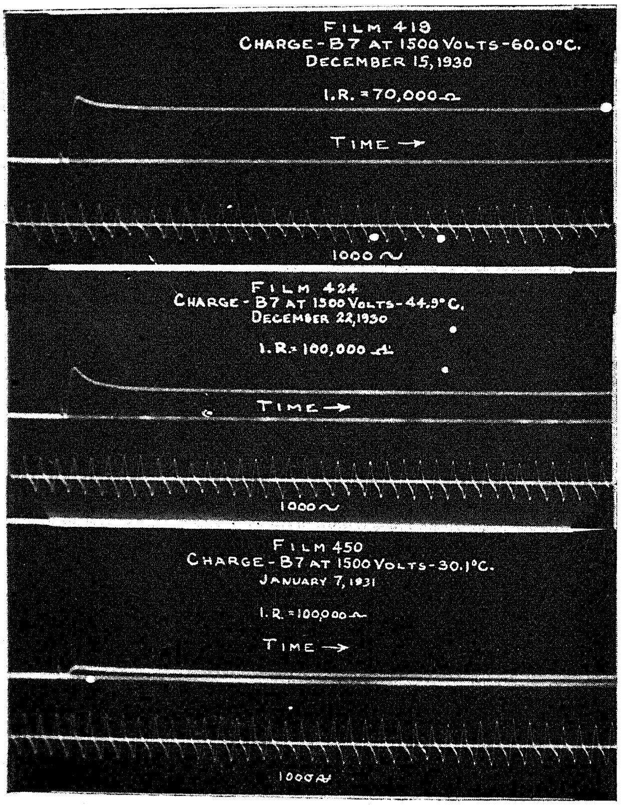

FIG. 16

tions of charge and discharge given in Fig. 20, where the irreversible conduction current is seen to be the *constant* difference between charge and discharge currents. This constant irreversible conduction current does not obey Ohm's law, as shown by Fig. 12, and its contribution to the power factor has already been computed from theoretical considerations and given in Table II.

Summary of Specimens. As an example of the insight afforded by this method of analysis, Fig. 21 gives the power factor as a function of temperature for the oil, paper, and impregnated paper specimens. The shaded areas represent the total increase in power factor with impregnation. Through the analysis of the discharge current-time relations it is possible to resolve this increase into two components:

1. The increase in reversible absorption brought about by impregnation.

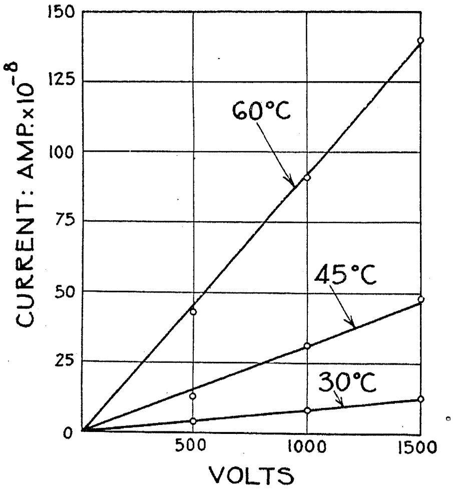

FIG.17—VARIATION OF INITIAL CONSTANT CURRENT WITH VOLTAGE. OIL SPECIMEN B7.

2. The creation of the additional contribution of irreversible conduction totally absent before impregnation and directly attributable to the conductivity of the impregnating oil.

ACKNOWLEDGMENT

Grateful appreciation is expressed to the Utilities Research Commission of Illinois, whose financial sup-

TABLE V—ANALYSIS OF POWER FACTOR AND DIELECTRIC LOSS AT 1,500 VOLTS AND 60 CYCLES BY COMPUTATION FROM D-C. CHARACTERISTICS

Oil Specimen B-7

	Measured at 60 cycles			Computed from d-c. characteristics: Reversible absorption			Irreversible conduction			Total	
Temp. deg. cent.	Charging current amp. × 10^{-6}	Power factor	Dielectric loss watt ×10^{-6}	In phase component of current amp. ×10^{-6}	Power factor	Dielectric loss watt ×10^{-6}	Initial constant current amp. ×10^{-6}	Power factor	Dielectric loss watt ×10^{-6}	Power factor	Dielectric loss watt ×10^{-6}
T	I''	$\tan \psi$	W	I_r'	$\tan \psi_r$	W_r	I_i'	$\tan \psi_i$	W_i	$\tan \psi_o$	W_o
60	420	0.00346	2,179	0.065	0.00016	98	1.309	0.00312	1,965	0.00328	2,063
	421	0.00346	2,184	0.145	0.00035	218	1.251	0.00297	1,880	0.00332	2,098
	422	0.00374	2,357	0.102	0.00024	153	1.349	0.00320	2,025	0.00344	2,178
45	425	0.00116	739	0.029	0.000067	43	0.475	0.001117	713	0.00118	756
30	430	0.00037	239	0.027	0.000064	41	0.124	0.000288	186	0.00035	227
	430	0.00034	219	0.018	0.000042	27	0.135	0.000313	202	0.00035	229
	429	0.00035	225	0.019	0.000043	28	0.120	0.000280	180	0.00032	208
	430	0.00032	206	0.022	0.000051	33	0.128	0.000298	192	0.00035	225
	431	0.00032	210	0.032	0.000074	48	0.105	0.000244	158	0.00032	206
Average	430	0.00034	220	0.024	0.000055	36	0.122	0.000285	184	0.00034	219

port has made possible the active prosecution of a research on the properties of impregnated paper from which the experimental results here reported have been taken. The authors also wish to acknowledge the interest and careful work of Dr. S. K. Waldorf in the development and operation of the amplifier oscillograph.

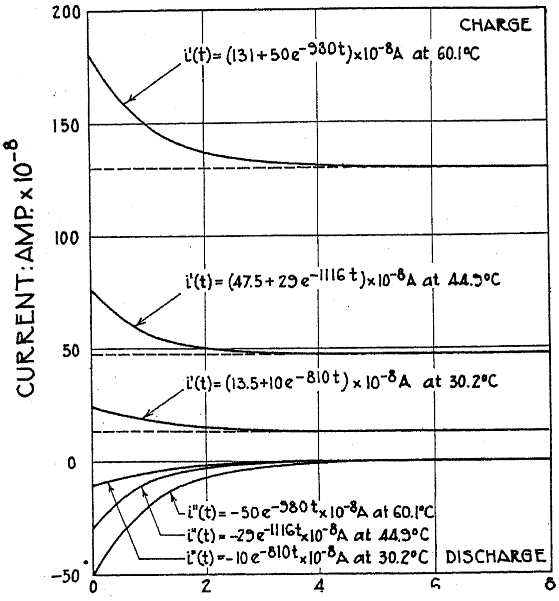

FIG. 18—VARIATION WITH TEMPERATURE OF CHARGE AND DISCHARGE CURRENT-TIME RELATIONS AT 1500 VOLTS. OIL SPECIMEN B7.

CONCLUSIONS

1. It is shown that for any type of dielectric the loss, power factor and capacity at 60 cycles may be accurately predicted from the charge and discharge currents under continuous potential.

2. The method developed is available at any frequency, provided that the continuous potential charge

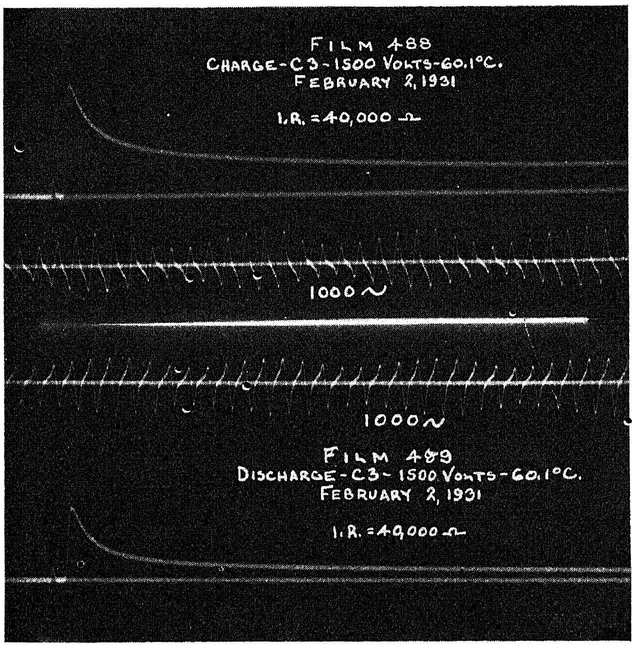

FIG. 19

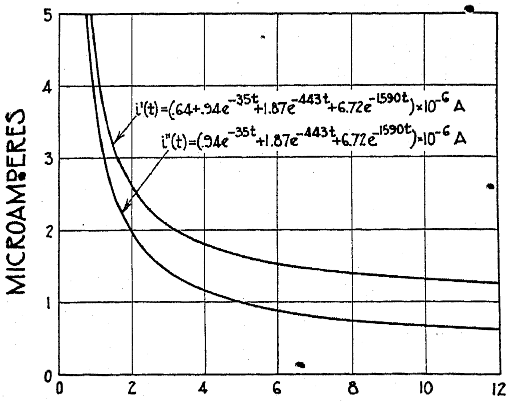

FIG. 20—CHARGE $i'(t)$ AND DISCHARGE $i''(t)$ CURRENT-TIME RELATIONS AT 1500 VOLTS, 60°C. IMPREGNATED PAPER SPECIMEN C7.

and discharge currents may be measured over initial time intervals comparable with the alternating period.

3. The nature of the relaxation function characteristic of reversible absorption in dielectric has been discussed from the experimental and analytical points of view.

4. It has been shown that agreement of the relaxation function with the t^{-n} expression is not general, but on the contrary it is confined to dielectrics showing negligible or no irreversible conduction. The inadequacy of the t^{-n} expression has been proved on experimental and analytical reasons. Chief among them is the failure of the t^{-n} expression to predict accurately the variation of dielectric loss with frequency.

5. The method of three exponentials has been developed as a convenient and sufficient expression for the relaxation function. The sufficiency and accuracy of three exponentials has been proved both analytically and experimentally.

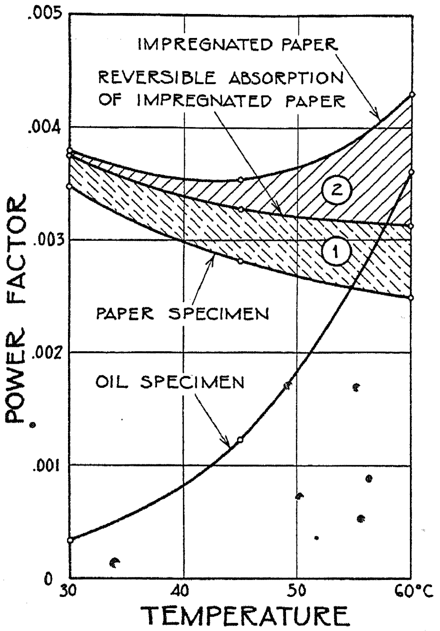

FIG. 21—ANALYSIS OF POWER FACTOR AT 1500 VOLTS. IMPREGNATED PAPER SPECIMEN C7
Increase in Power Factor with Impregnation
(1) Increase in Reversible Absorption
(2) Contribution of Irreversible Conduction

6. Irreversible conduction does not obey the principle of superposition. The usual forms of irreversible conduction encountered have been defined and classified. The case where the initial constant current does not obey Ohm's law has been considered analytically in its contribution to the a-c. behavior.

7. The manner in which the complete a-c. behavior of a dielectric is accurately predicted from suitable d-c. measurements has been illustrated by three typical sets of experimental data, paper, oil, and impregnated paper.

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Discussion

H. H. Race: The authors of this paper have presented some very interesting data but they suggest no physical explanation of their results. Therefore Fig. 5 interested me particularly because the variations of capacitance and loss with frequency are similar to corresponding curves calculated using a very definite physical picture, namely, Debye's theory of polar molecules. A comparison of these two separate lines of mathematical thinking, which lead to qualitatively similar results, shows that a correspondence should be expected since both are based upon an exponential relaxation function.

Dr. Whitehead's curves represent a-c. characteristics calculated from von Schweidler's theory using the first term of the relaxation function

$$\phi(t) = \beta_1 \epsilon^{-\alpha_1 t} + \beta_2 \epsilon^{-\alpha_2 t} + \text{etc.}$$

which represents the d-c. discharge current-time curve of the dielectric.

Likewise, Debye builds his theory on an assumption of an exponential relaxation function and τ, the relaxation time used by him, is the time necessary for a distortion to relax to $1/\epsilon$ of its original value. Therefore, since both theories start with the same assumption, the resulting relations should certainly be in qualitative agreement.

An exact quantitative agreement can be obtained and corresponding equations derived from the two theories can be shown to be identical if

$$\alpha_1 = \omega_m = 2\pi f_m \tag{1}$$

and

$$\beta_1 = \alpha_1 \left(\frac{\epsilon_0}{\epsilon_\infty} - 1 \right) \tag{2}$$

where α and β are respectively the exponent and coefficient in Dr. Whitehead's discharge current-time relation. ϵ_0 and ϵ_∞ are the measured effective dielectric constants at zero and infinite frequency and f_m is the frequency at which the maximum a-c. loss occurs.

Equation (1) can also be derived from Dr. Whitehead's work by setting $dB(w)/dw = 0$ in equation (11) in order to determine the frequency at which the maximum a-c. loss will occur. Dr. Whitehead has made this calculation and concludes that "only the exponentials with values of α in the neighborhood of (w) at the frequency at which correlation is intended contribute an appreciable amount to the loss component $B(w)$."

Since equation (1) also relates a-c. dielectric losses resulting from the relaxation of polar orientation to d-c. current-time curves resulting from the same relaxation, it seems that the following more general conclusions can be reached.

1. Any mechanism causing an exponential current-time discharge curve after the removal of a constant potential, will also cause an a-c. dielectric loss-frequency curve having a maximum at $f_m = (\alpha / 2\pi)$.

2. Inversely, if a maximum is observed in the a-c. loss-frequency curve, the exponent of a d-c. current time curve taken for the same material will be given by this same relation.

3. Conversely, if in a given range of frequency or relaxation time, one of these effects is not found the other will also be absent.

4. Through these relations, a correlation should be possible between Dr. Whitehead's d-c. and 60-cycle measurements and our high frequency data. However, for the mineral oils in which we have measured the dielectric loss, as a function of frequency, we have found no maxima at frequencies less than 10^4 cycles per second, even in high viscosity oils at temperatures as low as 0 deg. cent. Therefore, the d-c. relaxation curves corresponding to the maxima which we have observed should have become negligible in less than 1×10^{-3} second, which is the time at which Dr. Whitehead's measurements begin.

To check the above conclusions and obtain a better comparison between a-c. and d-c. measurements Dr. Whitehead has very kindly furnished us with a sample of his B-7 oil and we hope to present a paper in the near future containing the reasons leading to equation (1) and data on B-7 oil over a wide range of frequency and temperature.

F. M. Clark: In connection with the paper presented by Dr. Whitehead, it is interesting to note the procedure developed a few years ago in the Pittsfield Works Laboratory of the General Electric Company for the gaging of capacitor characteristics. With 5-kva. oil-filled capacitors designed for a-c. voltage application, ranging from 440 to 2,300, it was found that the relative a-c. characteristics could be successfully determined from the so-called "d-c. leakage." In determining this value, the capacitors were charged under d-c. voltage to the desired value, disconnected from the source of supply and allowed to stand under controlled conditions for a period ranging from one to several days, after which the remaining charge was measured. The difference between the original and final charge was used in gaging the a-c. characteristics of the capacitors. This so-called "d-c. leakage" appeared to be of greater value than insulation resistance measurements, probably because the time used to determine the latter was not of sufficient duration. The work was not carried sufficiently far to predict successfully the power factor from the leakage results obtained. With further investigation, it is believed this could be done.

W. A. Del Mar: The practical control of power factor of impregnated paper now lies with the paper tests. In 1922* the control was essentially a matter of oil resistivity.

The change occurred when we changed our paper from Manila to wood pulp and our oil from petrolatum to cylinder oil. The practical problem has long since been solved, but it would be interesting to have the authors explain these facts in the light of their admirably developed theories.

**Effects of the Composite Structure of Impregnated Paper Insulation on Its Electric Properties,* Hanson and Del Mar, A.I.E.E. TRANS., 1922, Vol. 41, p. 563.

Breakdown of Glass with Alternating Potentials

BY N. D. KENNEY,[1] A. M. LUERY,[1] and J. D. MORIATY[1]
Non-member Non-member Non-member

Synopsis.—*Previous tests on the breakdown of glass with direct potentials indicated the existence of three different regions of breakdown. The present paper describes similar tests on alternating potentials. The following conclusions are reached:*

1. Three distinct regions of breakdown are obtained on alternating current as well as on direct current.

2. In the disruptive and intermediate regions, breakdown occurs at an alternating peak voltage approximately equal to the direct breakdown voltage.

3. In the thermal region, breakdown occurs at an alternating peak voltage somewhat lower than the direct breakdown voltage.

* * * * *

INTRODUCTION

THE cause and reason for breakdown of solid dielectrics has resulted in the setting forth of many theories based on both experimental and mathematical research. Research by Kármán, Rogowski and Fock resulted in the establishing of the thermal breakdown theory. Inge and Walther, in their investigations of the breakdown of glass in 1928 found a region of puncture in which the potential gradient is independent of temperature and thickness—the so-called disruptive region. In 1929 Moon and Norcross concluded from their experimental data on glass, that there were three regions of breakdown.

Since the publication of the paper by P. H. Moon and A. S. Norcross[2] describing their experimental work on the breakdown of glass with direct potentials, there has been some question as to whether the same type of results would be obtained with alternating potentials. Our purpose was to determine if three regions of breakdown also existed for the a-c. case. We also hoped to find some relation between the alternating and direct breakdown voltages. Previous work by Hayden and Eddy[3] on the a-c. to d-c. ratio showed widely varying results, due perhaps to lack of knowledge of the types of breakdown and further complicated by edge effect. It was thought that careful tests made with edge effect eliminated, coupled with the present knowledge of different breakdown regions, might lead to a definite a-c. to d-c. ratio in each region.

For these tests it was decided to use glass because of its many advantages.

1. Previous tests have been made on glass, thus allowing a comparison of results.

2. Its characteristics are such that the tubing can be readily blown into spheres, thus eliminating edge effect.

3. Its homogeneous nature makes it desirable for breakdown investigations.

The disadvantages are:

1. Its physical structure is unknown.

2. Its chemical constitution is not exactly known for each batch of glass.

This paper describes the experimental results obtained at the Massachusetts Institute of Technology on the breakdown of glass with alternating potentials, and shows how these results compare with the d-c. tests of Moon and Norcross.

APPARATUS AND METHOD OF TEST

Alternating voltage (60 cycles) was obtained by means of a sine-wave motor-generator set and a 115/230-100,000-volt transformer. A motor-driven rheostat was used to vary the field voltage of the sine-wave set in order to obtain a uniform and easily-controlled rate of voltage rise. The transformer secondary voltage was read by means of a potential transformer and a Weston a-c. voltmeter.

Glass tubes 10 to 15 mm. in diameter and 300 mm. in length were sealed at one end and blown into spheres between 20 and 40 mm. in diameter. This procedure was similar to that used by Moon and Norcross and eliminated edge effect very nicely. These samples were placed in a mercury bath and filled on the inside with mercury. Contact was made to the internal electrode by means of a flexible wire suspended inside the tubing and to the external one by means of a steel container. (Fig. 1.) This was carefully lagged on the sides and top with asbestos and proved to be very effective in keeping the temperature uniform and constant. The voltage was impressed across the sample and increased by means of the motor-driven rheostat until puncture occurred. The sample was then carefully removed from the bath, all pieces being collected, cleaned, and measured. As it was not always possible to measure the thickness of the sample at the exact point of puncture, 5 to 10 micrometer readings were taken on the glass fragments near the puncture and the minimum value used as the approximate thickness at the point of breakdown.

The wave form of the generator was found to be practically sinusoidal; and since the transformer was operated at low saturation and with no series resistance in the primary circuit, it was assumed that the wave form of the voltage across the sample was also sinusoidal. The voltages read from the voltmeter were, of course,

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[1] Massachusetts Institute of Technology.

[2] "Three Mechanisms of Breakdown Obtained on Glass by Elimination of Edge Effect," *Journal of the Franklin Inst.*, 208, 1929, p. 705.

[3] *Dielectric Strength Ratio between Alternating and Direct Voltages*, Hayden and Eddy, TRANS. A.I.E.E., V. 42, 1923, p. 593.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

r.m.s. values; but a better comparison with d-c. tests was obtained by using peak instead of r.m.s. values. Thus all readings were multiplied by $\sqrt{2}$, and all results given in this paper are peak kilovolts and not r.m.s. kilovolts.

EXPERIMENTAL RESULTS ON G-1* GLASS

The experimental results are shown in Fig. 2, where each point represents one breakdown test. To show clearly the different regions of breakdown, the logarithm of breakdown voltage (peak kv.) is plotted against the logarithm of thickness in microns. ($1\ \mu = 0.001$ mm.) There is a certain amount of scattering of the points, as can be expected in the results of any breakdown tests. However, it will be noted that this variation is not at all bad, and thus the curves can be located fairly exactly.

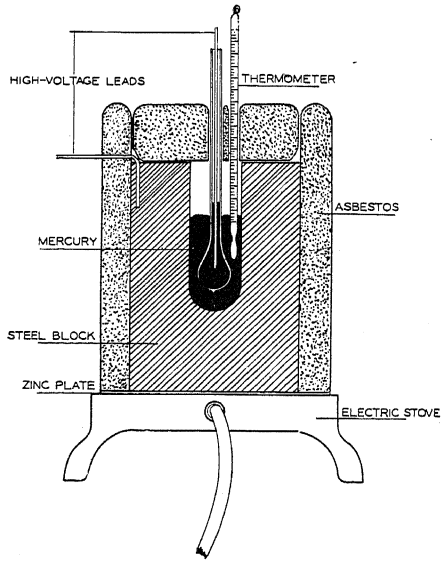

FIG. 1—SCHEME OF TEST APPARATUS

Fig. 2 shows clearly that with alternating potentials *there are three distinct regions of breakdown.* The tests at low temperatures and small thicknesses give a single curve of slope = 1. This represents the *disruptive region.* For somewhat higher temperatures, the curves depart from this straight line; and we have a set of parallel lines of slope = 2/3 breaking off from the disruptive-region line. The clusters of points for the 50 deg. and for 150 deg. curves, for instance, prove that these two are parallel or nearly so and that the slope is distinctly different from that of the 0 deg. curve. This is the *intermediate region.* At still higher temperatures we obtain another set of parallel lines having the slope of ¼. This is the *thermal region.* A comparison of the points at 150 deg. and 250 deg. shows very clearly that these two regions are distinct. The breaks in the 150 and 250 deg. curves may appear purely arbitrary, but they were found necessary in order to make a consistent family of curves when plotted against $(1/T)$. Thus the only questionable aspect of the curves as drawn is whether the transitions should be represented by sharp breaks or be slightly rounded. The accuracy of breakdown data does not warrant any statement as to which is correct. The present scheme, however, is the simpler and will be used in this paper.

In order to show more clearly the relations existing between breakdown voltage and temperature, curves of the logarithm of breakdown voltage (peak kv.) are plotted against the reciprocal of absolute temperature. (Fig. 4.) Here again three distinct slopes are noticed; zero slope for the disruptive region, slope = 430 for the intermediate and slope = 1,940 for the thermal.

How do the results of these a-c. tests compare with d-c. tests? To obtain a comparison, d-c. tests were also made on the same batch of lead glass. The results at room temperature (25 deg. cent.) are shown in Fig. 2 by the solid ▲'s. A very good check will be noted in both disruptive and intermediate regions. Thus we may say that on alternating current and in the disruptive and intermediate regions, *breakdown occurs at a peak voltage approximately equal to the d-c. breakdown voltage*

FURTHER DISCUSSION OF THE CURVES

The three regions of breakdown obtained by Moon and Norcross with G-1 glass and direct potentials are also found when alternating potentials are used. To facilitate comparison of our results with the previous ones, we have added Figs. 3 and 5 giving breakdown voltage for G-1 glass on direct potentials.[4] The a-c. curves (Figs. 2 and 4) are similar in every respect to the d-c. curves (Figs. 3 and 5).

For low temperatures and small thickness a linear relation (slope = 1) exists between breakdown voltage and thickness. The alternating breakdown gradient in this region is constant and has a magnitude of 3,550 kv. per cm., which is the same value obtained with direct current in this research.[5] The potential gradient is not affected by thickness or temperature.

With higher temperatures, an increase in temperature causes a decrease in the breakdown voltage for a given thickness. At intermediate temperatures (between the two previously mentioned regions) the voltage varies with the temperature according to some other relation, giving a third region which varies with the thickness as the 2/3 power.

The general form of the equation found to fit the curves is

$$V_b = K_d{}^n\,(10)^{\frac{b}{T}},$$

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*A lead glass made by the Corning Glass Works.

[4] *Three Regions of Dielectric Breakdown,* Moon and Norcross, TRANS. A.I.E.E., V. 49, 1930, p. 755.

[5] This magnitude differs from the value of 3,100 kv. per cm. obtained by Moon and Norcross (Fig. 3 and Fig. 5) by about 14 per cent. The discrepancy may be due to a slight difference in the chemical or physical properties of the glass used in the two sets of experiments, though both tests were carried out with G-1 glass.

where K is a constant dependent on the region, n is the slope of the *log kv.* vs. *log d* curves and b is the slope of the *log kv.* vs. $1/T$ curves. Table I gives the a-c. peak values for K, n, and b while Table II gives the d-c. values as found by Moon and Norcross.

TABLE I—(A-C. PEAK VALUES)

Region	K	n	b
Disruptive	3,550	1	0
Intermediate	25.5	2/3	430
Thermal	1.635×10^{-3}	1/4	1,943

TABLE II—(D-C. VALUES)

Region	K	n	b
Disruptive	3,100	1	0
Intermediate	21.3	2/3	470
Thermal	0.618×10^{-3}	1/4	1,540

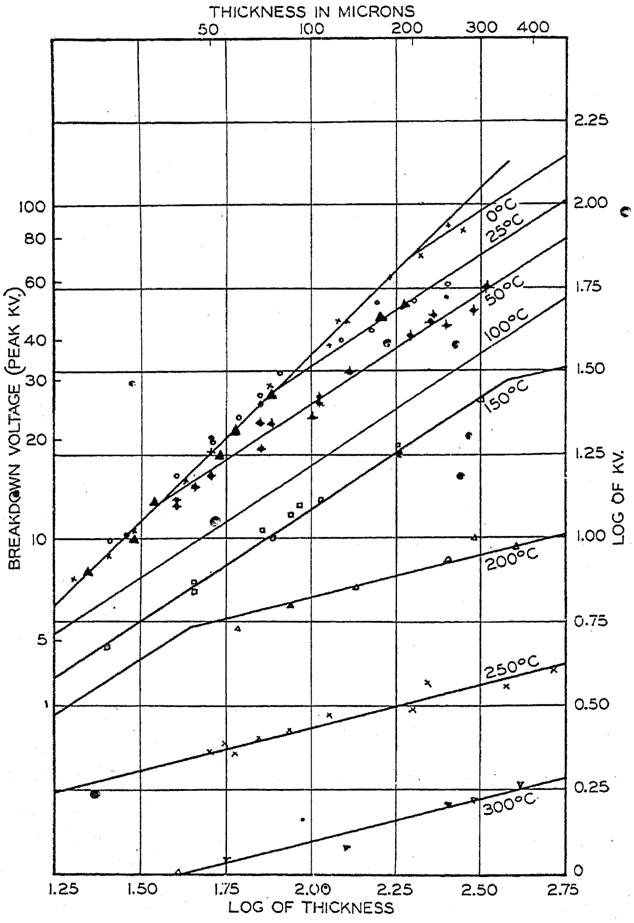

Fig. 2—Breakdown of Lead Glass Spheres (G-1) on 60 Cycle Alternating Current

▲ = d-c. test points at 25 deg. cent.

By comparing Fig. 2 with Fig. 3 it can be seen that in the thermal and intermediate regions the direct breakdown voltage is higher than the a-c. peak value. Comparison of Fig. 4 and Fig. 5 shows that the intermediate region for alternating current is longer than that for direct current. This is shown to better advantage in Fig. 6, where the a-c. and d-c. curves are plotted on the same sheet. For simplicity only the 20μ and 100μ curves are given. It is probable that some of the differences are due to unavoidable experimental variations and that still others are caused by slight differences in the glass used in the two investigations. In the intermediate region, however, the average difference between the a-c. and d-c. curves is only about 6 per cent, which is undoubtedly within the experimental error.

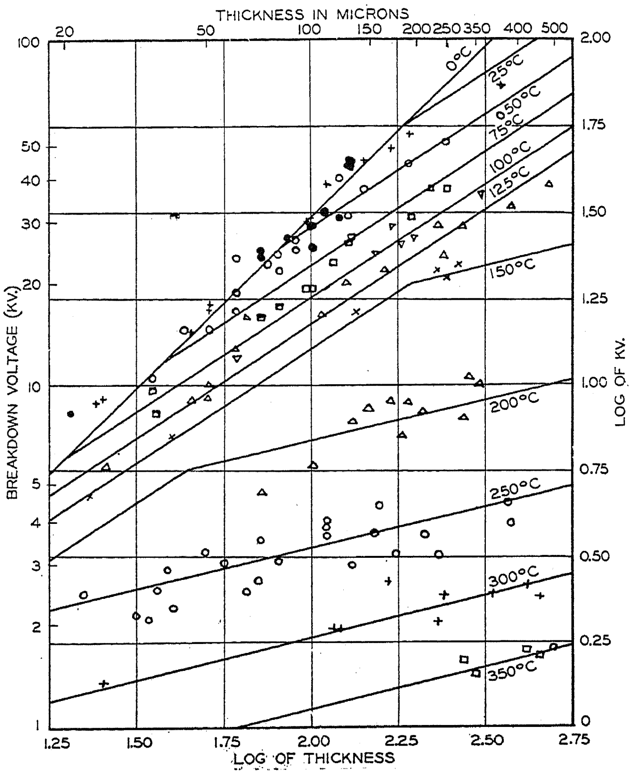

Fig. 3—Breakdown of Lead Glass Spheres (G-1) on Direct Current

(Moon and Norcross, *loc. cit.*)

At the higher temperatures in the thermal region, the d-c. breakdown voltage appears to be distinctly higher than the peak alternating voltage. If r.m.s. values had been used instead of peak values, this difference would of course be even more marked. In the thermal region there appears to be no simple relation between a-c. and d-c. breakdown voltages or between the slopes of the curves. Such a relation could hardly be expected if we consider that in the a-c. case the heating is due to the total loss whose nature is not understood, and that the results are undoubtedly complicated by the effects of electrolytic polarization which is present in the d-c. case but partially eliminated in the a-c. case.

DISCUSSION OF THE POSSIBLE CAUSES OF BREAKDOWN

Disruptive Region. In the disruptive region the rate of application of voltage has no effect on the breakdown.

Samples were punctured with rates of application varying between 4 kv. per min. and 40 kv. per min. and no change in potential gradient was noticed. Breakdown in this region may be due to submicroscopic cracks in the surface of the glass. This theory was first expounded by Horowitz[6] and seems more logical than the molecular disruption theory, which gives a breakdown value much higher than the experimental values. The equation for breakdown according to the submicroscopic theory is

$$V_b = 1{,}200\sqrt{\frac{\delta}{\mathcal{K}\lambda}} \text{ (volts per cm.)}$$

where

$\mathcal{K}$ = the dielectric constant
δ = the surface tension in dynes per cm.
λ = the depth of the crack in cm.

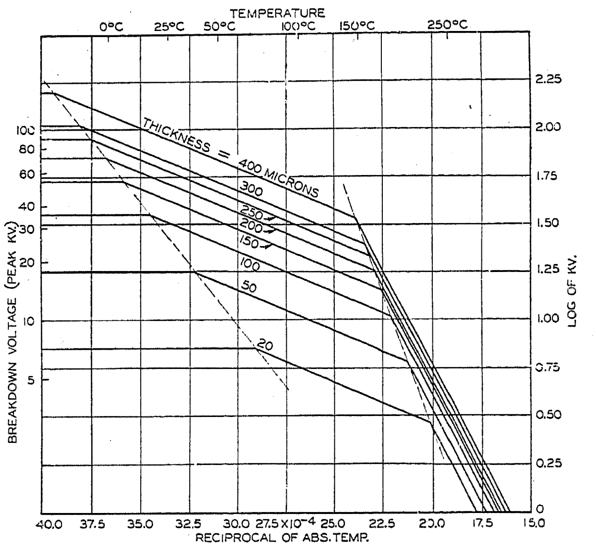

FIG. 4—BREAKDOWN OF LEAD GLASS SPHERES (G-1) ON 60 CYCLE ALTERNATING CURRENT

It will be noted that time should have no effect, according to this equation. Also, as the type of impressed voltage can have no effect on the depth of the crack, the dielectric constant, or the surface tension, the magnitude of V_b will be the same for alternating current and direct current. This was found to be true in our tests.

Intermediate Region. Unlike the disruptive region the intermediate has no constant potential gradient. From the experimental results obtained,

$$V_b = 25.51\, d^{2/3}\, 10^{\frac{430}{T}}.$$

It can be seen from this equation that the breakdown voltage is dependent on the thickness of the sample and the ambient temperature. The time factor appears to enter into this region and a study is to be made of its effect on the magnitude of the breakdown voltage.

Just what governs breakdown in this region is a question for debate. However, the ionization-by-collision theory seems to be a logical explanation of the results. This theory has been accepted for gases and liquids and it may be true also for solids.

Thermal Region. The thermal region, like the intermediate, has no constant breakdown gradient. The equation of this region was found to be

$$V_b = 1.635 \times 10\, d^{1/4}\, 10^{\frac{1940}{T}}.$$

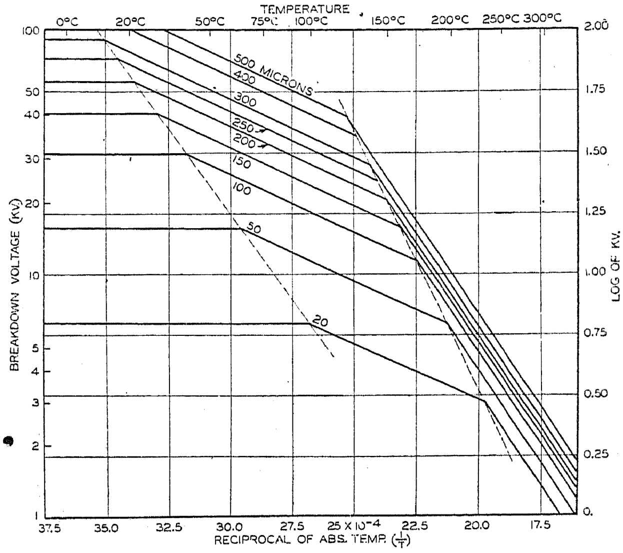

FIG. 5—BREAKDOWN OF LEAD GLASS SPHERES (G-1) ON DIRECT CURRENT

(Moon and Norcross, *loc. cit.*)

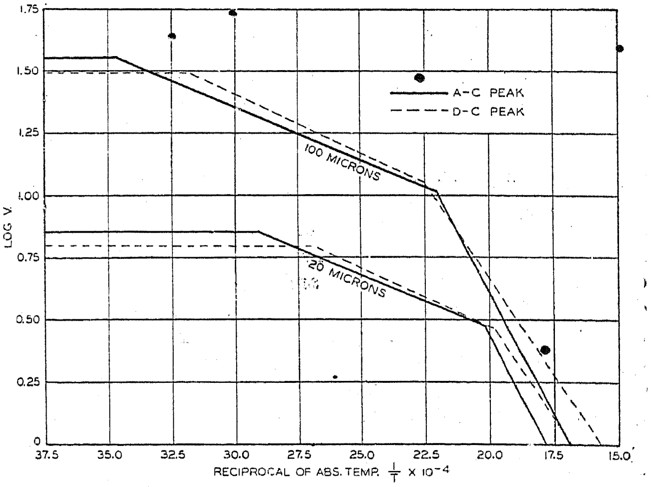

FIG. 6—COMPARISON OF D-C. AND A-C. RESULTS

Thermal breakdown is a function of thickness and temperature.

Heat is generated inside the sample by I^2R and other losses. It was noted during the tests that, by varying the rate of application of voltage, various values of breakdown voltage could be obtained. The rate which was found to be the most satisfactory was 300 volts per min., since further reduction in rate of voltage application had no effect on breakdown voltage.

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[6] Das Griffithsche Prinzip und Dielektrische Durchschlegspestigkeit, *Archiv. für El.*, 18, 1927, p. 535.

When a direct potential is impressed across a sample of glass the sodium ions drift under the action of the outer field and gradually produce a poorly-conducting layer near the anode. With alternating potentials, the effect of electrolytic polarization is very small. This means that the a-c. conduction current will be greater than the direct current and therefore the I^2R losses will be greater with alternating current than with direct current. Hence the r.m.s. alternating breakdown voltage will be less than the direct breakdown voltage. This is substantiated by the tests described above.

CONCLUSIONS

A consideration of all the data leads us to the following conclusions:

1. Three distinct regions of breakdown are obtained, both on alternating current and on direct current.
2. In the disruptive and intermediate regions, breakdown occurs at an alternating peak voltage approximately equal to the direct breakdown voltage.
3. In the thermal region, breakdown occurs at an alternating peak voltage which is generally slightly lower than the direct breakdown voltage.

ACKNOWLEDGMENT

The authors are indebted to Messrs. Moon and Norcross for their help and suggestions during the investigation and for their permission to use the data on direct-current breakdown.

Discussion

J. B. Whitehead: The sample used in these tests is an exceedingly ingenious one. The blowing of the glass bulb automatically introduces the increasing thickness of wall which is always necessary in high-voltage tests, in leaving the test region and approaching the ends. In this way, abrupt changes of field intensity are avoided without the complication of guard rings, end-bells and the like, as is often necessary in such experiments.

I have always found it difficult to accept the suggestion of Moon and Norcross and now of these authors, that the intermediate stage of breakdown should follow a definite law. The disruptive type of breakdown and the region of thermal breakdown are each subject to theoretical analysis and experimental test. There appears to me, however, to be no reason to suppose that the intermediate region is anything more than one in which conditions are such that neither the disruptive nor the thermal types of breakdown is in complete control.

Referring to conclusion (3), I believe that the explanation is to be found in the fact that owing to dielectric losses, the temperature inside the specimen is higher than that which is measured on the thermometer.

W. A. Del Mar: The authors' main thesis is the contention that there exist, with alternating currents, as claimed by Professor Moon for direct currents, three distinct regions of dielectric failure, instead of two, as commonly believed. The evidence of this is that when breakdown is plotted against the reciprocal of the temperature, the graph appears as three lines of different slopes. The authors do not give this evidence, as all the experimental points are omitted from the graphs shown in Figs. 4.

They give experimental points in their graphs of breakdown voltage *versus* thickness, but they are of no significance in this place.

An actual attempt to transcribe the points from Fig. 3 to Fig. 4 resulted in a very erratic looking graph which might have been either what the authors claim or a smooth curve.

The authors, in justice to their thesis, should give Fig. 4 with the experimental points plotted.

W. A. Hillebrand: Doubt has been raised about the existence of an intermediate region of breakdown as postulated by the authors.

This at once brings up the question of the nature of glass, the material used in these tests. Glass is a physical solution, an electrolyte, whose constituents are ionized. It has the high negative temperature coefficient of resistance characteristic of electrolytes. Furthermore, some of its physical properties are indefinite. For example, it will flow at continuously applied stresses much below its short time mechanical ultimate. It therefore seems to me not impossible that in such a material there should be a region of dielectric breakdown intermediate between the disruptive and thermal, partaking of the nature of both and representing a transition stage.

For the same reasons, glass would seem to be a very unsatisfactory material to use in fundamental studies of dielectric breakdown because it is too complex and indefinite in its structure.

F. M. Clark: In the study of insulating materials, it is well recognized that the heterogeneity of the materials and the difficulties of testing produces an erratic accumulation of results despite a well-organized and carefully planned attack. This has led most investigators to adopt an "average" curve to fit the scattered test findings. The curve in such cases is based on the average of several tests for each thickness determined. With most insulations, the dielectric strength—thickness relation—can be best approximated by the relation kva. = AT^n. The authors of this paper have based their conclusions on a series of individual test points. It is assumed that in drawing the conclusions regarding the existence of three distinct regions of breakdown as has been done, there is probably on file a large accumulation of experimental data. If such data are typical of that presented in the paper, however, it is difficult to accept the conclusions drawn. Rather, one might break the data up still further and assume a whole series of step-like characteristics of breakdown as a function of thickness. To accept from the data presented that "three distinct regions of breakdown are obtained" appears to be drawing a fine distinction not warranted in the light of the usual vagaries of insulating material.

In experimental work involving a material such as glass, it must always be borne in mind that one is dealing with a material which may vary from batch to batch and which in itself bears little, if any, relation to other materials. Being a super-cool liquid, largely electrolytic in its electrical conductivity, conclusions drawn cannot safely be extended to apply to pure crystals or ceramic materials on the one hand, or to the usual organic commercial insulation on the other. The conclusions drawn by the authors, even if accepted, should be carefully limited to glass of the type under discussion which in itself might be more clearly described than has been done in the paper.

N. D. Kenney: The question of there being an intermediate region as raised by Dr. J. B. Whitehead and Mr. W. A. Del Mar, can best be answered by an examination of Fig. 2 and Fig. 4. Unless puncture tests on dielectrics are made with edge effect eliminated the field is decidedly non-uniform and the data are such as to make it impossible correctly to interpret the effect of thickness and temperature. The use of glass spheres removes the error introduced by concentration of stresses along an edge and a decided increase in the potential gradient is obtained. An examination of either Fig. 2 or Fig. 3 will show that it would be impossible to draw a line through the intermediate points that would be parallel to either the disruptive or thermal line. In the thermal region, as Dr. J. B. Whitehead has brought out, for thick samples and high temperatures the internal temperature becomes greater than the ambient temperature at the instant of breakdown. In 1931 Professor P. H. Moon presented a very interesting paper on thermal breakdown.*

Mr. Del Mar questioned Fig. 4 because there are no points shown. The data for Fig. 4 were obtained by taking any given thickness such as 200 μ and reading the breakdown voltages for various temperatures from Fig. 2. These values were plotted in Fig. 4 so as to show more clearly the relations existing between breakdown voltage and temperatures for a given thicknes of glass. The points would lie along the curves because of the manner in which they were obtained and there was no need to show them.

It is to be understood that no claim is made that this intermediate region is characteristic of all dielectrics. The physical construction of a dielectric is an important factor of its breakdown strength. This region seems to be characteristic of glass, quartz, and celluloid as consistent tests have shown and its physical form no doubt is closely related to its cause.

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*_The Theory of Thermal Breakdown of Solid Dielectrics_, by P. H. Moon, A.I.E.E. Trans., Sept. 1931, p. 1008.

Magnet Steels and Permanent Magnets

Relationships Among Their Magnetic Properties

BY K. L. SCOTT*
Member, A.I.E.E.

INTRODUCTION

A GENERAL study of the magnetic properties of magnet steels and permanent magnets embraces a number of related problems. Of chief interest to the designer of magnets is the problem of determining the relationship between the open-circuit remanence of a permanent magnet and the various factors which determine its value. These factors include the magnetic characteristics or properties of the steel, as displayed by its hysteresis loop, and the shape and dimensions of the magnet. Of interest to the manufacturer of magnet steel are the questions of how chemical composition, melting practise, and rolling practise affect the magnetic and mechanical properties of the steel. The manufacturer of magnets is interested in the effect of the necessary manufacturing operations, such as heating for hot forming and annealing to increase machinability, upon the flux obtainable in the finished magnet; on the tendency of the steel to warp and crack upon quenching; and on the proper hardening treatment to use.

These interests overlap, of course, and include many special subjects, among which may be listed the manner in which various influences may affect the state of magnetization of a magnet, the correlation of the microstructure and mechanical properties of a magnet steel with its magnetic properties, the determination of a suitable criterion of magnetic quality, the development of accurate and convenient testing equipment, and various other matters of greater or less importance.

It is the purpose of this paper to present data relating to some of the above topics, which have been collected by the writer during the course of several years connection with the manufacture of permanent magnets, both in the laboratory and in the shop.

SYMBOLS AND NOTATIONS

The following symbols and notations will be used. See Fig. 1. All values of B are intrinsic or ferric induction.

B_{max} = The value of magnetic induction corresponding to the tip of a given hysteresis loop, in gauss.

H_{max} = The value of H corresponding to B_{max}, in oersteds.

B_s = The saturation value of flux density corresponding to an infinite magnetizing force.

B_r = Residual induction, the magnetic induction in a ring or infinitely long straight bar after the value of H has been reduced from H_{max} to O. The value of B at the intersection of the hysteresis loop with the B axis, in gauss.

H_c = Coercive force, the value of H required to reduce B from B_r to O in a ring or infinitely long bar. The value of H at the intersection of the hysteresis loop with the H axis, in oersteds.

B_{rem} = Remanence, the magnetic induction at the magnetic equator of a permanent magnet with no external magnetizing or demagnetizing force. Values of B_{rem} given in this paper are without pole pieces on the magnets, in gauss.

L = The actual developed length of a magnet.

A = The area of cross section of a magnet.

D = The equivalent diameter of a magnet $= 2\sqrt{A}/\sqrt{\pi}$.

L/D = The dimension ratio of a permanent magnet.

$(BH)_{max}$ = The maximum value of the product of the coordinates of the demagnetization curve for a given steel. The demagnetization curve is the portion of the hysteresis loop between B_r and H_c.

T = Hardening temperature, absolute scale.

T_0 = Optimum hardening temperature, absolute scale.

FACTORS THAT DETERMINE THE REMANENCE OF A PERMANENT MAGNET

Of the subjects enumerated in the introduction, the first is the one that has been of greatest technical interest to the writer. During the course of several laboratory investigations connected with shop problems, advantage was taken of the opportunity to secure data which might be used for the purpose of making generalizations regarding relations among magnetic properties of magnet steels and permanent magnets.

The specific problem was to find out exactly the way in which the remanence of a magnet is related to its shape and dimensions as well as to the magnetic properties of the material of which it is made. Although it requires the complete family of hysteresis loops for a given piece of magnet steel to portray all of its various magnetic characteristics, it was felt that possibly the

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*Western Electric Company, Inc., Hawthorne Station, Chicago, Ill.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

combined influence upon the remanence of a magnet of the sum of these characteristics could be expressed in terms of the values of B_r and H_c for the material. Such an assumption, if valid, would simplify the problem considerably.

It is true that in the literature on the subject there are several papers[1,5,6] describing more or less accurate methods by means of which a designer may predict the flux of a given magnet provided the demagnetization curve for the steel is known. But these methods are laborious, and unless a complete redesign is worked out for each case, they do not answer such questions, for example, as how much a certain change in B_r or H_c will affect the remanence of a given magnet, other factors remaining constant.

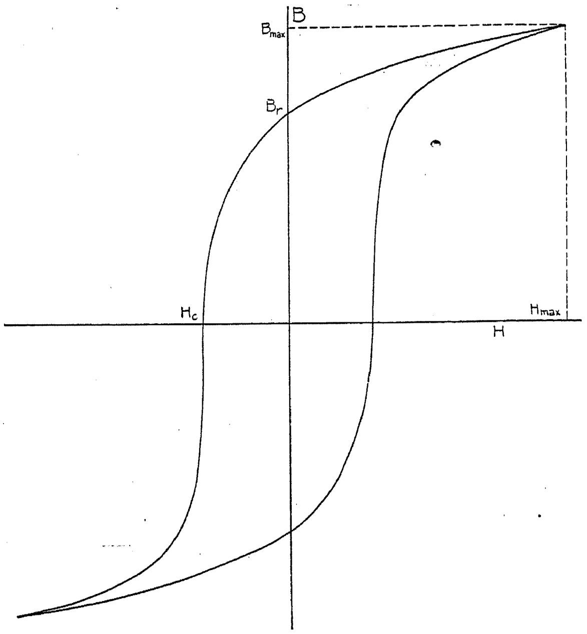

Fig. 1—Hysteresis Loop of Magnet Steel

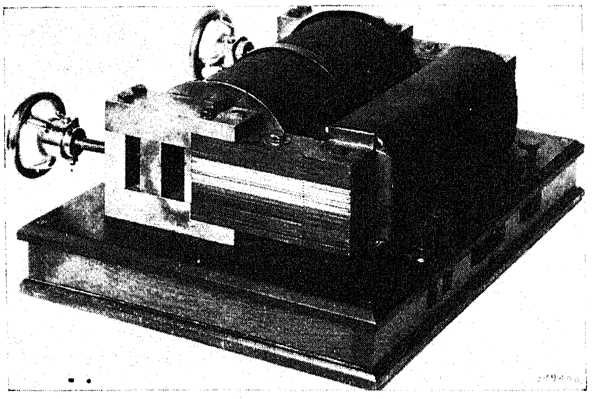

Fig. 2—The Babbitt Permeameter

Used for making magnetic measurements on any kind of magnet steel

Because the flux value of a magnet is affected by shape as well as other factors, and because magnets are made in a bewildering variety of shapes, it was decided to confine the investigations to straight bar magnets and leave the matter of the effect of shape on flux for a later study.

Test Specimens. Accordingly, a large number of straight bars of magnet steel was used for test specimens. Each of the bars was 12 in. in length. In the lot were bars of 1/8 by 1-1/2 in., 1/4 by 1/4 in., 3/16 by 3/4 in., 1/4 by 1-1/4 in., 3/4 by 5/8 in., and 3/8 by 1/2 in. cross sections. All of the types of steel available were used, including 0.9 to 3.5 per cent chromium steel, 0.85 per cent manganese steel, 5 per cent tungsten steel, and 20, 25 and 36 per cent cobalt steels. Along with this range in composition a range in coercive force of from 40 oersteds to 260 oersteds was available, accompanied by a variation in residual induction of from 6,000 gauss to 11,000 gauss.

Experimental Procedure. The test bars were hardened in the manner appropriate to each type of steel, except that some bars were purposely quenched from a high temperature in order to secure low values of B_r. After hardening, the demagnetization curve for each bar was determined by means of permeameter measurements, using the Babbitt[7] permeameter and a Grassot fluxmeter with lamp and scale. The control circuit of this permeameter is the conventional circuit for a ring test, as shown in Fig. 3. The magnetic data are all comparable since accurate measurements may be made with this permeameter on all of the above mentioned types of magnet steel.

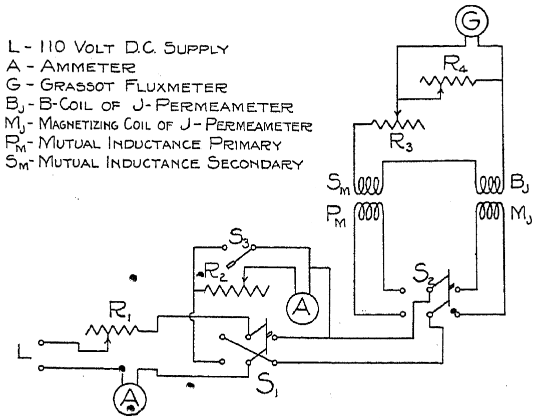

Fig. 3—Control Circuit of the Babbitt Permeameter

Note that the connections are those of a simple ring test

Following the permeameter measurements, each bar was magnetized as a straight bar magnet in an air core solenoid. Field strengths of over 1,000 oersteds were applied to each bar. Upon removal from the solenoid the flux density at the middle of the bar was measured by means of a search coil and the Grassot fluxmeter with lamp and scale. It is the flux density measured in this way that is referred to in this discussion as the remanence of a magnet, or B_{rem}.

After the measurements at 12-in. lengths were com-

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[1] For references see end of paper.

pleted, each bar was cut in succession to 10, 8, 6, 4, and 2-in. lengths by removing 1 in. of material from each end except that in the case of some of the bars of larger cross-section the lengths of 10, 6, and 2 in. were omitted.

Remanence measurements were made at each of the above lengths. The bars were remagnetized for each new measurement of B_{rem}.

The foregoing experiments made available for analysis a wide range of values of the associated variables B_{rem}, B_r, H_c, length L, and cross-sectional area A.

Analysis of Data. The values of remanence *versus* length for a number of the straight bar magnets are plotted in Fig. 4 to show the range of values existing in different bar magnets of the same length, when fully magnetized. The differences in B_{rem} for a particular length of bar are due to the differences in B_r, H_c, and cross-section of the various samples.

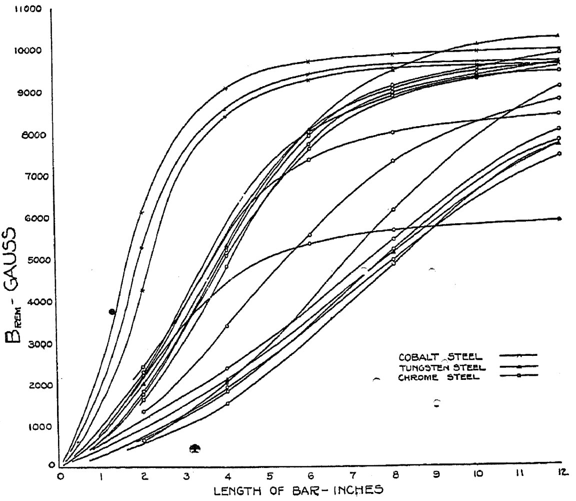

FIG. 4—CURVES OF REMANENCE *vs.* LENGTH FOR STRAIGHT BAR MAGNETS OF DIFFERENT CROSS-SECTIONS AND MATERIALS

The chief characteristic that the curves of Fig. 4 have in common is a resemblance in shape to a normal induction curve and an asymptotic approach to limiting values of B_{rem} as the lengths increase.

In Fig. 5 is shown the result of plotting B_{rem}/B_r *vs.* $(L/D)\sqrt{H_c}(T/T_0)^2$ for the same set of bars. This combination of variables was arrived at purely by a cut and try method, but as shown, a surprisingly good correlation is obtained.

It should be mentioned in connection with Fig. 4 that it was found that the data relating to some of the bars did not fit the curve satisfactorily if B_{rem}/B_r was plotted against $L\sqrt{H_c}/D$, but in each case where this was so the bar had been quenched from above or below the optimum hardening temperature. It was found, however, that the data from these bars could be correlated with those from the properly hardened bars if B_{rem}/B_r were plotted against $(L/D)\sqrt{H_c}(T/T_0)^2$ in which T is the actual, and T_0 is the optimum hardening temperature on the absolute scale. The ratio T/T_0 is, of course, equal to unity for bars properly hardened. The definition of the optimum hardening temperature will be given later.

After this empirical correlation was obtained, it was suggested that B_{rem}/B_r *vs.* $L\sqrt{H_c}/D\sqrt{B_r}$ would be preferable as a choice of variables, from theoretical considerations based upon the assumption of uniform magnetization in the magnets. Although it can be shown that this assumption is not fulfilled in the actual case of straight bar magnets, this method of plotting gives as good a correlation as is shown in Fig. 5. Such a result might be expected, since the values of B_r commonly encountered range from 9,000 gauss to 11,000 gauss, thus differing from 10,000 gauss by 10 per cent at most. The square root of B_r, therefore, differs from 100 by not more than 5 per cent in most cases, whence

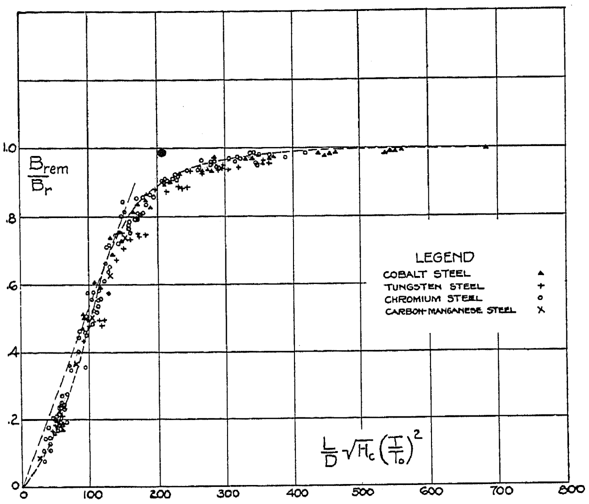

FIG. 5—THE SAME DATA AS IN FIG. 4

Showing the correlation between B_{rem}/B_r and $(L/D)\sqrt{H_c}$ for bars of any kind of material, magnetic properties, length or cross-section

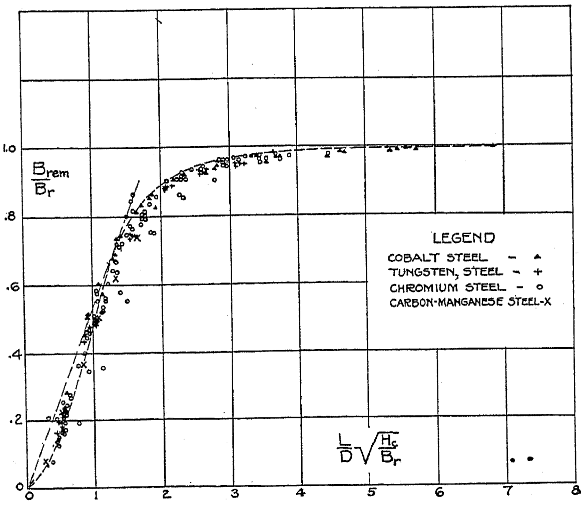

FIG. 6—THE DATA OF FIG. 4 WITH B_{rem}/B_r PLOTTED AGAINST $L\sqrt{H_c}/D\sqrt{B_r}$

it follows that dividing $L\sqrt{H_c}/D$ by $\sqrt{B_r}$ shifts the relative position of the abscissas of the points in Fig. 5 by not more than 5 per cent in most cases. Fig. 6 illustrates the results obtained by the second method of plotting.

Dimensional considerations favor the use of the quantity $L\sqrt{H_c}/D\sqrt{B_r}$, however, since this quantity, like B_{rem}/B_r, is dimensionless, whereas $L\sqrt{H_c}/D$ is not. For this reason it is felt that the method of plotting shown in Fig. 6 is to be preferred to that of Fig. 5 although the correlation obtained in Fig. 5 appears to be as good as that shown in Fig. 6. Further, the necessity for injecting the variable of temperature into the picture is done away with.

Inasmuch as the data from a large number of bars of widely different compositions, magnetic properties, dimensions and heat treatment are all found to cluster quite closely along a single curve, it is felt that it is legitimate to use the curve as a basis for some generalizations, even though the curve was arrived at largely by empirical processes.

The type of curve that is obtained when B_{rem}/B_r is plotted against $L\sqrt{H_c}/D\sqrt{B_r}$ indicates that other factors being constant, the remanence of a magnet is roughly proportional to $\sqrt{B_r}$ and, for large values of $L\sqrt{H_c}/D\sqrt{B_r}$, in other words, for large values of dimension ratio or coercive force, the remanence is practically equal to B_r. This fact is of interest from a design standpoint.

It was to be expected, of course, that the values of B_{rem} would approach B_r as a limit for high values of length of bar, since the end effect diminishes as the length increases and the condition of an infinitely long bar or a closed ring is simulated. The functional relationship which the quantity $L\sqrt{H_c}/D\sqrt{B_r}$ bears to the ration B_{rem}/B_r is not known. A fairly good fit of the observed data is given by the expressions:

$$r = \frac{q^{1.5}}{2.15} \qquad q = O \text{ to } q = 1.25 \text{ and}$$

$$r = e^{-\frac{0.8}{q^{2.75}}} \qquad q = 1.25 \text{ to } q = \infty$$

in which $r = B_{rem}/B_r$ and $q = L\sqrt{H_c}/D\sqrt{B_r}$, but aside from the direct proportionality of B_{rem} and B_r, the equations appear to have little meaning. However, they do indicate that for a given dimension ratio, there is a practical upper limit to H_c, beyond which very large increases in H_c are necessary to produce small increases in the ratio of B_{rem} to B_r. Considerable interest attaches to the fact that the dimension ratio L/D and $\sqrt{H_c}/\sqrt{B_r}$ are of equal weight in affecting the remanence of a magnet. It is also worth noting that the value of B_{rem} is independent of the contour of the cross-sectional area. It is possible that this would not hold for dimension ratios less than one, but it does appear to hold for dimension ratios of practical importance. As shown in Fig. 5, a line drawn through the origin and tangent to the dotted curve of that figure has its point of tangency at values of B_{rem}/B_r and $L\sqrt{H_c}/D\sqrt{B_r}$ of approximately 0.65 and 1.25 respectively. It will be shown later that this is the point of maximum efficiency, *i.e.*, the point at which are obtained the highest values of B_{rem} or external magnetic energy per unit volume of steel.

It follows, if this is true, that magnets should be designed so that $L\sqrt{H_c}/D\sqrt{B_r} = 1.25$, and with this as a basis, the nomogram of Fig. 7 was laid out. The use of this chart to design a magnet for maximum efficiency, is illustrated by the dotted lines of the figure. In the case shown a total flux in the magnet of 3,000 maxwells is desired, and a steel with a B_r of 10,000 gauss and a coercive force of 54 oersteds is assumed. The correct values of A and L are found to be 0.461 cm.^{2} and 13 cm., respectively, and the dimension ratio is 17.

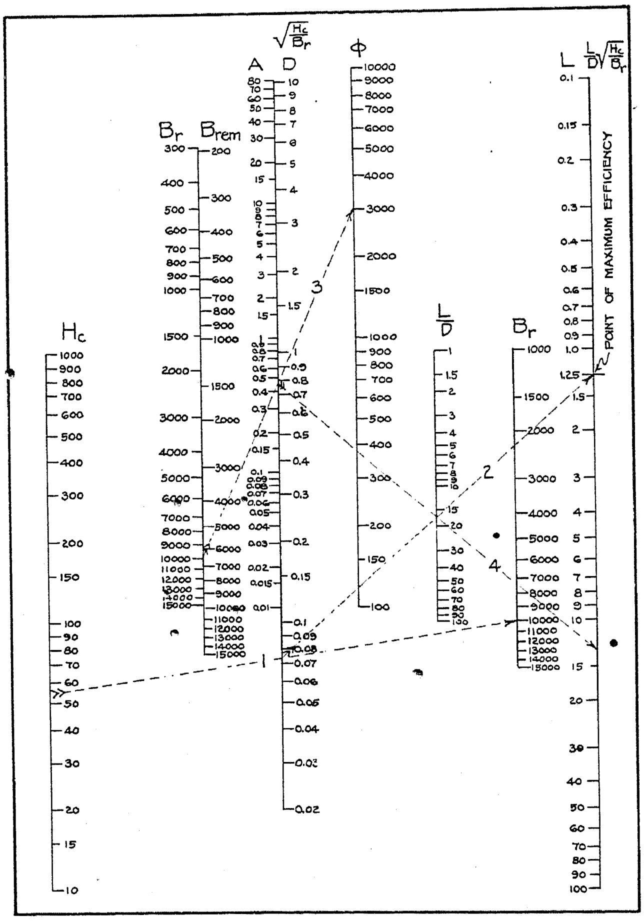

Fig. 7—Nomogram Chart for Permanent Magnet Design for Maximum Efficiency. Based on the Curves of Figs. 6 and 10

It should be borne in mind throughout this discussion that Figs. 4, 5, 6 and 7 apply directly only to straight bar magnets. Formed magnets with short air-gaps with or without pole pieces, will have higher remanence values than are shown by the curve of Fig. 6. This is because, compared with straight magnets, their effective lengths are greater than their actual lengths. The difference between effective and actual lengths is negligible, however, except for magnets whose poles are brought quite close together, either actually or by means of pole pieces of soft iron. Watt-hour meter damping magnets are good examples of magnets whose poles are brought thus closely together without the aid of pole pieces.

On the other hand, the general conclusions drawn from the curve of Fig. 6 are applicable to magnets of any shape if L is assumed to stand for the effective length of the magnet. Fig. 7 also is perfectly general on the same basis. By effective length is meant the length of a straight bar magnet of equal cross-section and magnetic properties, having the same remanence as the given magnet.

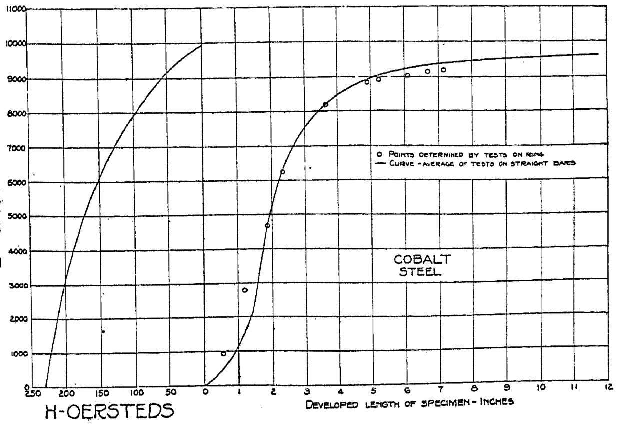

FIG. 8—COMPARISON OF B_{rem} *vs.* LENGTH CURVES FOR STRAIGHT BAR AND 2-IN. I.D. RING OF 1/4-IN. x 1/4-IN. COBALT STEEL IN WHICH $B_r = 9{,}980$ GAUSS AND $H_c = 230$ OERSTEDS

CORRELATION BETWEEN STRAIGHT AND BENT MAGNETS

Subsequent to the establishment of the relationships shown in Fig. 6, a limited amount of data were secured which have a bearing upon the effect on B_{rem} of departure from the form of a straight bar magnet.

To get these data, one 1/4 by 1/4 in. rod each of cobalt steel and tungsten steel were formed into rings of 2-in. inside diameter, with the ends touching. Companion test bars were cut adjacent to each end of the rods used in forming the rings. All three pieces of each kind of steel received as nearly identical heat treatments as possible, even to heating and cooling the straight test bars at the time of forming the rings. After hardening, demagnetization curves were obtained on each of the two test bars of tungsten steel and of cobalt steel. Then data for B_{rem} *vs.* length curves were obtained from both the test bars and rings by the process of cutting off the ends and remeasuring B_{rem} already described.

The demagnetization curves and the B_{rem} *vs.* length curves, for the two straight cobalt steel bars are practically identical. Inasmuch as these bars were cut adjacent to the ends of the bar from which the ring was formed, and heat treated with it, it is safe to assume that the material in all three samples is alike magnetically, and that the demagnetization curve for the ring is the same as for the straight bars. The same conclusions hold for the tungsten steel, as shown in Fig. 9.

It follows then that it is valid to compare the B_{rem} *vs.* length curves for the straight bars with those for the rings, and conclude that all differences in the curves are due to differences in shape. It is interesting to note that the ring for cobalt steel yields the same curve for B_{rem} *vs.* length as the straight bar, but that, in the case of the tungsten steel, the proximity of the ends of the ring has an appreciable effect on the value of B_{rem}, causing the curve for B_{rem} *vs.* length to turn upwards for those lengths corresponding to short air gaps.

Additional data along these lines would be very useful, and should include measurements on magnets with pole structures having well defined air gaps.

CRITERIA OF MAGNETIC QUALITY OF PERMANENT MAGNET STEELS

A number of quantities determinable by magnetic measurement has been proposed at various times by different investigators for use as criteria of magnetic quality of magnet steels. Among these quantities may be listed $(BH)_{max}$(1), the maximum product of the coordinates of the demagnetization curve; and the quantities $B_r\, H_c$(2), B_r/B_{max}, $B_r\, H_c/B_{max}$ and B_r/H_c(3). High values of all of the proposed criteria except the last were considered desirable.

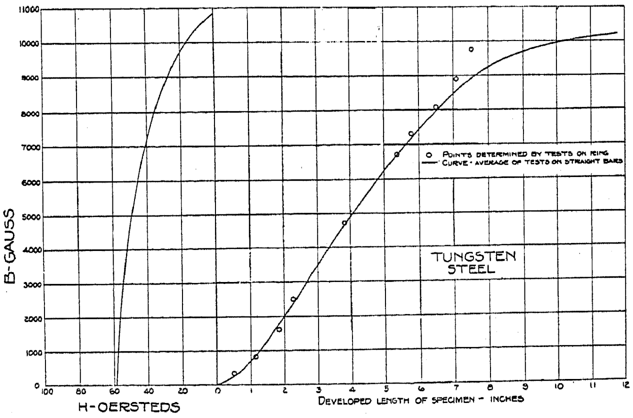

FIG. 9—COMPARISON OF B_{rem} *vs.* LENGTH CURVES FOR STRAIGHT BAR AND 2-IN. I.D. RING OF 1/4-IN. x 1/4-IN. TUNGSTEN STEEL IN WHICH $B_r = 10{,}830$ GAUSS AND $H_c = 59$ OERSTEDS

The form B_r/H_c has decided disadvantages, since a low value of the criterion is obtained when B_r is small as well as when H_c is large, and low values of B_r are distinctly undesirable in view of the fact that B_{rem} is directly proportional to B_r. The ratio B_r/B_{max} was found to be practically constant for some types of steel, regardless of the value of H_c, and thus was of no value as a criterion. The quantity $B_r\, H_c/B_{max}$ would accordingly amount to a constant times H_c and would give no actual weight to the value of B_r. These three criteria would, therefore, fail either to indicate a choice between different types of magnet steel or to aid in the selection of an optimum heat treatment for a given kind of steel.

This leaves the quantities $(BH)_{max}$ and $B_r\ H_c$ as the only ones not obviously faulty. The first was proposed by S. Evershed in a noteworthy paper in the *Journal* of the Institution of Electrical Engineers for September 1920, in which he showed that the quantity $(BH)_{max}$ is a measure of the maximum amount of external magnetic energy which can be supported per unit volume of a given magnet steel, and that this in turn defines the term "magnetic quality" as applied to magnet steel.

Evershed's derivation of his criterion of magnetic quality, given in the article noted above, is complete and convincing, but it was thought desirable to obtain experimental verification. The data from the bars of Fig. 4 were used to this end. In Fig. 10 is shown a number of curves each corresponding to a different type of magnet steel. The demagnetization curves for the bars are shown plotted in the usual way, and to the right of these are plotted the curves of (BH) *vs.* B. To the right of these are plotted further the experimentally determined curves of B_{rem} *vs.* length for the bars in question.

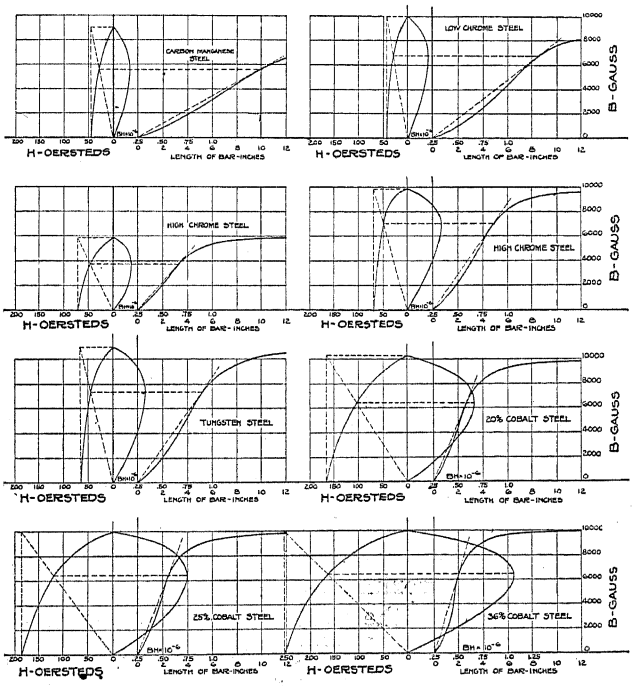

FIG. 10—CURVES SHOWING EXPERIMENTAL VERIFICATION OF VALIDITY OF EVERSHED'S CRITERION FOR MAGNETIC QUALITY OF PERMANENT MAGNET STEELS

Fig. 10 shows that if tangents to the curves of B_{rem} *vs.* length are drawn through the origin the point of tangency in each case indicates the value of B_{rem} and length at which the ratio of B_{rem} per unit length or volume of steel is a maximum, and it is clearly demonstrated that in each case these values of B_{rem} coincide closely with the values of B for which the product (BH) is a maximum. The writer feels that this constitutes a sufficiently convincing verification of the correctness of Evershed's criterion.

The chief objection to the use of Evershed's criterion is that a number of points on the demagnetization curve for a piece of magnet steel must be known before the numerical value of the product $(BH)_{max}$ can be determined. The quantity $B_r\ H_c$ is more easily obtained than $(BH)_{max}$ but its validity as a criterion cannot be established from theoretical considerations. Accordingly, the values of $(BH)_{max}$ and $B_r\ H_c$ were determined by actual measurement for a large number of bars of different kinds of magnet steel. In Fig. 11 are shown the values of $B_r\ H_c$ plotted against the corresponding values of $(BH)_{max}$. It is felt that these points lie closely enough along the straight line in that figure to warrant the use of $B_r\ H_c$ as a criterion of magnetic quality of magnet steel for all practical purposes. In cases where very accurate comparisons are required, it may be necessary to go to the greater trouble of determining $(BH)_{max}$.

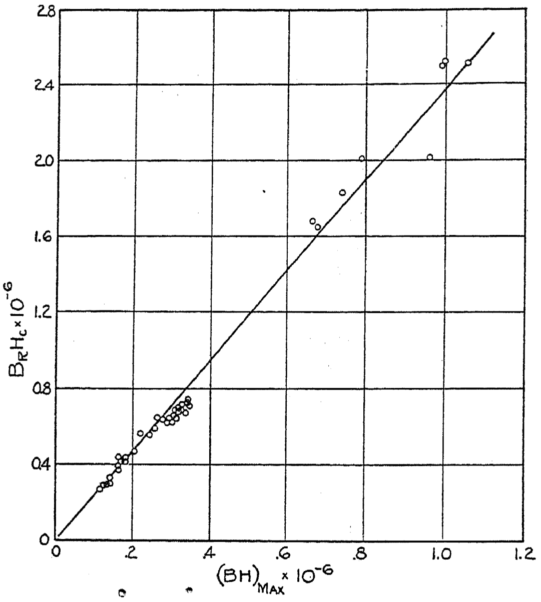

FIG. 11—RELATION BETWEEN THE QUANTITY $B_r \times H_c$ AND EVERSHED'S CRITERION, $(BH)_{max}$

Referring to a previous mention of optimum hardening temperatures, it will be understandable now to state that the optimum hardening temperature for a given type of steel is that temperature at which a maximum value of $(BH)_{max}$ is obtained, or practically speaking, a maximum value of $B_r\ H_c$.

FIT OF DEMAGNETIZATION CURVE BY HYPERBOLA

It will be noted that the points on the demagnetization curves for which the product (BH) is a maximum are given quite accurately in each case by the intersection of the demagnetization curve with a line through the origin having the slope B_r/H_c. That this should be so follows from the fact that any demagnetization curve for magnet steel may be closely approximated by a rectangular hyperbola whose equation is $B = a - k/(H + b)$ in which a, b, and k are parameters of each particular curve. It is a mathematical property of the foregoing rectangular hyperbola that the coordinates whose product is a maximum are located by a line through the origin having a slope equal to the ratio of the intercepts of the hyperbola. This property of the hyperbola is a property of the demagnetization curves for magnet steel in so far as these curves can be closely fitted by appropriate hyperbolas. That the fit is quite good in all cases is shown by Fig. 12 in which hyperbolas calculated for each demagnetization curve are shown by dots and the observed points by a full line. The graphical method of Fig. 10 for determining the point on each demagnetization curve the product of whose coordinates is a maximum is more accurate than the method of plotting the curve of (BH) vs. B, because the latter curve is usually quite flat-topped and its maximum is hard to locate exactly. The fact that the curve is flat-topped explains why many methods of magnet design give good results. It is because about equal efficiencies are obtained in any case over a fairly wide range of values of B_{rem}.

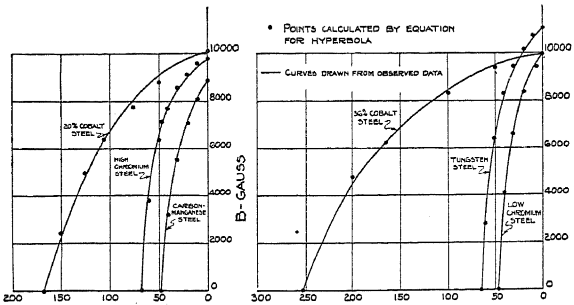

Fig. 12—Curves Showing Approximation of Actual Demagnetization Curves by Points on the General Hyperbola, $B = a - k/(H + b)$

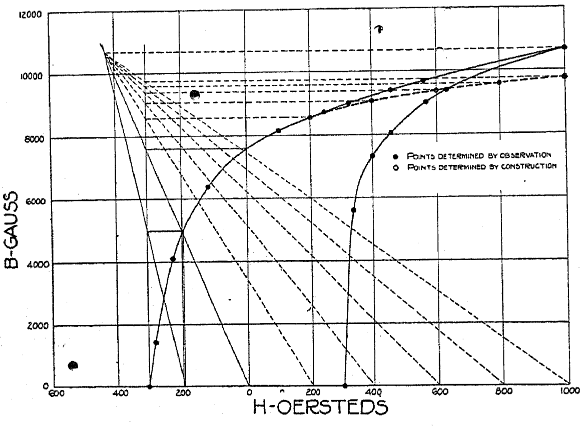

Fig. 13—Comparison of Graphical Construction of the Descending Portion of the Hysteresis Loop with Actual Measured Data

Note the lack of agreement for positive values of H

A further application of the idea that the demagnetization curve can be represented by a hyperbola is made by Mr. E. A. Watson.[4] Watson applies a graphical construction for a hyperbola to the demagnetization curve and extends the construction into the region of positive values of H, thus providing a means of extrapolating the hysteresis loop to the saturation value of B. It has been the writer's experience that Watson's graphical construction cannot be legitimately extended into the first quadrant because the curve thus predetermined almost always lies considerably under the true curve, and yields too low a value for the saturation flux density. Fig. 13 illustrates the case in point.

If, instead of potting B vs. H for the points of the demagnetization curve, we plot B/B_r vs. H/H_c, it is found that all test specimens yield points fitting fairly well the single curve represented by the equation $y = A\ (x + 1)/(x + A)$, in which $y = B/B_r$, $x = H/H_c$ and $A = \sqrt{2}$ approximately. It is possible to derive from this equation the relationship that $(BH)_{max} = 0.423\ B_r\ H_c$ which checks very well with the line drawn in Fig. 11. It can also be seen from the equation $y = A\ (x + 1)/(x + A)$ that the representation of the demagnetization curve by a hyperbola cannot be extrapolated to give the correct value for B_s since for values of H approaching infinity, the equation yields the result that $y = B_s/B_r = A = \sqrt{2}$ whence $B_s = \sqrt{2}\ B_r$, while values of B in excess of $\sqrt{2}\ B_r$ are commonly encountered at ordinary values of H.

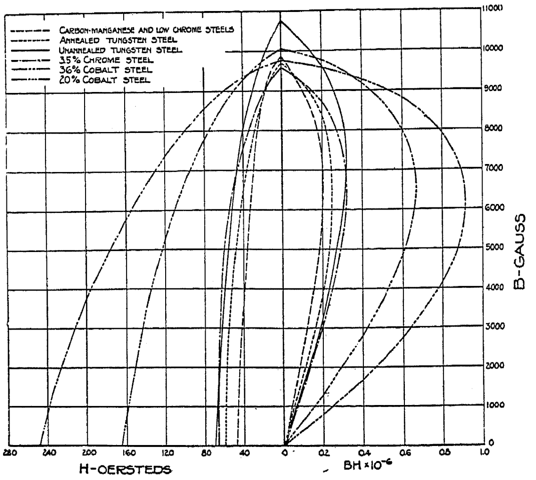

Fig. 14—Typical Demagnetization Curves and Curves of BH vs. B for Various Kinds of Magnet Steel

Characteristic Curves for Magnet Steel

In Fig. 14 are shown demagnetization curves and (BH) curves for the various types of magnet steel with which the writer has had experience. It should be remembered that while these curves are more or less typical of the steels in question, there is no one curve which can be given as the curve for the material. There will be as many curves as there are samples tested, because magnetic characteristics of a piece of magnet steel, or any other magnetic material depend upon the whole previous thermal history of the sample from the time of melting to the time of testing, the amount of hot and cold working the sample has received, and the conditions of stress in the material at the time of testing. The relation of all of these variables to the remanence of a permanent magnet, fortunately, is focussed, as it were, in the values of B_r and H_c for the material, and the relation of these variables to B_{rem} is as shown by Fig. 6.

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4. In discussion, *loc. cit.*(1), *Journ. Inst. Elect. Eng.* **58**, 829 (1920), also *ibid.* **61**, 641 (1923).

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Discussion

W. A. Codd: The paper quite evidently checks experimentally Evershed's criterion of magnetic quality for magnet steels and apparently offers a considerably simplified design method in the nomogram presented.

In Fig. 9, from the data given on the closed ring sample, 2 in. inside diameter and ¼ in. by ¼ in. section, together with the maximum length plotted on the curve, it would appear that the developed length of the ring sample has been taken as the circumference on the outside diameter of the ring rather than the mean length which appears to be more logical. Is there a reason for this or is the 2 in. inside diameter of the ring merely an approximate figure.

Also in connection with Fig. 9, what is the reason for the difference of the effect of proximity of the ends of the magnet for Tungsten steel as indicated in this figure as compared to this effect for the Cobalt sample as shown in Fig. 8. It would appear that the reason for this lies in the facts brought out in the paper in regard to the relatively small effect of increases of H_c, or equivalent changes, as B_{rem}/B_r approaches unity. Is this correct?

There is one other question in regard to the data taken on the ring samples. How was the measurement of the B_{rem} flux value made on the ring sample for maximum length with the ends touching?

It is understood that the measurements of B_{rem} were made, after the sample was magnetized, by use of a search coil. In this case to measure the total flux of the magnet at this neutral section it would be necessary to draw the coil from the neutral section off the end of the magnet. Otherwise only the leakage flux would be measured, of course.

In regard to the nomogram shown in Fig. 7, for use in permanent magnet design, this apparently offers a decided simplification over Evershed's step-by-step method of calculation. However, for use in connection with bent magnets, as the author points out, it is necessary to use the effective length rather than the actual length of the magnet. Figs. 8 and 9 evidently offer a method of obtaining this effective length for a given shape of bent magnet. Is it necessary to make such an experimental determination for each particular shape which is to be used, or is there a general method for obtaining the effective length? If such a method of readily calculating this effective length were available, the design nomogram would be extremely valuable.

The author mentions variations between samples due to differences of the thermal history of the sample. In connection with this, Evershed in his article of Aug. 1925 on "Permanent Magnets in Theory and Practise" discusses these thermal effects in considerable detail emphasizing in particular two effects which he refers to as ultra-heating and spoiling.

Ultra-heating, he states, results in reduced values of B_r due to failure of some of the iron molecules to return to the α or magnetic state at room temperatures. He suggests a curing process consisting of heating to a moderate temperature (1,380 deg. fahr.) followed by slow cooling and states that this process should always be used by the magnet maker before forming magnets. This does not seem to agree with the process recommended by some steel manufacturers which indicates that the total amount of heating used by the magnet maker should be kept at a minimum.

In regard to "spoiling," Evershed states that this consists of breaking up of the carbide molecules which results in reduced coercive force. This, he states, should be cured by heating to about 2,370 deg. fahr. for several minutes. This he indicates should be done at the mills before rolling into bars. If this is not done the magnet maker will receive steel low in coercive force.

Evershed gives the above particularly in regard to Tungsten steel. Has the author any experience in regard to these points?

The author has made numerous measurements on a large number of samples of each of the various kinds of magnet steel available. Does he find that any of these steels show any better uniformity than others when exercising equal care in control of heat treatment?

K. L. Scott: The ring magnets, about which Mr. Codd has asked were formed hot about a mandrel roughly two inches in diameter. The developed length, as given in the paper, represents the actual mean length of the ring. Mr. Codd's interpretation of Figs. 8 and 9 is correct. A gap was ground between the ends of the ring magnets, large enough to admit a small search coil, before any flux measurements were undertaken, although an initial value of B_{rem} could have been obtained before this was done by following the procedure for a standard Rowland ring test.

A general method of determining the effective length of a magnet, with or without pole pieces, has not been worked out to the writer's knowledge. Such a general method would probably involve the calculation of internal and external reluctances for a given design of magnet and pole pieces, and the calculation of effective length from these reluctances either by reference to an empirical curve or formula.

The writer has had no first hand experience with steel mill practise in the manufacture of tungsten magnet steel. His own experience does bear out the recommendation that the total amount of heating by the magnet maker should be kept as low as possible. The writer's experience does not enable him to differentiate between the various kinds of magnet steel as regards uniformity of magnetic quality when exercising equal care in control of heat treatment. It appears to the writer that exact control of the heat treatment of magnetic materials is yet to be attained.

An Induction Motor with Paralleled Rotor and Stator

BY A. G. CONRAD[1]
Associate, A.I.E.E.

and

R. G. WARNER[1]
Member, A.I.E.E.

Synopsis.—*A wound rotor induction motor may be operated with the stator and the rotor independently magnetized, (neither supply frequency corresponding to the slip frequency). If the stator and rotor are paralleled the pull-out torque as a motor is approximately doubled and the continuous horsepower output may be increased, and as an induction generator, the maximum kva. output is increased. It may be operated simultaneously as a motor and as a generator.*

This method of operation was new to the authors and to several eminent induction motor designers and as no reference has been found to others employing this principle, the theory has been developed and is presented with the experimental results of tests.

INTRODUCTION

WITH machines wired as in Fig. 1 it is possible to transfer power from rotor to stator or from stator to rotor at high frequency and while this is taking place, the machine can either receive power or deliver power at its shaft. Using more general terms, if stator and rotor are independently excited then power can be delivered to the machine through three channels namely, the stator, the rotor or the shaft. When energy is supplied through two of these channels it can be taken from the third and when it is supplied to one it can be taken from one or both of the other two.

The rotor and stator voltage and frequency can be adjusted to the same values; thus it is possible to parallel stator and rotor and make the machine operate either as a double motor or double generator.

PRINCIPLE OF OPERATION

Fig. 1 shows a wiring diagram of a wound rotor induction motor with the rings of the motor connected to a synchronous machine. The armature of the synchronous machine serves to provide a path for the secondary current and consequently the induction motor can be started. The induction motor will behave much the same as it would with its rings short-circuited. If the phase rotation of the stator was in the order of 1-2-3 the voltages induced in the rotor would be a low-frequency voltage with a phase rotation of *a*-*b*-*c*. This will cause three currents to flow in the synchronous machine armature that are 120 deg. out of phase and in the order of *A*-*B*-*C*. This means, of course, that if the windings are placed on the machine as shown in the diagram that the induction motor will rotate in a clockwise direction.

With the stator disconnected from the supply and short-circuited directly or through transformers, and power supplied to its rotor from the alternator in Fig. 1, the induction machine may be started again. If the phase rotation of the alternator is *C*-*B*-*A* the induction machine will rotate in the same direction as above and the low-frequency currents in the stator will be in the direction 3-2-1.

If the stator is now supplied as usual and at the same time the rotor is supplied from the alternator with phase rotation *C*-*B*-*A* the supply transformers will act as a low impedance path for the slip-frequency currents in the stator and the alternator will complete the path for the slip-frequency currents of the rotor. The magnitude of the voltage impressed on the rotor is dependent on the field excitation of the synchronous machine. The frequency of the voltage depends upon the speed of the synchronous machine. By adjustment of the input of the d-c. machine the frequency in the rotor circuit can be made equal to the stator frequency. By the field adjustment of the synchronous machine the rotor voltage can be made equal to the stator voltage. When the rotor and stator have the same voltage and frequency they can be synchronized and electrically connected through the switch S_1 of Fig. 1. The circuit to the synchronous machine can now be opened at S_2. The induction motor will continue to operate with its rotor and stator electrically and magnetically connected; both the stator and rotor will receive power from the line. The machine will rotate at a speed less than synchronous speed. The high-frequency current in the stator will be in the order of 1-2-3 and the low-frequency currents in the order of 3-2-1. The high-frequency currents in the rotor will be in the order of *c*-*b*-*a* and the low frequencies in the order of *a*-*b*-*c*. The machine when

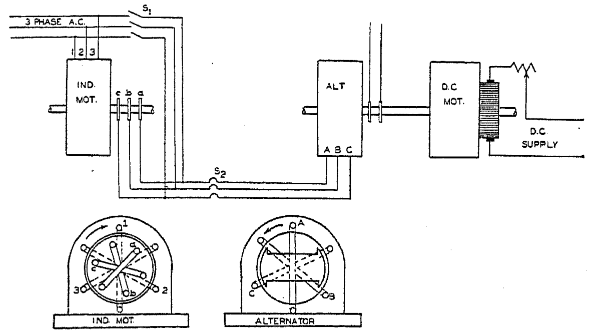

FIG. 1

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1. Asst. Prof. of Elec. Engg., Yale University.

Abridgment of a thesis submitted by A. G. Conrad to the Graduate School of Yale University, June 1931.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

so connected has characteristics similar to those of the normal induction motor. When loaded the speed decreases slightly.

When driven above synchronous speed the machine acts as an induction generator. The low-frequency currents in the stator and rotor are opposite in phase rotation to what they are as a motor and are in the order of 1-2-3 in the stator and *c-b-a* in the rotor.

The characteristics of the motor with its rotor and stator paralleled can be determined by applying ordinary induction motor theory. Using slip as parameter it is possible to determine the torque, stator, copper loss and rotor loss, etc., by the power fed into the stator and for the same slip determine value for the same items which are produced by the power fed into the rotor.

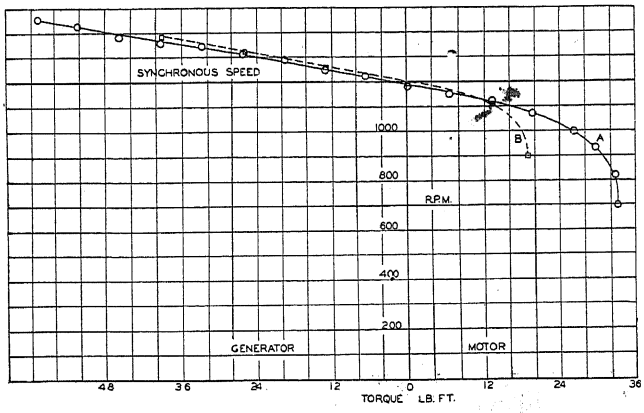


FIG. 2

Curve *A*—Speed torque curves of machine with rotor and stator paralleled
Curve *B*—Speed torque curves of machine with rotor short-circuited

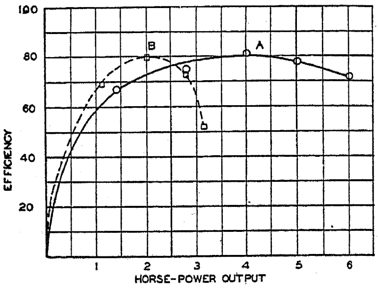


FIG. 3

Curve *A*—Efficiency of the motor with its rotor and stator paralleled
Curve *B*—Efficiency of the motor with its rotor short-circuited

EXPERIMENTAL INVESTIGATIONS

Calculations and analysis of operation were checked experimentally. The machine selected for tests was a Westinghouse wound rotor induction motor rated as follows:

Horsepower...... = 10 Stator volts....... = 220
Cycles........... = 60 Rotor volts....... = 206
Three-phase...... 1,140 r.p.m.
Serial No. 4191333

The motor has nearly a 1 to 1 ratio of stator to rotor turns.

In order to keep within the limitations of the laboratory circuits, the voltage on each phase was dropped to 110 volts, 60 cycles and the characteristics were determined with stator and rotor paralleled. The machine was operated both above and below synchronous speed to determine the characteristics as a motor and as a generator. In this machine the voltage supplied to each phase is a fundamental voltage while the current will contain not only a fundamental component but also a low-frequency component.

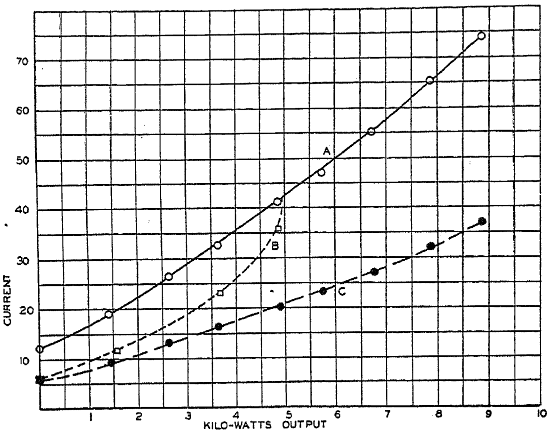


FIG. 4

Curve *A*—Line current delivered by generator with rotor and stator paralleled
Curve *B*—Current in stator phase of the generator with rotor short-circuited
Curve *C*—Current in stator phase of the generator with its rotor and stator paralleled

The results of these tests are shown as curve *A* in Figs. 2 to 5. For comparison curve *B* in each figure shows the characteristics of the motor with normal connections.

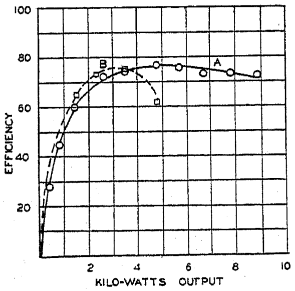


FIG. 5

Curve *A*—Generator efficiency with rotor and stator paralleled
Curve *B*—Generator efficiency with rotor short-circuited

Fig. 2 shows that the breakdown torque of the motor can be approximately doubled by using the stator and rotor in parallel; likewise that the generator capacity and maximum output can be increased by the parallel connection.

The relative efficiencies for the two methods of operation are shown in Figs. 3 and 5. With the parallel connection high efficiencies are obtained for large loads on account of reduced copper losses per unit output but the efficiencies are low for light loads because of larger iron losses and copper losses produced by the additional exciting current. The maximum efficiency for the two cases seemed to be approximately the same for both the condition of motor and generator action.

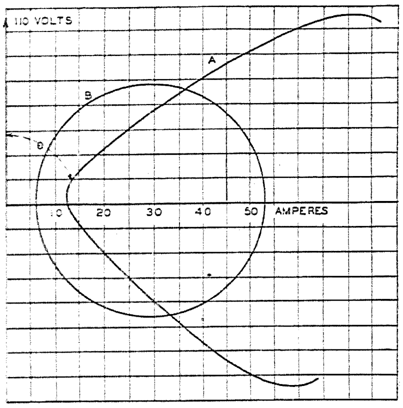


Fig. 6

Curve *A*—Locus of line current with rotor and stator paralleled
Curve *B*—Locus of line current with rotor short-circuited

The heating in the machine due to the copper losses will always be less for heavy loads when parallel connected than when the rotor is short-circuited. In Fig. 4 is shown the line current for different loads on the generator for each of the methods of operation. It can be seen that the phase current with the parallel connection will be less than the phase current when the generator is supplying the same load with its rotor short-circuited.

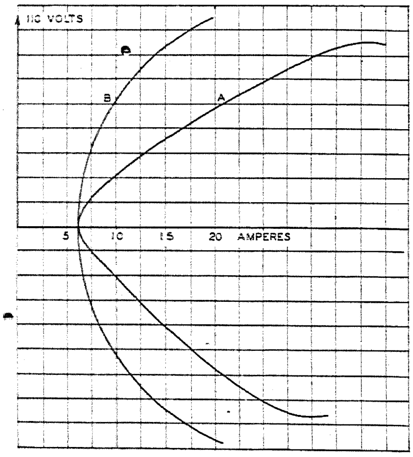


Fig. 7

Curve *A*—Locus of current in stator phase with rotor and stator paralleled
Curve *B*—Locus of current in same state phase with rotor short-circuited

In Fig. 6 is shown the circle diagram of the motor when the rings are short-circuited. The no-load losses for this machine are very small but it can be seen that when the rotor and stator are paralleled that the no-load line current contains a larger power component. This is because of the increased iron losses due to the high-frequency rotor flux and also the super-position of the two fluxes traveling in the iron at different speeds. In addition to this the exciting current in the rotor causes copper losses that increase the no-load input.

It will also be noted that the maximum power component of the input current is a little less than twice the maximum power component of the motor when its rings are short-circuited. This is caused by a slight terminal voltage drop with these heavy loads. It can be assumed that the heart-shaped diagram would have had a slightly higher maximum power component if the voltage had remained constant. The arithmetical sum of stator and rotor current was almost equal to the line current. If the rotor and stator constants were the same each current would be one-half the line current and when compared with the current with the rings short-circuited we would have current loci per stator phase as shown in Fig. 7.

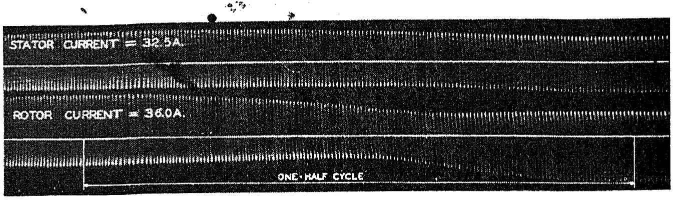


Fig. 8—No-Load Currents of Machine Operating with Rotor and Stator in Parallel. Voltage Applied = 245

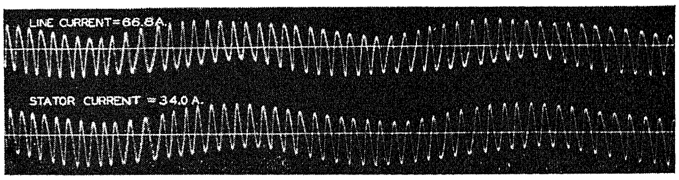


Fig. 9—Line and Stator Current for Condition of 220 Volts Applied to the Rotor and Stator in Parallel. Horsepower Output = 10 (100 Per Cent Load)

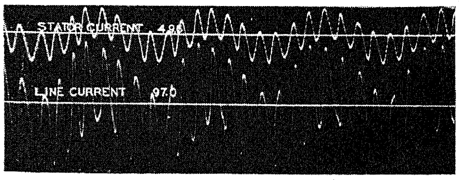


Fig. 10—Stator and Line Currents Taken by Motor with Its Rotor and Stator Paralleled Supply Volts = 220; Speed = 1040; Horsepower Output = 20 (200 Per Cent Load)

The exciting current taken by the motor is indicated in the oscillogram of Fig. 8. It can be seen in Fig. 9 that some of the high-frequency waves have pointed peaks and the other maximum value of the same wave has a rounded point. This indicates a high saturation during part of the cycle.

When the machine is loaded, the high-frequency currents are slightly out of phase and the low-frequency currents are slightly out of phase. This is caused by different constants in the rotor from those in the stator, *i. e.*, the ratio of stator inductance to stator resistance is different than the ratio of rotor inductance to rotor resistance. It can also be seen that the low-frequency current is slightly less than the high-frequency current in the same phase as is shown in Figs. 9 and 10.

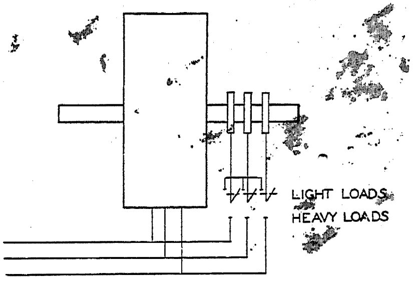
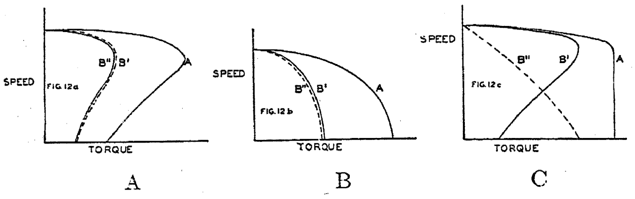


FIG. 11—A SUGGESTED CONNECTION BY WHICH OPERATING CHARACTERISTICS MIGHT BE IMPROVED

CONCLUSIONS

When the practical value of the connection is considered it can be seen that this connection increases the break-down torque approximately 100 per cent. The efficiency for heavy loads is higher than that of the same motor operated with its rings short-circuited. (See Figs. 3 and 5.) For light loads the efficiency of the motor with the rotor short-circuited is higher and the total excitation current is one-half or less depending upon the degree of saturation. When the machine is to be subjected to a wide range of loads it is possible to use a switching arrangement in the rotor that will short-circuit the rings when the load is light and connect the rotor and stator in parallel when the load is heavy. (See Fig. 11.)



A B C

FIG. 12

Curve *A*—Torque developed by motor when rotor and stator are paralleled
Curve *B*—Torque developed by motor with rotor short-circuited and power supplied to stator
Curve *C*—Torque developed by the motor with stator short-circuited and power supplied to the rotor

The advantages of this parallel connection are:

1. The rating of a given motor can be increased over what it would be if the rings were short-circuited.
2. Initial cost of motor required to drive a given load will be less and with the suggested switching arrangement smaller motors may be utilized giving the additional advantages.
3. The no-load losses may be made smaller.
4. The all day efficiency of the motor when subjected to a wide range of loads will be higher than that of a larger motor operating with its rings short-circuited.

The tests made on the machine as a motor and as a generator indicated two disadvantages of the parallel connection. The machine was noisy and the exciting current was high. This noise might be eliminated by special designing.

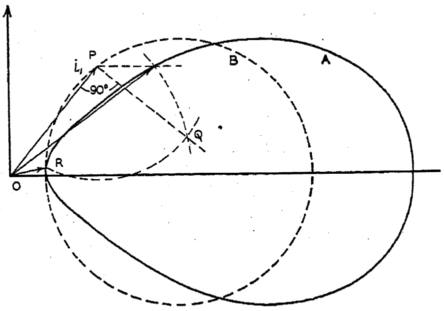


FIG. 13—CONSTRUCTION OF CURRENT LOCUS DIAGRAM

Due to the super-imposed fluxes rotating in opposite directions the iron of the motor was saturated as is indicated in Fig. 14. This difficulty could be minimized by a special design.

It is possible to operate a wound rotor induction motor with its rotor and stator paralleled when ratio is not 1 to 1, by connecting the low-voltage rotor to the high-voltage line through an auto-transformer. The behavior of the machine under these conditions should be better than when no transformer is used, since the transformer will by-pass part of the low-frequency current that ordinarily would flow through the supply transformers.

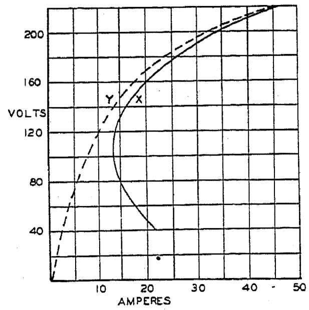


FIG. 14

Curve X—No-load current taken by the machine at different voltages with its rotor and stator paralleled
Curve Y—Estimated saturation curve

By designing the machine so that the rotor and stator phases have the same constants it is possible to obtain speed torque curves as shown in Fig. 12A, the total torque being the sum of the action and reaction. By putting resistance in each phase a characteristic can be obtained as in Fig. 12B. If either the rotor or stator are designed with a high resistance and those of the other element with a low resistance it is possible to obtain a total characteristic as shown in Fig. 12C, obtaining both high starting torque and also good regulation.

The effects of the low-frequency currents on the behavior of the line will depend to a certain extent on the ratio of the motor rating to the rating of the transformers supplying the load. It is obvious that if this ratio is large the efficiency might be reduced.

Discussion

Wesley B. Hall: The same performance as the authors obtained may be secured from a normally connected induction motor by running it on $\sqrt{2}$ times normal voltage. This can be done to any induction motor even if its rotor and stator are wound for different voltages or if it has a squirrel-cage rotor. The torque produced is uniform, and the flux density also uniform at $\sqrt{2}$ times normal density, while the authors' connection produces a torque pulsating periodically from zero to twice load torque, and a flux density pulsating from zero to twice normal density. The performance, losses, and efficiency are in general the same. A transformer may be needed to supply the increased voltage, which may not be required for the authors' scheme.

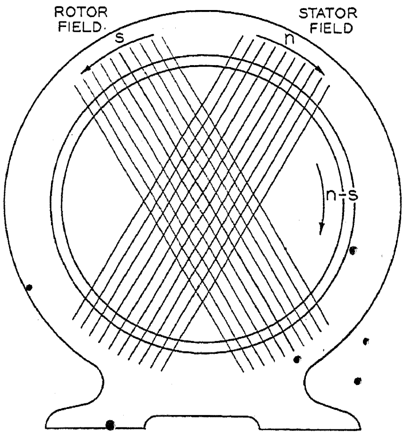


Fig. 1

With the rotor and stator paralleled as described in the paper, the air gap flux may be considered as made up of two components; a field set up by the stator, which will be of normal value and rotate at synchronous speed with respect to the stator, cutting the rotor at slip speed; and a field set up by the rotor, of equal value, which will rotate at synchronous speed with respect to the rotor and cut the stator at slip speed. Each of these fields will be of uniform constant value. This is shown in Fig. 1, where *n* indicates synchronous speed, *s* indicates slip speed, and *n-s* indicates rotor speed.

If the load on the motor is twice normal torque, it may be considered that the field produced by the stator carries normal torque and the field produced by the rotor carries an additional normal torque (although it will be shown later that this involves only the average value of a pulsating torque). Then the stator carries normal current at line frequency to maintain its own field and carries an additional normal current at slip frequency caused by the rotor's field. The true stator load current is the sum of these components and is shown in Fig. 2. The r.m.s. value of this sum is, of course, $\sqrt{2}$ times normal current in the stator winding. Similarly the current in the rotor winding is of exactly the same character and has an r.m.s. value of $\sqrt{2}$ times normal. This is the same value as would be present if the motor were normally connected and carrying twice normal torque on $\sqrt{2}$ times normal voltage. The similarity exists at all loads, so that for the two methods of connection the I^2R losses are equal, the efficiencies are the same, and the slips alike for any given load, neglecting differences in exciting currents.

The torque obtained by the authors' connection, however, is not the uniform sum of two uniform torques obtained from these two uniform rotating fields. The fields are periodically aiding or opposed, so that the resultant air gap flux is periodically zero or twice normal, at a frequency equal to line plus slip frequency, as may be seen by an inspection of Fig. 1. This result is shown in Fig. 3. When the fields are opposed the torque is zero, when aiding, it is twice load torque (four times normal in the specific case assumed). The periodic occurrence of twice normal flux density calls for very large magnetizing current, and the torque pulsation causes very evident vibration.

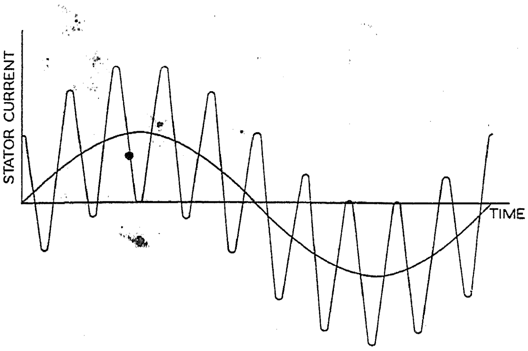


Fig. 2

The average value of torque equals the sum of the individual torques of the two component fluxes (twice normal in the case assumed).

R. E. Hellmund: The paper by Messrs. Conrad and Warner discusses an exceedingly interesting operating arrangement for induction motors, which like many other special arrangements undoubtedly will find some practical applications. There are other cases known where the interesting phenomenon of two induction motor actions takes place within a single motor. Examples of this are the various internal cascade arrangements and the operation of an induction motor with a single-phase secondary, which is equivalent to an internal cascade arrangement. To my knowledge, however, the particular arrangement described in the paper has not been covered previously in literature.

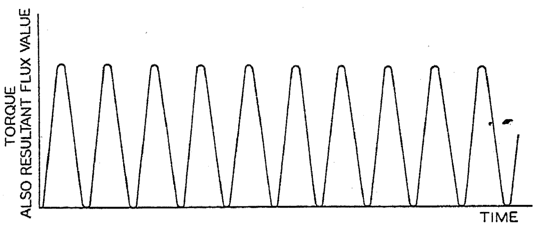


Fig. 3

Like many other special arrangements, the one covered in the paper probably will not find application for general purpose work because the same results can usually be obtained with the ordinary induction motor. This is evident from the following simple considerations. In the proposed scheme, double full-load torque, for instance, can be obtained by having a full-load current of slip-ring frequency superimposed upon full-load current of line frequency in each member. This means twice the full-load losses, because with two currents of different frequency superimposed, the losses are equal to the sum of the losses caused by the individual currents. On account of the superimposition of the two fields in the core portions, the maximum magnetic densities, however, are doubled. This, of course, will mean correspondingly high core losses and perhaps difficulties with noise, etc. If, on the other hand, the same motor were operated with 41 per cent higher voltage and 141 per cent load current, it would also give twice the output with twice the copper losses and only 41 per cent increase for the core densities. Such an arrangement would further obviate the necessity of a wound rotor and permit the use of the much simpler squirrel-cage winding.

In employing the arrangement suggested in the paper care must be taken if lights are operated from the same circuit, because low frequency flowing in the primary circuit is likely to cause flickering of the lights. This condition is similar to that obtained with an induction motor operated with single-phase secondary, which, except for this reason, might have found more general application.

Although for the reason stated the general application of the proposed arrangement is unlikely, it may nevertheless be useful in special cases, as, for instance, where speed torque characteristics as shown in Fig. 12c are desirable, and therefore the publication of the results obtained by the authors should be considered a welcome contribution to the art.

C. L. Fortescue: In dealing with rotating apparatus, by the method of symmetrical coordinates, the usual convention is to consider all phase sequences with reference to the stator. With this convention a symmetrically-wound induction motor running at normal slip will have only positive sequence currents in both stator and rotor. The same motor when driven above synchronous speed will have positive phase sequence currents in the stator and negative phase sequence currents in the rotor. Analytically this is taken care of by the fact that the negative phase sequence currents and e.m.fs. in the rotor are conjugates of the positive phase sequence quantities that appear in the analysis and which have negative indexes. The analytic method has no discontinuities but the solution is obtained, partly in terms of positive sequence quantities having the conventional positive indexes and partly in terms of positive sequence quantities having negative sequence indexes which in the final statement of the solution may be replaced by their conjugate negative sequence equivalents with positive indexes.

In dealing with problems in which the rotor is coupled with the primary circuit inductively or conductively the phase sequence relations of the motor with respect to the primary circuit must be considered with respect to these conventions, in order to obtain the correct analytical solution.

In the present problem from the standpoint of the stator and the circuit connections thereto the circuit connections to the rotor must be made negative sequence in order to give the same direction of rotation. Therefore, in the rotor the primary currents will be negative sequence as far as the motor reactions are concerned and the resulting secondary currents will likewise be negative sequence. But whereas the primary current in the circuit will be positive sequence for both stator and rotor, the secondary currents will both be negative sequence with respect to the primary circuit.

$S' \check{I}_{a1}$ is the positive sequence system of currents in the stator
$S^2 \check{I}_{u2}$ is the negative sequence system of currents in the rotor
$S^2 \check{I}_{a2}$ is the negative sequence system of currents in the stator
$S' \check{I}_{u1}$ is the positive sequence system of currents in the rotor

The equations of the induction motor operated with both primary and secondary connected to the same source of power are:

$$S' \check{E}_{a1} = S' \left\{ (R_a + j\omega_0 L_a)\, \check{I}_{a1} + j\omega_0 M \check{I}_{u1} e^{\,j(\omega_0 - \omega_1)t} \right\}$$

$$\begin{aligned} O &= S^2 \left\{ (R_a + j\omega_1 L_a)\, \check{I}_{a2} + j\omega_1 M \check{I}_{u2} e^{-j(\omega_0 - \omega_1)t} \right\} \\ O &= S' \left\{ (R_u + j\omega_1 L_u)\, \check{I}_{u1} + j\omega_1 M \check{I}_{a1} e^{-j(\omega_0 - \omega_1)t} \right\} \\ S^2 \check{E}_{a1} &= S^2 \left\{ (R_u + j\omega_0 L_u)\, \check{I}_{u2} + j\omega_0 M \check{I}_{a2} e^{\,j(\omega_0 - \omega_1)t} \right\} \end{aligned}$$

To simplify the problem assume $R_u = R_a$, $L_u = L_a$, then $\check{I}_{a1} = \check{I}_{u2}$ and $\check{I}_{a2} = \check{I}_{u1}$. The slip frequency is given by ω_1 and the synchronous frequency by ω_0. The two systems of primary currents, that is to say those supplied to the stator and rotor respectively, are entirely unrelated in the induction motor circuits as they are throughout without any mutual coupling. The two fields will be independent and each be determined by the impressed e.m.f., $S'\check{E}_{a1}$ as if the other did not exist. If B be the maximum value of the induction in the air gap due to the stator field, it may be represented by the rotating vector

$$S' \check{B} = S' B\, e^{\,j\omega_0 t}$$

The rotating field to the rotor will be rotating in the opposite sense to the actual mechanical rotation and since the mechanical rotation is $e^{j(\omega_0 - \omega_1)}$ the actual rotation of this induction in the air gap will be $e^{-j\{\omega_0 - (\omega_0 - \omega_1)\}}$ or $e^{-j\omega_1}$. Its value will also be B the same as for the stator so that the total induction will be

$$B\,(e^{j\omega_0 t} + e^{-j\omega_1 t})$$

So that in less than every complete cycle there will be a doubling up of the induction.

For the same output the heating due to copper loss is very much reduced. To get at this let us take I_a as the normal full load current when the machine is operating in a normal induction motor. We may assume the I_u is approximately equal to I. Then we shall have

$$I^2R \text{ loss} = (I_a^2 + I_u^2)\, R_a = 2I_a^2 R_a$$

Assume the two-winding scheme, and let the two currents supplied by the primary be mI_a, mI_u, etc. Then

$$I^2R \text{ loss} = 2\,(m^2 I_a^2 + m^2 I_u^2)\, R_a = 4m^2 I_a^2 R_a$$

For the same loss we have

$$4m^2 I_a^2 R_a = 2I_a^2 R_a$$

and therefore

$$m^2 = \frac{1}{2}$$

$$m = \frac{\sqrt{2}}{2} = 0.707$$

The current that may be supplied by the primary circuit to give the same I^2R loss will therefore be 41.4 per cent higher than for the straight induction motor. That is to say, its output for the same copper loss will be increased 41 and a fraction per cent.

To offset this the induction is doubled though the actual iron loss will not be so greatly increased in each element by the doubling of the induction but the rotor iron loss and the stator iron loss will have approximately the same magnitude, so that we may say that the iron loss for the same applied voltage will be approximately double that in the regular induction motor.

Some Considerations in the Design of Damper Windings for Synchronous Motors

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Associate, A. I. E. E.

Synopsis.—*As the application of synchronous motors is extended to include more specialized fields, the design of damper windings for special starting performance becomes more of an exact art. Methods have been presented before the Institute by which the starting performance can be determined with good accuracy. The purpose of this paper is to present certain possibilities and limitations of the performance which may be obtained by variation of the damper winding design.*

For purposes of consideration, damper windings are arbitrarily classified as low reactance and high reactance types. It is pointed out that low reactance damper windings are well suited to give a high ratio of torque to kva., especially during the initial part of the starting period. Many other types of performance, including a fairly uniform torque over the starting period or, a low starting torque with high pull-in torque—all with a relatively low kva. can be obtained with high reactance windings.

INTRODUCTION

THE constant effort of the designers of synchronous motors to improve the starting characteristics of their machines has led to marked advances in the art of predetermining their performance. Putman[1] and Linville[2] have given methods of calculating the performance of the motor throughout the entire starting period. By their use the designer may calculate the starting performance with reasonable accuracy provided, first, that he can calculate the machine constants correctly, and second, that he has an appreciation of the possibilities and limitations of the various combinations of these constants. It is the purpose of this paper to point out certain aspects of this second condition of successful design.

MACHINE CONSTANTS

For purposes of comparison, the machine constants may be classified as stator reactance (x_1) and resistance (r_1), damper winding reactance (x_b) and resistance (r_b), total rotor reactance (x_2) and resistance (r_2) and the magnetizing admittance (H). Often the individual damper bars do not all have the same reactance and resistance. Putman used an equivalent reactance and resistance for the damper circuit, in which the constants of the individual bars are combined into the constants of a damper winding made up of equivalent bars. Linville reduced the motor circuits to an equivalent network, placing each bar with its individual constants in a separate branch.

In this paper all constants are referred to the stator and expressed in per cent.

CHARACTERISTICS OF DAMPER WINDINGS

A good criterion for the best starting performance for a synchronous motor appears to be that the motor shall develop the required torque throughout the starting period with the lowest kva. that can be obtained without impairing the operating characteristics at synchronous speed.

The design of special damper windings for induction motors has been studied for years and probably most of the practical combinations have been used in them. In studying the problem of starting windings for synchronous motors, many of the steps of the induction motor designers doubtless are retraced.

Types of Windings. For the purpose of discussion, it is convenient to divide damper windings arbitrarily into two classes. The first is called "low-reactance windings." This includes all windings which are made up of round or rectangular bars of such size that skin effect has only a negligible effect on the distribution of the current in the bar. Such windings are so arranged in the pole head that the ratio of specific permeance to effective specific resistance of the winding as a whole is only slightly affected as the motor comes up to speed. The second class is called "high-reactance windings." This includes all forms of double deck windings, such as those having two distinct bars placed one over the other and all windings made entirely or in part of L bars or inverted T bars as well as plain rectangular bars of such a shape and material that they are subject to an appreciable skin effect. Moreover, the second class includes all single deck windings which are so arranged that the characteristics of the winding change noticeably as the motor comes up to speed.

Low-Reactance Windings. Most of the ordinary single deck damper windings are of the low-reactance type, although it will be shown in Appendix III that certain of them actually have a considerable change in characteristics as the motor comes up to speed.

In order to obtain the highest ratio of torque to kva. throughout the starting period, it is necessary to have the rotor circuits of high resistance and low reactance. Very high-resistance damper windings have the serious practical objection that they produce low torque at the higher speeds unless the motor has a low stator reactance. This difficulty becomes quite marked in low-speed motors. However, low-reactance

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[1] For references see Bibliography.

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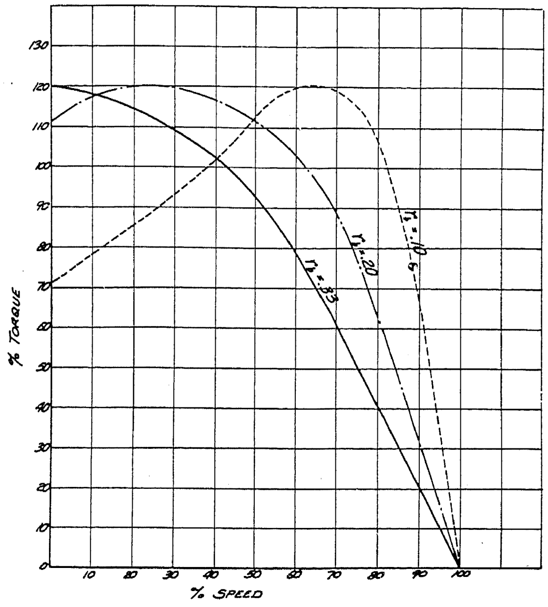
damper windings are well suited for low-speed motors with 40 to 50 per cent pull-in torque accompanied with a starting torque as high as 75 per cent. Higher torques with approximately the same ratio of pull-in to starting torque can be built with an increase in frame size. Most high-speed motors will produce 100 per cent pull-in torque with 150 to 175 per cent starting torque with low-reactance dampers.



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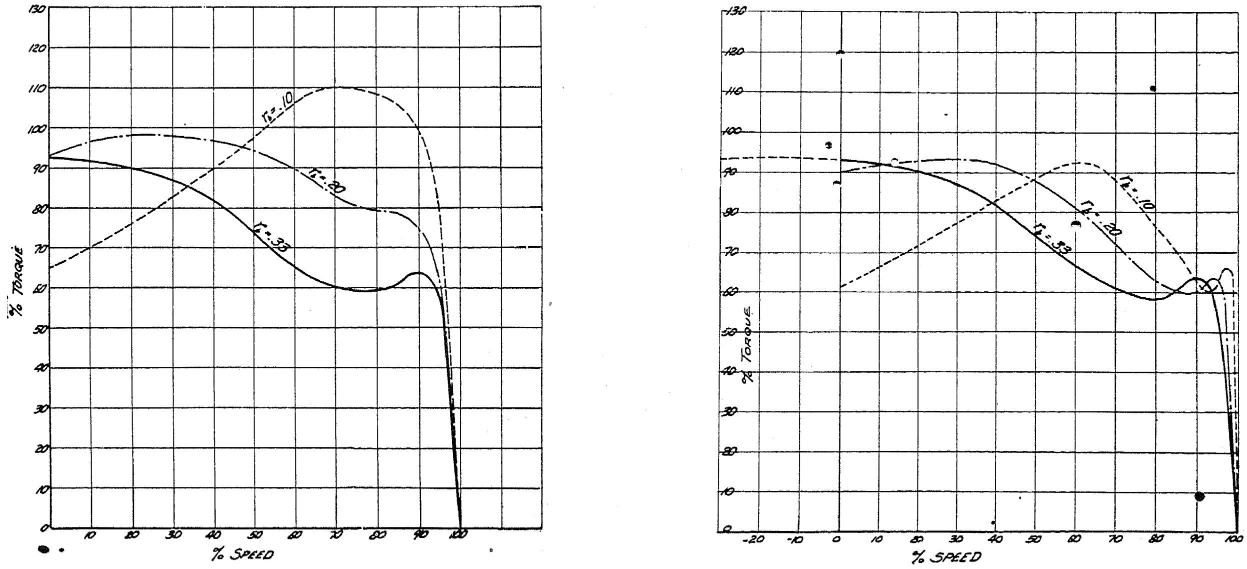

B

C

FIG. 1—EFFECT OF CHANGE IN DAMPER WINDING RESISTANCE ON TORQUE 2,000-HP., 150-R.P.M., 100 PER CENT POWER FACTOR SYNCHRONOUS MOTOR

A. Field open
B. Field closed through normal starting resistor
C. Starting resistor varied in same ratio as damper winding resistance

The effect of changing the damper winding resistance is shown in Fig. 1. Fig. 1A shows the case with the field open. The effects are the same as with an induction motor. Fig. 1B shows the same conditions except with the field winding closed through a starting resistor. It will be noted that the curve with the lowest resistance has the highest maximum value. This is because the maximum of the torque from the damper winding for this condition coincides most nearly with the maximum of the torque from the field winding. The effect of the field winding has been covered in a previous paper.[1] Fig. 1C shows as a matter of interest the results obtained when the starting resistor is varied in the same proportion as the damper winding resistance.

The per cent starting current for these cases is shown in the following table:

FIG.	1A	1B	1C
$r_b = 0.10$	360	400	400
$r_b = 0.20$	320	370	370
$r_b = 0.30$	260	320	320

An increase of the damper winding reactance decreases the torque at all slips. Probably most designers of synchronous motors have noted that increasing the damper winding reactance for a given value of resistance in some cases results in an actual increase in starting kva. The reason for this becomes apparent upon examining an approximate equation for starting current which was derived originally for the induction motor. The equation is:

$$I = \frac{\sqrt{1 + 2Hx_2}}{Z_0}$$

where Z_0 is the total impedance of the motor. Differentiating this expression with respect to x_2, it is found that the current has its maximum value when the following relation is fulfilled:

$$X_2 = \frac{-\frac{1}{H} + \sqrt{\left(\frac{1}{H}\right)^2 - 4\left(\frac{x_1}{H} - x_1^2 - r_0^2\right)}}{2}$$

where r_0 = sum of stator and rotor resistance.

An excellent example of a low-reactance winding in which a high ratio of torque to kva. is maintained at all times is found in a recently developed salient-pole motor with a phase wound damper winding.[4] This motor uses a single deck winding of round bars with a low permeance slot over each bar. The winding is brought out through slip rings to external resistance grids. By means of suitable control equipment, the resistance in the damper circuit can be maintained at all times as high as the torque requirements will permit.

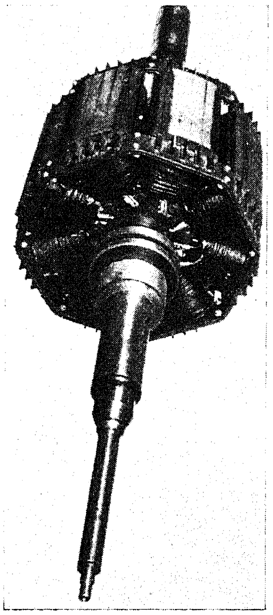

FIG. 2—ROTOR FOR GENERAL PURPOSE SYNCHRONOUS MOTOR

High-reactance inverted T bar and low-reactance round bar

It has long been recognized that leaving the field circuit open during all or a part of the starting period is an effective means of increasing the rotor circuit resistance and decreasing its reactance. Recently a number of applications calling for a particularly high ratio of torque to kva. has been filled with motors which had the field circuit open during the greater part of the starting period. Large and medium size slow-speed motors can be built readily for open field starting. They may be excited from standard 125 or 250-volt circuits by the use of a field sectionalizing switch mounted on the rotor.[4] Such switches have been successfully applied to low-speed motors both with a normal single cage winding and with the phase-wound damper. In high-speed motors of small and medium size, it has been found more convenient to use reduced voltage excitation, so that the voltage across the rings at starting will be kept within the limits of a reasonable amount of insulation on the field coils, the collectors and the control equipment. The reduction of kva. by the use of open field varies from 20 to 30 per cent in high-speed motors and from 8 to 15 per cent in the low speed. These kva. reductions are accompanied by an increase in starting torque of from 3 to 15 per cent.

An example of the possibilities of a single-cage, high-resistance, low-reactance damper winding is found in a 250-hp., 900-r.p.m., 100 per cent power factor motor for driving a hammer mill. The rotor was designed for low reactance. The motor was built for open field starting and tested 245 per cent starting torque with 110 per cent pull-in torque and 425 per cent starting kva.

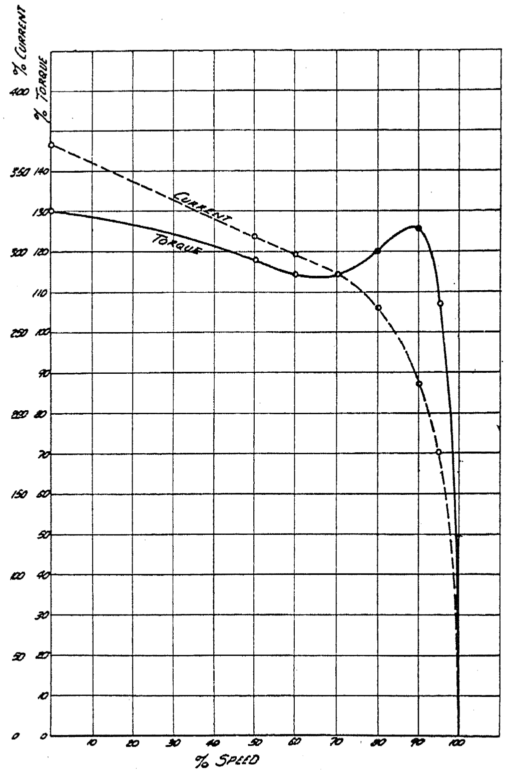

FIG. 3—SPEED-TORQUE AND SPEED-CURRENT CURVES FOR GENERAL PURPOSE TYPE OF SYNCHRONOUS MOTOR

250 hp., 900 r.p.m., 80 per cent power factor

Where a high ratio of torque to kva. is required, especially during the early part of the starting period, and where the starting torque itself must be high with a relatively low kva., low-reactance damper windings are well suited.

High-Reactance Windings. It is well recognized that to get any appreciable change in the effective resistance of any type of a double deck damper bar it is necessary for at least a portion of the material of the bar to be of low resistance. In high-speed, high-torque motors, where the damper winding must furnish a major portion of the impedance to limit the starting current, a straight double deck winding defeats its own purpose. If its reactance is high enough to limit the starting current, it is apt to be low in starting torque. And if the resistance of the winding is high enough to contribute materially to the impedance of the machine, it probably will not have enough decrease in resistance at the higher speeds to give the required torque.

Double deck windings of the conventional induction motor type, with two distinct bars placed one over the other, have been used rather commonly on slow-speed synchronous motors which require nearly a uniform torque throughout the starting period. A recent A.I.E.E. paper[6] discusses applications of this type and presents a set of performance curves for a rubber mill motor showing 120 per cent starting torque and 100 per cent pull-in torque. Many rubber mill applications require a somewhat higher starting torque with the same pull-in torque.[5] On these latter applications motors with the simpler low-reactance winding give good performance.

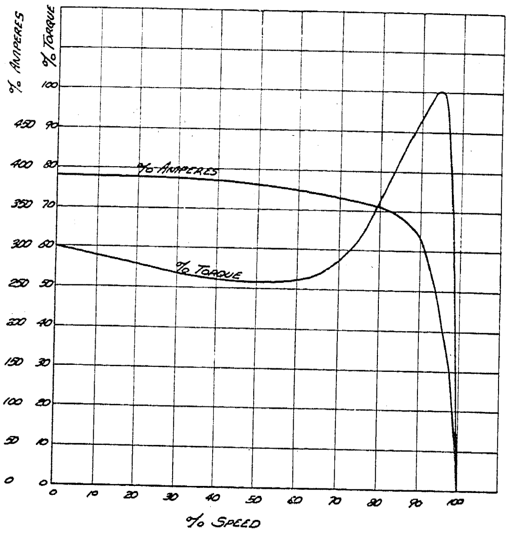

FIG. 4—SPEED-TORQUE AND SPEED-CURRENT CURVES FOR A 700-HP., 900-R.P.M., 100 PER CENT POWER FACTOR SYNCHRONOUS MOTOR FOR CENTRIFUGAL PUMP DRIVE

One field in which high-reactance rotors may be used to advantage is the general purpose class of motors and those of somewhat higher horsepower rating, which usually have general purpose characteristics. These motors require 100 per cent pull-in torque and only a little higher starting torque. Due to the usual small physical size of these motors, it has been found that bars of an L or inverted T section are well suited. The use of a few high-reactance bars, combined with low-reactance bars in the same pole head, presents no serious manufacturing difficulties and gives sufficient impedance at the start to limit the current, yet low enough resistance at low slips to give the required pull-in torque. The constants of such a winding are given in Appendix II. Fig. 2 shows an 8-pole rotor for a general purpose motor of this type.

Curves of the tested starting performance of a 250-hp., 900-r.p.m., 80 per cent power factor motor are shown in Fig. 3.

An interesting example of the possibilities of special forms of high-reactance rotors is found in the design of high-speed motors for centrifugal pump and blower drive. This is a good example of an application where a low torque per kva. is quite acceptable. The present day requirements are at least 100 per cent pull-in torque with 50 to 60 per cent starting torque and the starting current between 400 and 450 per cent.

One solution of the problem which has been quite successful is the two winding motor described in a recent A.I.E.E. paper.[6] This method often gives the required starting performance from motors of practically standard design. However, there are cases, especially with small high-voltage motors, where it is difficult to obtain the required number of parallels. And the control equipment must include an auxiliary contact or breaker for closing the second half of the winding.

For this reason, a high-speed motor with a simple type of high-reactance rotor and a normal, high efficiency stator has been developed, which permits the use of standard full voltage starting equipment.

This motor employs a single-cage damper winding of the conventional type, except of quite high resistance, combined with one or two low resistance and medium reactance bars in each pole. In this type of a winding, the resistance and reactance are both high at standstill. However, the impedance of a majority of the bars is mostly resistance. As the motor comes up to speed, the impedance of the high-resistance bars remains approximately constant. The impedance of the low-resistance bars, however, decreases materially at the higher speeds, and thus effects the necessary reduction in total rotor impedance.

A set of test curves on such a motor rated at 700-hp., 900-r.p.m., 100 per cent power factor is shown in Fig. 4.

These tests were made with the field circuit closed through a proper starting resistor. With the field open, the starting current is reduced to 275 per cent, with a slight increase in starting torque.

As the applications of synchronous motors become more specialized, there will doubtless be a marked improvement in the results obtained from special, yet simple, forms of high-reactance damper windings.

ACKNOWLEDGMENT

The author acknowledges the aid of Mr. M. R. Lory in calculating several of the examples given in this paper.

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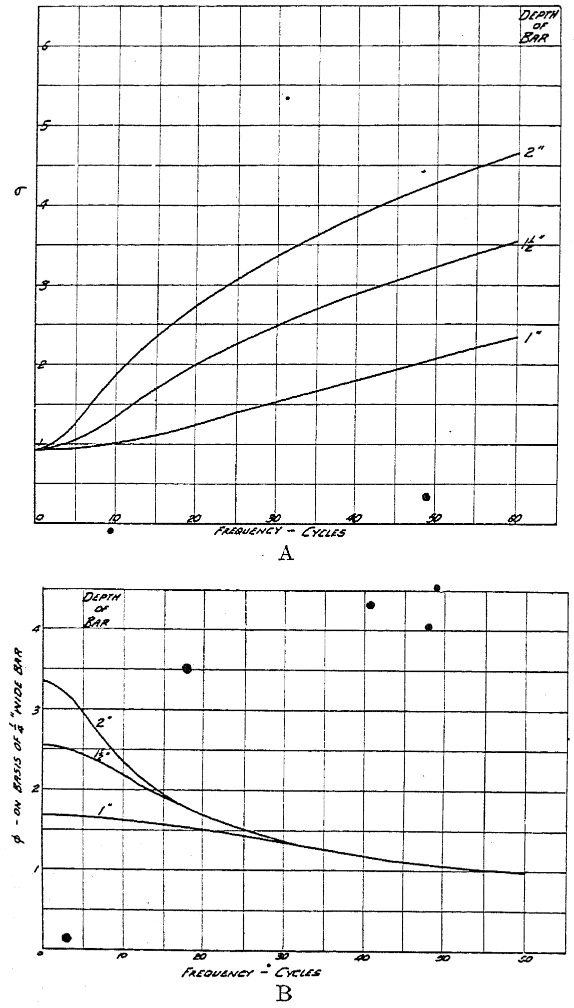


A

B

FIG. 5—CHARACTERISTICS OF RECTANGULAR BARS

A. Resistance σ = ohms per cu. in. × 10^6
B. Reactance φ = specific permeance per cm. length

Appendix I

CHARACTERISTICS OF VARIOUS TYPES OF DAMPER BARS

The constants of a number of types of damper bars in common use are presented here for reference and comparison. Much of the material has been presented before. See Fig. 7 for reference to dimension letters.

1. Round Bar. The resistance of the round bar is ordinarily determined by its cross section and the specific resistance of the material without reference to skin effect. Mr. Fechheimer[7] showed that the permeance is

$$1.26\left[0.62+\frac{d}{W}\right]$$

The figures were obtained by integration in polar coordinates based on flux lines going straight across the round slot. Careful flux plots have later shown the value of the constant inside the bracket to be 0.66 instead of 0.62.

2. Square Bar. In this class are included all bars of square or approximately square cross section. Here, also, the resistance usually may be calculated without regard to skin effect. The permeance is given by the well known expression

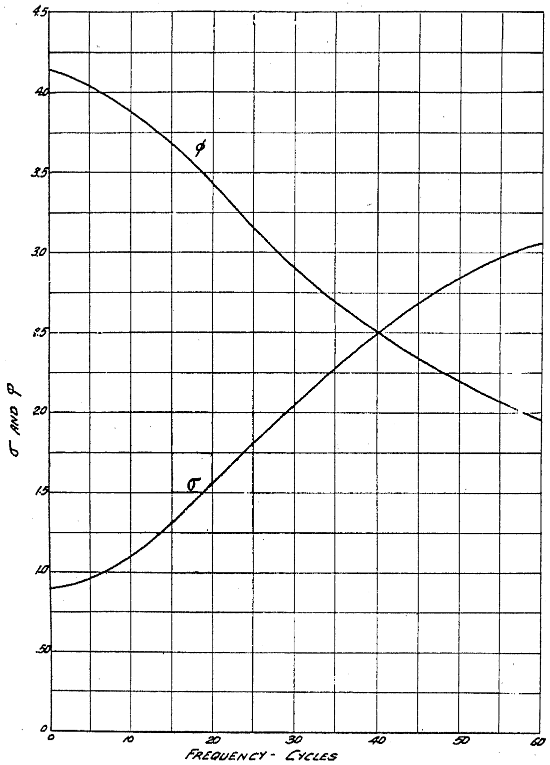


FIG. 6—CHARACTERISTICS OF INVERTED T BARS

σ ohms per cu. in. × 10^6
φ = specific permeance per cm. length

$$1.26\left[\frac{d}{3w}+\frac{d^1}{w^1}\right]$$

It is apparent at once that a square bar has a better power factor than a round one for the same resistance.

3. Rectangular Bar. In determining the constants of a rectangular bar the displacement of the current in the bar itself at the higher frequencies must be taken into account. The expressions may be conveniently derived by the use of integral equations. A more general formula has already been presented[1] which reduces for a rectangular bar to

$$Z = \frac{r(A + jA)}{ab}\left[\frac{\cosh 2A \cos 2A + j \sinh 2A \sin 2A}{\sinh 2A \cos 2A + j \cosh 2A \sin 2A}\right]$$

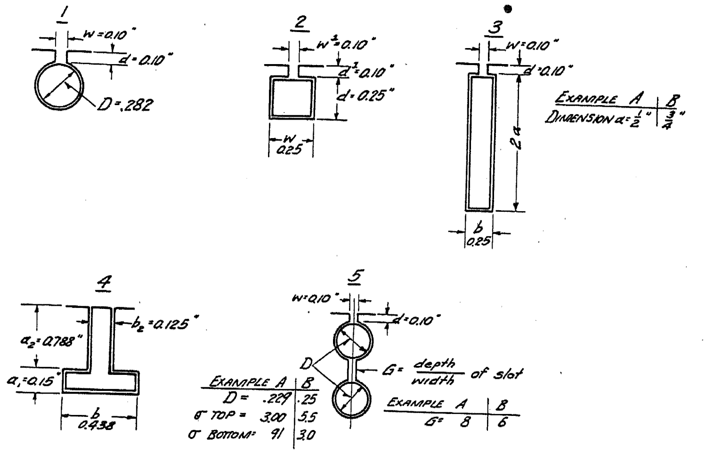
where

$$A = 2\pi a \sqrt{\frac{f}{r}}$$

r = specific resistance—absolute units

f = frequency

The impedance Z will be given in absolute units. The expression does not include the permeance of the slot above the bar. A set of curves covering various depths of 1/4 inch wide bar is shown in Fig. 5A and 5B.



Bar No.	σ		φ	
	60 ~	3.5 ~	60 ~	3.5 ~
1	3.00	3.00	2.09	2.09
2	3.00	3.00	1.68	1.68
3A	2.35	0.92	2.24	2.94
3B	3.55	0.95	2.29	3.81
4	3.10	0.91	1.95	4.15
5A	2.06	0.69	5.13	9.06
5B	2.7	1.96	5.32	6.00

σ = Ohms per cu. in. × 10^6
φ = Specific permeance per cm. length.

FIG. 7—CHARACTERISTICS OF SEVERAL COMMON BARS

4. Inverted T Bars. The theory of the calculations for the inverted T bar has been presented in a previous paper.[1] The impedance formula in hyperbolic notation is

$$Z = \frac{r(A + jA)}{b_2}\left[\frac{R \cosh (S + jS) - \cosh (D + jD)}{R \sinh (S + jS) - \sinh (D + jD)}\right]$$

where $R = \dfrac{b_1 + b_2}{b_1 - b_2}$, $s = A\,(a_2 + a_1)$, $D = A\,(a_2 - a_1)$.

Fig. 6 gives a set of curves for a copper bar of the dimensions shown in Fig. 7.

5. Double Deck Bar. Perhaps the best known form of double deck bar for induction motors is that consisting of two bars, one above the other. They have been used to a certain extent on synchronous motors, although the limited depth of pole head presents a handicap to their application.

In general, bars of this type are of such a shape and size that the skin effect in the conductor itself is negligible. The impedance of the type shown in Fig. 7 is given by

$$Z = \frac{r_a\, r_b - 105\, w^2(1 + G) + jw\,[r_a(24.3 + 12.6G) + r_b\, 8.32]}{r_0 + jw\, 4\pi\, (1.025 + G)}$$

where

r_a is resistance of top bar in absolute units per centimeter length

r_b is resistance of bottom bar in absolute units per centimeter length.

$r_0 = r_a + r_b$

$w = 2\pi f$

G = ratio of depth to width of slot between bars.

The impedance Z is given in absolute units. It does not include the permeance of the slot above the bars.

The above list is not inclusive of all types of bars in use, but fairly covers the range of common practise.

Fig. 7 shows sketches of the various types of bars together with a comparison of their characteristics.

Appendix II

CHARACTERISTICS OF A COMBINATION OF INVERTED T BARS AND ROUND BARS FOR A GENERAL PURPOSE MOTOR

In this appendix examples are given of the constants which can be obtained by the use of high and low reactance bars in the same pole. The desirable characteristics are a high impedance with fairly high power factor at the start and an appreciable decrease in resistance as the motor comes up to speed. A sketch of a pole using 2 high-reactance bars of the inverted T type combined with 4 low-reactance bars is shown in Fig. 8.

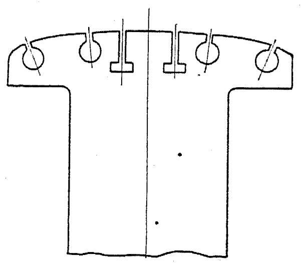


FIG. 8—POLE PUNCHING FOR A GENERAL PURPOSE MOTOR

Instead of handling each bar separately in calculating the starting performance, it is convenient to find the resistance and permeance constants of an equivalent bar of such characteristics that the winding may be considered to consist of 6 (in the case of Fig. 8) such equivalent bars per pole. The impedance of the equivalent bar may be found by multiplying the actual impedance of all bars in parallel by the number of bars.

The following table gives the specific resistance and specific permeance of the equivalent bar for the pole shown in Fig. 8 with various resistance materials in the round bars. The inverted T bar is copper in all cases. The equivalent bar will have the same area as one of the inverted T bars.

		⊥ bar				Equivalent bar			
Round bar		60 cycles		3.5 cycles		60 cycles		3.5 cycles	
σ	ϕ	σ	ϕ	σ	ϕ	σ	ϕ	σ	ϕ
1.87	2.51	2.07	2.89	0.831	4.09	2.10	2.62	1.45	4.23
2.84	2.51	2.07	2.89	0.831	4.09	2.84	2.69	1.74	5.34
5.50	2.51	2.07	2.89	0.831	4.09	4.20	3.08	2.08	7.23
12.82	2.51	2.07	2.89	0.831	4.09	6.21	4.68	2.44	9.70

σ = resistance per cu. in. $\times 10^6$
ϕ = specific permeance per cm. length.

It will be noted that the highest resistance round bars give the greatest decrease in resistance as the motor comes up to speed. A 250-hp., 720-r.p.m. motor using the last combination listed calculated 14.8 per cent damper circuit reactance and 17.3 per cent resistance at standstill.

Appendix III

CONSTANTS OF SINGLE-CAGE WINDING ON ROUND BARS

In some cases where the winding is made of round or square bars, all the same size, the slots above the bars are deeper at the center of the pole than near the tips.

Consider a pole with five 3/8 inch round bars. The slots over the middle 3 bars are 3/16 inch deep by 1/8 inch wide and over the two end bars they are 1/16 inch deep by 1/8 inch wide. In case it is necessary to use bars of different materials to meet the necessary resistance, an appreciable change of constants may occur as the motor comes up to speed. For bars all of the same material, the change is quite small. Three examples are illustrated in the following table:

				Equivalent bar			
Center bars		End bars		60 cycles		3.5 cycles	
σ	ϕ	σ	ϕ	σ	ϕ	σ	ϕ
5.5	2.72	5.5	1.46	5.56	2.16	5.51	2.21
1.98	2.72	5.5	1.46	3.04	2.93	2.66	3.05
5.5	2.72	1.98	1.46	3.30	2.08	3.18	2.12

Discussion

M. R. Lory: Synchronous motors are being applied to a great variety of applications which differ widely in starting requirements. The popularity of full-voltage starting with its cheaper and simpler control has increased the importance of limiting the kva. drawn by the motor during the starting period. The problem of the designer is to meet the torque requirements of the load and still keep the current drawn from the line to a minimum. In his paper, Mr. Shutt has given a comprehensive summary of the various means by which the designer can accomplish this result.

In Fig. 7 and Appendix I, Mr. Shutt has given the constants for several types of damper bars including two double-deck arrangements. Mr. Shutt pointed out the difficulty of using double-deck windings where the depth of pole head is limited. As a matter of interest, I have calculated these two examples with the two bars in parallel but in separate slots, making the permeance of the slot above each the same as the permeance which it has in the arrangement given in Fig. 7. Calculated on the same basis, the values of the constants for the two bars are as follows:

	σ		ϕ	
	60 cycles	3.5 cycles	60 cycles	3.5 cycles
5-A	1.83	0.713	3.62	8.16
5-B	2.66	1.95	3.83	4.93

In many cases the above characteristics are not as desirable as those given by the conventional arrangement with both bars in the same slot. A given change in resistance is accompanied by a greater change in reactance. However, in some cases the simpler mechanical construction will offset the slight disadvantage in performance. When a number of bars is used per pole, the high reactance of the low-resistance damper may be obtained by concentrating it into a fewer number of bars than the high-resistance damper. This materially decreases the depth of slot which must be put over the low-resistance bars. This principle was utilized by Mr. Shutt in the machine he describes for use with pumps or blowers.

This same principle is in evidence when bars of different resistivity are used in a single cage winding, as shown in Appendix III. Appendix III does not mention the fact that the slots above some of the bars do not have to be deeper than the others to get a change in resistance with frequency when the difference of the resistivity of the individual bars is large. So as an example, take five bars of the proportions of bar No. 1 of Fig. 7 and make one bar of resistivity 0.91 and four of resistivity 13.6. The constants of the equivalent bar show a pronounced double-deck effect. They are as follows:

σ		ϕ	
60 cycles	3.5 cycles	60 cycles	3.5 cycles
4.46	3.59	6.08	6.6

I believe that the formula for the value of X_2 for maximum inrush is more of theoretical interest than of practical value. I find that with the constants normally found in design this formula gives either negative values or positive values which are so small that they are below what would be obtained in a machine.

R. E. Hellmund: Mr. Shutt's paper is further evidence of the excellent progress which has continuously been made by American designers of synchronous motors in the development of motors suitable for the many commercial applications. As a result of such work, the American market has been kept free of the numerous more complicated and therefore less desirable excitation and phase-advancing schemes for induction motors which we find so extensively discussed and produced in Europe. In this country these various arrangements have been investigated and built experimentally, but it was found that economically the synchronous motor designs were always just one step ahead of anything which could be accomplished with the various possibilities of excitation of induction motors. By the use of the simplest design features and through painstaking design work, many desirable characteristics have been obtained with synchronous motors, and at the same time design proportions have been worked out which have resulted in very favorable cost comparisons with domestic induction motors and which for many years have been away ahead of any European synchronous motor designs which have come to my attention.

Part of the credit for keeping the more complicated induction motor arrangements off the American market is also due to the central station engineers through their permitting starting-current and kva. values in excess of those usually permitted in other countries.

These remarks may answer the question frequently raised as to why various exciter and phase-advancer schemes for induction motors have not been commercially exploited in this country.

C. C. Shutt: The intensive study of the starting performance of the synchronous motor has been carried on for a comparatively short time when compared to its predecessor in the alternating current field—the induction motor. The designer of synchronous motors can obtain a great amount of benefit from a study of what has been tried and adopted—or discarded—in the starting windings of induction motors.

In commercial design work the designer is somewhat handicapped by the fact that present methods of calculation, which accurately take into account the effects of the field winding, are rather laborious. It is expected that improvements will be made in the near future in the methods of calculating the starting performance.

In regard to Mr. Lory's comment on the practical value of the formula for the value of X_2 which gives the maximum inrush, an inspection of the equation as given in the paper will show that for high values of resistance X_2 may have an appreciable magnitude. In actual design work this "critical" value of rotor reactance will be noticed particularly on high-speed motors where the starting current must be limited largely by the impedance of the rotor. In these cases the damper winding resistance is often made quite high.

Mr. Hellmund's remarks are particularly applicable to the low-speed field. The operating advantages of the low-speed synchronous motors permit the use of many low-speed direct-connected motors in industrial applications which would otherwise require high-speed induction motors, which have relatively high full load power factor and efficiency, and reduction gears or belts.

Film Characteristics of Electrolytic Condensers

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Synopsis.—Electrolytic condensers of the film-forming type have some interesting characteristics which have never been satisfactorily explained. Extremely thin oxide or hydroxide films formed on the surface of the metal anodes seem insufficient in themselves to explain the complete action. Likewise, an electromotive equilibrium is insufficient to explain the action.

A résumé of former research reveals a great deal of information of large practical value in building electrolytic condensers but little of value from a theoretical standpoint. Reviewing the prominent papers, it appears that aluminum has been the chief subject of discussion for condenser anodes. Such anodes show small leakage currents in selected electrolytes at low temperatures. Increasing voltage, temperature, or electrolyte concentration cause increases in leakage; the addition of impurities also has a deleterious effect.

The oxide-gas theory of rectification as applied to electrolytic condensers by Guntherschulze predicts changes in capacity for variations in pressure. Experiments do not substantiate the presence of gas. The oxide layer alone is unable to fulfil the necessary conditions found.

Electrolytic condensers with extremely low applied voltages show large variations in measured capacity from high-voltage measurements. The results indicate a film in which ion penetration plays a large part.

The presence of a film saturated with water as the dielectric overcomes a number of former obstacles, and lessens the necessity for the very thin films formerly assumed, due to the high dielectric constant of water.

* * * * *

ELECTROLYTIC condensers have been of general interest to electrochemists since 1855 when Wheatstone[1] presented a paper on the polarization of aluminum in electrolytes.

A number of metals possesses the property of forming on their surfaces protective coats when immersed in electrolytes. The skin formed is extremely thin normally; but if the metal is made electrically positive with respect to the electrolyte the thickness will increase until the layer is readily apparent. This film has a high electrical resistance, rising in some instances to several thousand megohms per square cm. of surface, the resistance remaining only as long as the metal is the anode in the electrolyte.

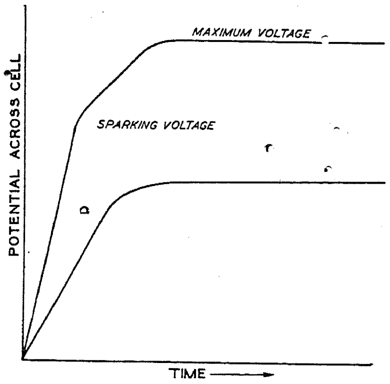

FIG. 1—THE FORMATION OF AN ELECTROLYTIC CONDENSER ANODE WITH CONSTANT CURRENT THROUGH THE ELECTROLYTE

Upper curve high current density, lower curve low current density

This film assumes the role of the dielectric in a condenser,[2] the plates of which are the metal on the one side and the electrolyte on the other.

The metals which form protective films in electrolytes include aluminum, antimony, bismuth, magnesium, zinc, niobium, vanadium, colombium, zirconium, tantalum, their alloys, and a number of other metals. These metals all can be used in electrolytic condensers. Copper and silver exhibit ionic valve action as opposed to the electronic valve action of film forming metals.[3]

The choice of electrolytes is narrow when aluminum is used as the electrode, but tantalum exhibits valve action in all electrolytes except hydrofluoric acid. The usual practise in selecting an electrolyte for aluminum is to choose the salt of a weak acid and a strong base which will nearest meet all specifications. A sufficient amount of free acid is usually added to the solution to insure an acid reaction to retard the precipitation of aluminum hydroxide.

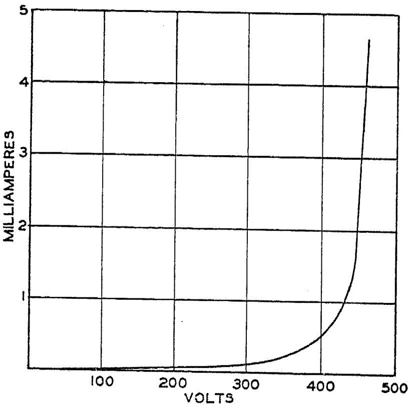

FIG. 2—LEAKAGE CURRENT OF 8-MICROFARAD CONDENSER FOR RADIO FILTER CIRCUITS, FORMED TO 450 VOLTS, 25 DEG. CENT.

If a strip of pure aluminum is made the anode in a suitable electrolyte and a constant current is caused to flow, the voltage across the cell will rise with increasing time as indicated by the curve of Fig. 1. The initial portion of the curve is straight, but as the voltage is raised, the rate decreases and abrupt bends appear in

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Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

the curves. Close examination will disclose a faint luminescence at the surface of the anode after the first bend; and upon further increases small sparks appear at the surface of the aluminum and finally heavy sparking takes place between the anode and the electrolyte and large current increases result from small increases in voltage. This maximum voltage varies widely under different conditions and it is known as the breakdown voltage; but the useful operating range of the condenser is below the first bend in the curve. The lower curve of Fig. 1 is for a lower forming current; in this case the maximum voltage is not approached.

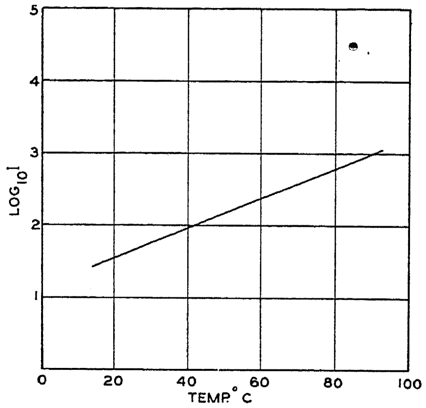

Fig. 3—Leakage Current of Aluminum Anode Plotted as a Log Function against Electrolyte Temperature. (Bairsto and Mercer)

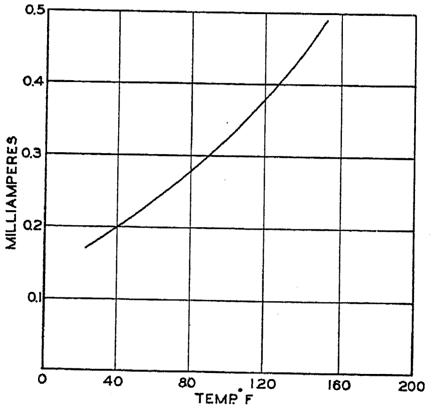

Fig. 4—Leakage Current of 8-Microfarad Aluminum Condenser with 350 Volts Impressed, for Different Operating Temperatures

Fig. 2 is the characteristic leakage current of an eight-microfarad aluminum condenser formed to 450 volts. The area of the anode is 72 square inches, and the solution is saturated sodium borate with the addition of sufficient boric acid to insure an acid reaction from the electrolyte. The leakage current of a condenser depends not only upon the voltage, but also the materials used and the operating temperature. The effect of temperature on the leakage current of aluminum is expressed by the empirical equation.[4]

$$I = A\epsilon^{\alpha T}, \qquad (\log_\epsilon I = \log_\epsilon A + \alpha T$$

where A is a constant, α is a constant dependent only on the equivalent weight of the anion in solution, and T is the temperature. When the current density is high the rise of current with temperature is more rapid than predicted. Fig. 3 shows the leakage of an aluminum anode plotted as a log function against temperature. The variation of film resistance with impressed voltage[5] is illustrated in Fig. 5 which shows clearly the asymmetrical characteristic of the cell.

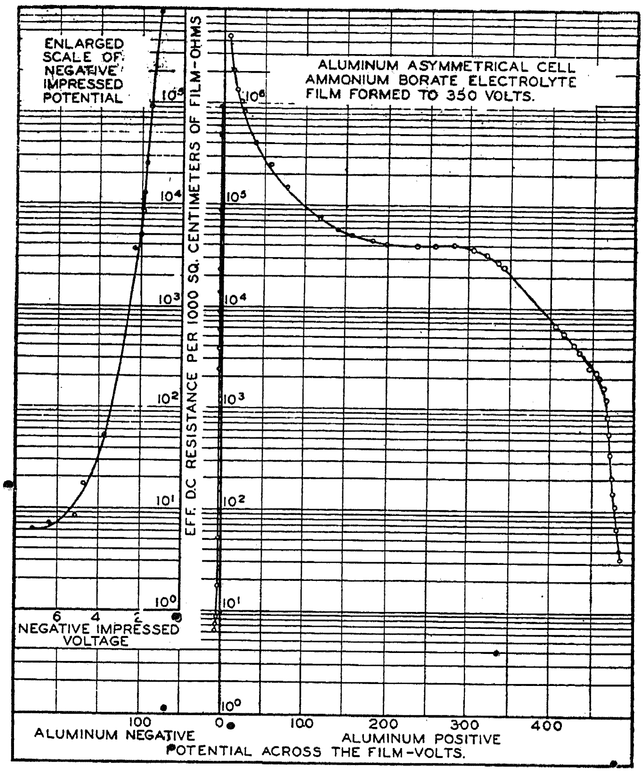

Fig. 5—Variations of Film Resistance with Impressed Voltage in Aluminum Electrolytic Condenser. (Siegmund)

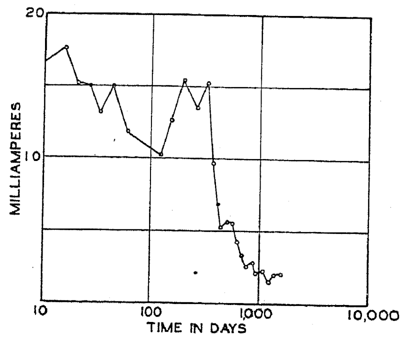

Fig. 6—Decrease in Leakage Current of Electrolytic Condenser with Time. (Siegmund)

After an anode has had high voltages applied the leakage currents are found to be slightly higher for normal voltage values upon reduction of the voltage. After a relatively short time has elapsed, practically the same values of leakage current are obtained. Over a long period of time condenser characteristics change appreciably,[5] particularly when the condenser is formed initially to a voltage different from the operating voltage, Fig. 6.

With metals that are soluble or slightly soluble in the electrolyte, leakage current may increase rapidly with concentration of the solution. Using aluminum as the anode, the addition of very slight amounts of chlorine will increase leakage current many fold. In general, the addition of impurities to the electrolyte seriously affects the leakage when the impurities themselves are incapable of acting as film forming electrolytes.

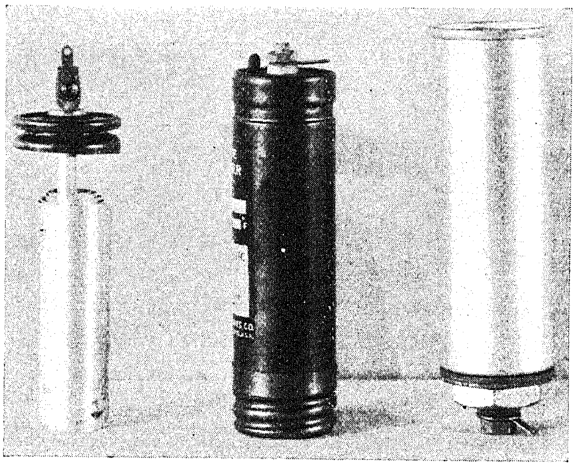

Fig. 7—Aluminum Condensers for Radio Filter Circuits. Anode Construction Shown in View on Extreme Left

The electrostatic capacity of the film formed between the metal and the electrolyte is found to be almost independent of the solution, changes only slightly with variations in temperature, and is inversely proportional to the voltage of formation of the anode, Fig. 8.

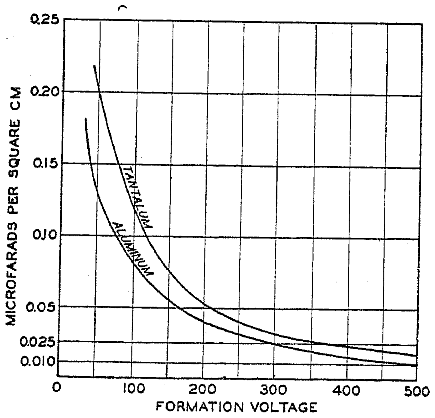

Fig. 8—Capacity-Voltage Relations in Aluminum and Tantalum Electrolytic Condensers

The variation of capacity with frequency is slight but due to the fact that most of the present forms of electrolytic condensers have comparatively long resistance paths through the electrolyte to the film, the effective capacity decreases with increases in frequency, Fig. 9.

The breakdown voltage of a condenser depends upon the electrolyte and the anode metal. An aluminum anode has been formed to 1890 volts.[6] The usual commercial aluminum condenser is formed to 500 volts. Tantalum condensers are formed commercially to 150 volts, with possibilities of units to withstand 600 volts. The other metals and alloys are not in general use and only a small number are suitable for commercial applications.

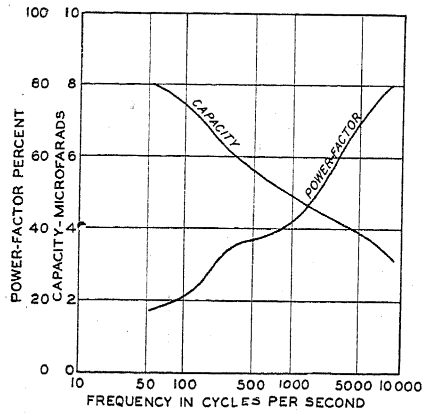

Fig. 9—Effect of Increasing Frequency on Effective Capacity of Electrolytic Condenser with High Resistance Electrolyte. (Sprague)

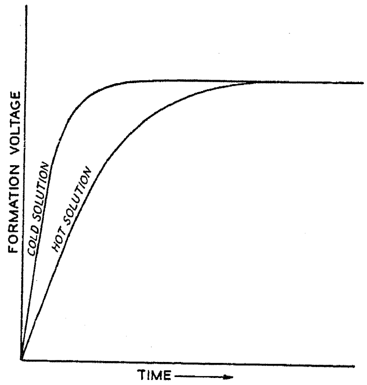

Fig. 10—Formation of Aluminum Anodes in Hot and Cold Forming Solutions

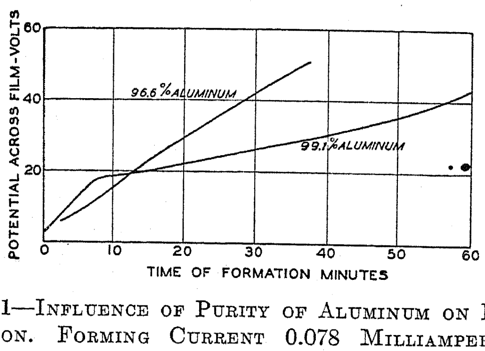

Fig. 11—Influence of Purity of Aluminum on Rate of Formation. Forming Current 0.078 Milliamperes per Sq. Cm. (Siegmund)

The rate of formation of a film varies widely under different conditions. In general the gradient of the curve in Fig. 10 is independent of the concentration of the electrolyte, and decreases considerably with increasing temperatures as indicated by the lower curve. The rate of formation in volts per second is almost directly proportional to the current density at high current densities, so that equal formation voltages are produced by equal quantities of electricity through the cell. Pure aluminum forms at a lower rate than impure aluminum, Fig. 11.

Commercial aluminum condensers formed to 450 volts approximately are formed in hot electrolytes; current densities range from 5 to 15 milliamperes per square inch. After the formation period, the condensers are given an aging period of two to eighteen hours to further reduce leakage current. Tantalum condensers will form with a current density of 15 milliamperes per square inch at a rate of 60 volts per minute in cold sulphuric acid solution.

An empirical equation for aluminum[6] gives a decrease in the breakdown potential with increasing ionic concentration,

$$V = (514.51 \log D\text{-}330) \text{ volts}$$

where V is the breakdown voltage for small ionic concentrations and D is the dilution in litres per gram equivalent of anion. Apparently the cation has very little or no influence upon the maximum voltage.

Table I compares the breakdown voltage of tantalum with that of aluminum in 0.1 N. solutions after 24 hours of formation.

TABLE I

Solution	Aluminum	Tantalum
Sodium chloride	— volts	390 volts
Oxalic acid	—	445
Potassium cyanide	275	520
Potassium permanganate	112	105
Ammonium chromate	122	147
Sodium silicate	445	540
Sodium sulphate	40	490
Ammonium citrate	470	550
Borax	480	540
Diamonium phosphate	460	575
Ammonium bicarbonate	425	525
Citric acid	536	605

Some metals will "form" in molten, water-free salts.[7] Aluminum will form to approximately 80 volts in molten potassium nitrate. However, the effective film thickness is found to be as high as forty times as much as in aqueous solutions for aluminum, and up to five times as thick for tantalum.

The maximum voltage is affected also by the nature of the anion. In case of anions containing metals, a specific maximum voltage is given by that anion for any stated concentration and the maximum is dependent upon the metal of the anion. Table II for tantalum in 0.05 N. solutions of chromium gives practically the same breakdown voltage in each case.[8]

TABLE II

Metal	Cation	Anion	Maximum voltage
Chromium	K	Cr_2O_7	131
		CrO_4	141
		$Cr(C_2O_4)_3$	136
		$Cr(CN)_6$	126
			133 Mean.

The following list of the specific maximum voltages applies for 0.05 N. solutions of the electrolytes,[8] Table III.

TABLE III

Pd	19.5	Cr	133	H	448
Pt	20	Ni	181	Bi	450
Rh	23	Sn	190	Hg	453
Au	24	Co	243	Cd	478
Ir	32	Cu	335	Al	660
Mn	46	W	363	Sb	435
Fe	84	Ag	422		

Only the metals seem to have a definite voltage but a few of the other elements seem to give fairly consistent results,[8] Table IV.

TABLE IV

Nitrogen	470	Silicon	490
Arsenic	465	Fluorine	505

After voltage is removed, the rate at which the film dissolves depends upon the anode metal and the electrolyte. Antimony and tantalum are very sensitive to current interruptions, but bismuth, magnesium, and aluminum films dissolve very slowly. With a reduction in thickness of the film there is also an increase in capacity. A condenser originally formed to a voltage higher than the operating voltage will gain capacity until it stabilizes at the new value of voltage, the time depending upon the condenser and the voltage difference. Capacities will increase more rapidly for condensers on reduced voltage when the temperature is high.

The uniformity of capacity measurements for films formed in different electrolytes preclude the possibility of an effective film resulting from the reaction of the metal and the anion. Likewise, different metals have different capacity-voltage ratios, which indicates that the anode metal is very actively concerned in the film structure with film thicknesses ranging from 10^{-6} to 5×10^{-4} cm.

The existence of a film on the surface has been questioned,[9] and due to the fact that chlor-ion concentrations destroy valve action in aluminum and also due to the fact that an amalgamated electrode of aluminum will not form a film, it has been proposed that an electromotive equilibrium exists.[10]

Interference measurements[11] of the thickness of films have given results of 5 × 10^{-6} to 5 × 10^{-4} cm. for aluminum. A method of removing the aluminum and leaving the film intact has been tried;[12] the thickness of these films as measured with a direct vision spectroscope was approximately 7 × 10^{-4} cm. Film thicknesses were also measured by computing the weight of anion deposited in the formation period by measuring the forming current in coulombs; and also by the increase in weight,[12] Fig. 12. By assuming a dielectric constant for the film equal to unity, the effective film thickness can be computed from capacity measurements and comparisons have been made for a number of anode metals and electrolytes,[13] Fig. 13.

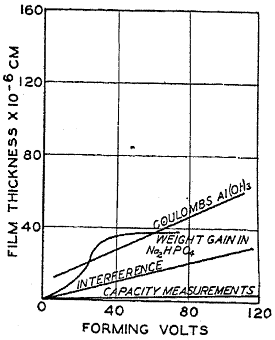

Fig. 12—Film Thickness on Aluminum Anodes as Measured by Capacity Measurements, Interference Methods, Increase in Weight of Formed Anode over Unformed Anode, and by the Formation Current. (Mott)

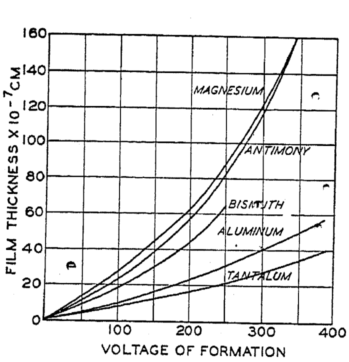

Fig. 13—Relative Film Thickness of Different Metals; Dielectric Constant Unity. (Guntherschulze)

Relative thickness of films for aluminum and tantalum in aqueous solutions and in fused, water-free salts are given in Table V.

TABLE V—RELATIVE THICKNESS AT 50 VOLTS AFTER 30 MINUTES FORMATION. (THICKNESS × 10^{-7} CM. FOR DIELECTRIC CONSTANT UNITY)

Electrolyte	Tantalum	Aluminum
Dilute aqueous	4.10	6.36
Pure sulphuric acid	6.69	10.65
Fused water-free		
NH_4NO_3	6.26	—
$KHSO_4$	6.98	13.10
$AgNO_3$	7.12	220.
$TiNO_3$	10.10	—
$LiNO_3$	7.93	—
$NaNO_3$	12.50	133
KNO_3	11.10	142
$K_2Cr_2O_7$	18.20	30

The electromotive equilibrium theory has been advanced as a complete explanation of the action at an anode surface;[10–14] evidence has also been presented for the presence of a film which forms when the aluminum is the anode and dissolves when the aluminum is the cathode.[15] However, oscillograms do not show such action during rectification, current passing immediately when the voltage reverses across the film. On high voltage rectifiers, the weight of the supposed aluminum hydroxide film which would have to be formed, dissolved, and reformed many times a second is prohibitive to such a theory. One investigator[16] was convinced by a microscopic examination of the luminous points, that they constitute an immense number of Wehnelt interrupters in parallel.

The first satisfactory theory accounting for alternating current rectification was advanced in 1902 by Guthe.[17] This was the oxide-gas film theory which proposed that a solid oxide or hydroxide film is formed, proportional in thickness to the formation voltage. Accompanying the oxide layer a thin film of oxygen formed on the solid film increases the leakage resistance.

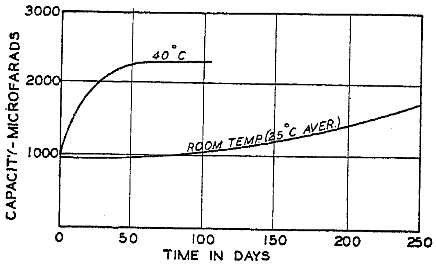

Fig. 14—Aluminum Condensers Operated at Half of the Formation Voltage. Effect of Temperature on the Rate of Capacity Change. (Siegmund)

Rectification was attributed to the high mobility of free electrons when penetrating the films as compared to the relatively heavy and slow moving anions of the electrolyte. The electrons are able to easily penetrate the films even under very low accelerating fields, but the ions are not able to penetrate the oxide film deeply enough to reach the anode metal. Since there is an abundance of free electrons in the metal and none in the electrolyte, rectification will take place.

This theory has undergone considerable expansion. The gas layer was thought to carry the principal burden so far as voltage drop was concerned due to the low specific inductive capacity of oxygen; and the oxygen film was supposed to constitute the real capacity of the anode. Fig. 15 shows the relative thickness of the gas and oxide layers on an aluminum anode. This convenient mechanism was employed to explain many of the former discrepancies observed for electrolytic rectifiers and condensers. Guntherschulze[18] calculated potential gradients as high as 8,000 volts per micron in such very thin films.

In an experiment to detect the presence of gas,[18] an aluminum anode was formed to 400 volts, and the voltage was suddenly reduced to 4 volts. The calculated reduction in pressure was from 2,100 to 0.21 kg. per cm.^{2}; but instead of an expansion of gas and reduction in capacity, the capacity increased slightly. Fig. 16 is illustrative of the supposed arrangement of oxide layer and gas film adsorbed in the pores of the oxide film. Hydrostatic pressure applied to the electrolyte of rectifiers caused decreases in rectification.[19] Pressures up to 20 atmospheres were applied, and results indicated that the gas-oxide theory was correct.

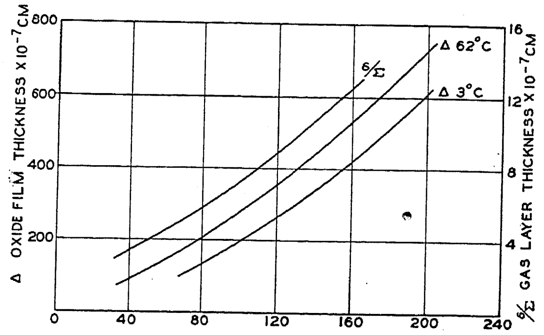

Fig. 15—Relative Thickness of Gas Layer Compared to Oxide Film on Aluminum Anode. (Guntherschulze)

An attempt was made by the writer to correlate the discrepancies found, and a condenser was constructed with provisions for applying pressure up to one hundred atmospheres. Both electrodes were of aluminum and of the same area of 350 sq. cm. The condenser was formed to 100 volts on a 60 cycle circuit, and oscillograms were made of the applied voltage and current at 80 volts 60 cycles for pressures ranging from atmospheric to 1,500 lb. per sq. in. The r.m.s. value of current was found to be 0.297 ampere in every case, and the oscillograms showed no change in current wave shape when sine wave voltage was applied.

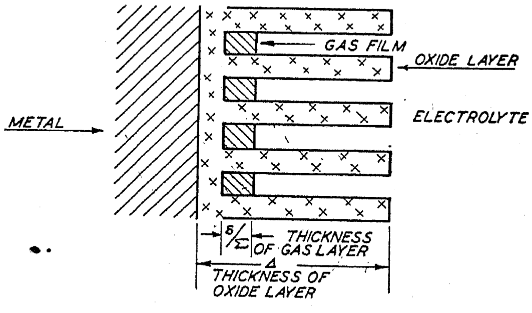
GAS FILM
OXIDE LAYER
METAL
ELECTROLYTE
δ/Σ THICKNESS OF GAS LAYER
Δ THICKNESS OF OXIDE LAYER

Fig. 16—Roughly Equivalent Structure of Film on Anode Showing Gas Layer in the Gross Film, and the Continuous Oxide Layer

If the experiments of Carman and Balzar[19] on the effects of increased pressure on rectification are discounted, the reduced voltage experiment of Guntherschulze and the writer's work with condensers under pressure both indicate that there is no gas film present. There may be oxygen present; but if so, it is undoubtedly present as an adsorbed gas in the oxide film, relatively free from external disturbances and not obeying the gas laws. Oscillograms from an electrolytic condenser under pressure are shown in Fig. 17.

The results of some measurements by the writer on electrolytic condensers operating in alternating current circuits at very low voltages are shown in Fig. 18. For both aluminum and tantalum the effective capacity decreased rapidly when the applied voltage fell below five millivolts. Both condensers were symmetrical condensers for a-c. operation formed to 60 volts a-c. at 60 cycles, and all measurements were made at 60 cycles.

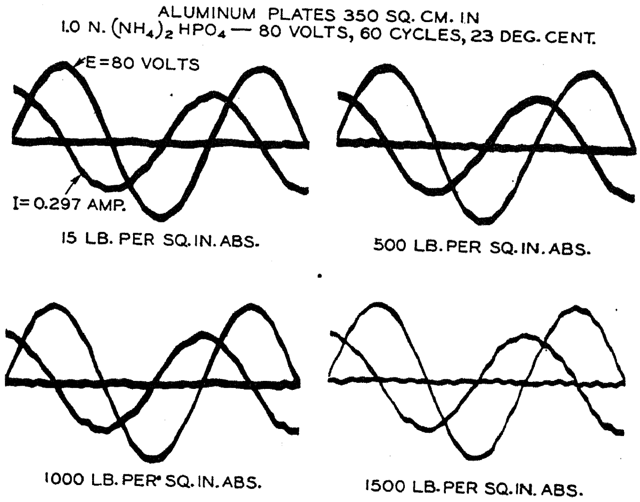

Fig. 17—Aluminum Electrolytic Condenser under Hydrostatic Pressure

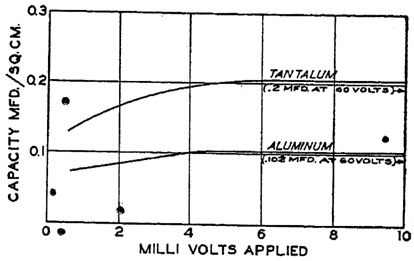

Fig. 18—Electrolytic Condensers with Very Low Applied Voltages on 60 Cycles A-C.

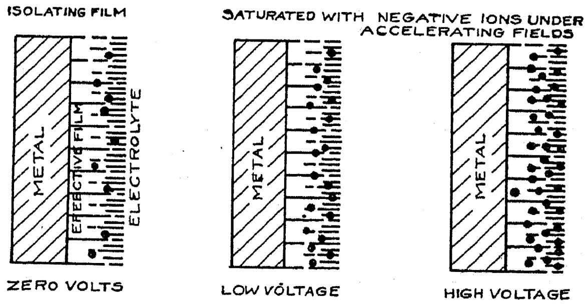

Fig. 19—Ion Distribution in the Oxide Film of an Electrolytic Condenser with Various Applied Voltages

An explanation of the reduction in effective capacity can be made on the basis of diffusion of ions throughout the oxide film under accelerating fields. Referring to Fig. 19, there are shown in an illustrative manner cross-sections of an oxide film under three conditions of impressed voltage. For the case of zero applied voltage very few ions from the electrolyte penetrate the oxide film, and the effective thickness of the film approaches the gross thickness. When low voltages are impressed more ions penetrate the film under the accelerating fields, and the effective thickness decreases with a consequent increase in capacity. As the voltage increases further, the ions penetrate deeper until the layer next to the anode metal is reached.

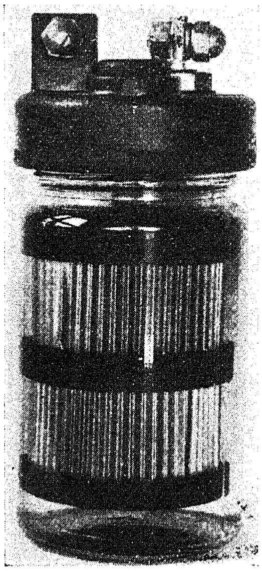

FIG. 20—TANTALUM ELECTROLYTIC CONDENSER FOR BATTERY CHARGING CIRCUITS. (FANSTEEL PRODUCTS CO., INC.)

Any ions penetrating this film constitute leakage current; the film therefore preserves a fairly constant thickness of effective film by natural growth resulting from leakage current. Thus after a certain critical voltage is reached for a given film the effective capacity of the film can change only slightly with variations in voltage.

Summarizing the foregoing, it appears that the normal film in electrolytic condensers is an oxide or hydroxide of the anode metal which is of a greater gross thickness than the effective film, this effective film maintaining itself relatively impermeable to ion penetration by the expedient of natural growth of film wherever leakage currents flow. Both the effective film and the gross film are probably saturated with sufficient water from the electrolyte to account for the extremely high capacities encountered by virtue of the high dielectric constant of water, approximately 80. A dielectric constant approaching this value for the effective film makes possible films of sufficient thickness that the voltage gradients encountered formerly may be materially reduced.

Recent work of the writer has shown that polarization capacity may account for a large part of the measured capacity, this polarization probably occurring in the outer layers of the gross film. A future paper discusses the influence of polarization in electrolytic condensers.

The writer wishes to express here his appreciation of the assistance received from the members of the Electrical Engineering Department of Sheffield Scientific School of Yale University, a large portion of the preparatory work having been done at Dunham Laboratory of Electrical Engineering. Particular appreciation is expressed to Prof. Wesley B. Hall for much constructive criticism, and to former Prof. A. E. Knowlton for valuable assistance in the preparation of this paper.

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Discussion

For discussion of this paper see page 449.

Power Losses in Electrolytic Condensers

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Synopsis.—*Electrolytic condensers on a-c. circuits have higher power losses than would be expected from the leakage currents and electrolyte resistances. Likewise, losses change with the applied voltage, with temperature of the cell, with the frequency of the applied voltage, and with external circuit conditions governing current wave shape in such ways that it is almost impossible to predict results in advance.*

The current wave usually contains large harmonics when a sine wave of voltage is impressed, indicating a non-linear function in the impedance of electrolytic condensers. Increases in voltage, frequency, electrolyte temperature, and concentration of the electrolyte cause increases in the harmonics in the current wave.

Dielectric losses in an oxide film or a gas-oxide film are too low to account for the total losses in electrolytic condensers; and in general the losses in a polarization cell are higher than those found in electrolytic condensers. Polarized cells show some characteristics which are similar to the peculiarities of electrolytic condensers; a number of low-voltage condensers very closely approach the action of polarization cells.

Electrolyte potentials of symmetrical condensers on a-c. circuits are lower than the theoretical values. These discrepancies can be accounted for only by a voltage distribution in the electrolyte. Controlling the potential from outside sources has small influence on power losses except for extreme values of potential.

An equivalent circuit can be drawn for a condenser from the losses, the applied voltage, and the electrolyte potential. On the basis of dielectric theory, the principal portion of the losses is attributed to the voltage drop above the rectifying region. The losses now approach those for a polarization cell.

A film on the anode surfaces saturated with the electrolyte solvent would explain the characteristics as measured, and also make it possible to predict characteristics with some accuracy. Ion penetration of the outer regions of the film and a comparatively continuous film near the anode explain the non-linear characteristics of electrolytic condensers.

The variation of the electrolyte potential can be predicted on the basis of the above theory, and power losses are also more easily predictable.

ELECTROLYTIC condensers consist of electrodes immersed in proper electrolytes with means for obtaining electrical contact to the electrodes and the electrolyte. The action of electrolytic condensers is due to films formed on the surfaces of the anodes. One class of condenser known as the dry type utilizes a hygroscopic, viscous electrolyte which will not flow from between two parallel electrodes separated by a suitable spacer,[1] such condensers have comparatively short electrolyte paths, and consequently much lower electrolyte resistance. They possess the further advantage that it is very simple to roll the electrodes and the electrolyte into a compact cartridge similar to the impregnated paper condensers. One such condenser is now available having 2.5 μf. of capacity per cubic inch of space, with a rated voltage of 450 volts.

Three of the important film theories include the electromotive equilibrium theory, the solid oxide film, and the gas-oxide film theory. The writer's adsorbed liquid theory supplements the older theories.[2] The solid oxide film theory considers that the entire electrical stress is distributed across the oxide film as the dielectric.[3] The gas-oxide theory postulates an inactive film of oxide and hydroxide which serves to support an extremely minute layer of gas carrying the major portion of the electrical stress.[4] The electromotive equilibrium theory states that the oxide film is only supplementary to the principal action of the condenser which is entirely electrochemical in nature.[5] The writer's adsorbed liquid theory[2] more recently advanced proposes that the oxide film is the carrier of an adsorbed volume of liquid, the solvent of the electrolyte, which carries ions the signs of which will vary with their positions with respect to the anode and the applied voltage. This theory takes advantage of the fact that the solvent, in most cases water, has a dielectric constant over 80.

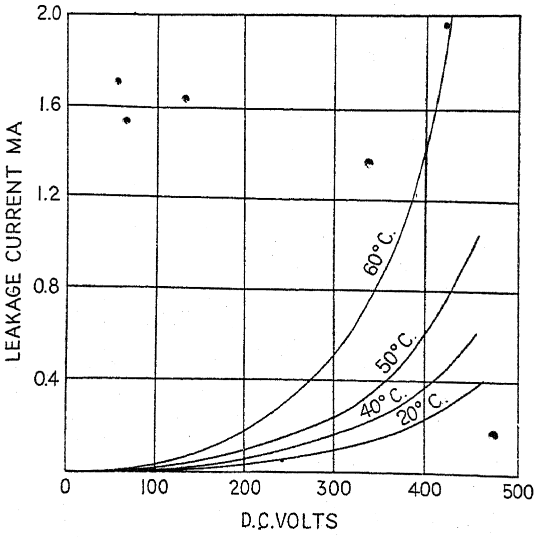

FIG. 1—LEAKAGE CURRENT OF 8-MICROFARAD FILTER CONDENSER FORMED TO 500 VOLTS D.C.

Aluminum anode in saturated borax solm

As a source of power loss, the leakage current of the majority of the electrolytic condensers is negligible, Fig. 1. Leakage current will in most cases account for less than one-tenth of one per cent of the total losses. A redissolving film[6] which would dissolve when current was passed in the permeable direction and would again reform when it was attempted to pass current in the impermeable direction would account for high losses, but this theory is open to many objections.

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[1] For references see Bibliography.

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Resistance of the electrolyte can be controlled by the temperature of the condenser, and by the composition of the electrolyte; the majority of suitable electrolytes have high electrical resistance. However, the power factor of an electrolytic condenser increases with rising voltage, Fig. 2. Increases in temperature should reduce power losses, but measurements show increased losses with increasing temperatures, Fig. 3. The anode construction of Fig. 4 gives rise to a temperature power factor curve such as shown in Fig. 5 due to the long current conduction paths to a large portion of the anode. This defect has been overcome to some extent by special shaping of anodes and venting in suitable places.[7]

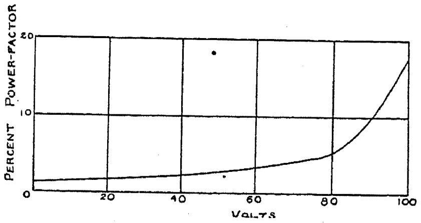

FIG. 2—DEPENDENCE OF POWER FACTOR ON APPLIED VOLTAGE

Aluminum condenser in 1.0 N. diammonium phosphate formed at 100 volts 60 cycles per sec., 23 deg. cent.

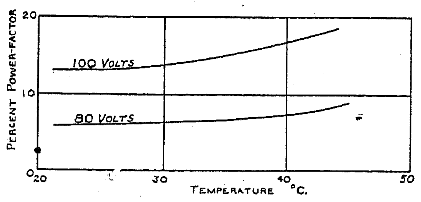

FIG. 3—POWER FACTOR *vs.* TEMPERATURE OF ELECTROLYTE

Aluminum condenser in 0.9 N. diammonium phosphate formed at 100 volts, 60 cycles per sec.

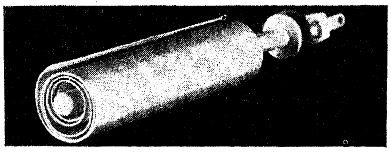

FIG. 4—ANODE CONSTRUCTION OF 8-MICROFARAD ALUMINUM FILTER CONDENSER FOR RADIO USE, 450-VOLT RATING

Typical electrolyte resistances range from 400 ohms to 100 ohms per cm. cube for aluminum condensers, and may go as low as 3 or 4 ohms per cm. cube for low voltage tantalum condensers. The total resistance of the electrolyte of the condenser of Fig. 2 and Fig. 3 is approximately 0.8 ohms, decreasing possibly 50 per cent for the higher temperatures of Fig. 3. The maximum power loss in the electrolyte of this condenser accounts for a power factor slightly under one-half of 1 per cent, much lower than the over-all values which range from 1.25 to 20 per cent. Tantalum in sulfuric acid emphasizes the inadequacy of the resistance of the electrolyte to explain the power losses in electrolytic condensers, Fig. 6; the total resistance of the electrolyte is only a few hundredths of an ohm, and the power factor would be less than one-tenth of one per cent were the losses found in the electrolyte.

When a sine wave of alternating voltage is applied to an electrolytic condenser, large variations in current wave shape are found (Fig. 7); the oscillograms show the current waves 180 deg. out of phase. The wave shapes for aluminum anodes are different in detail from tantalum, but of the same general form, Fig. 8.

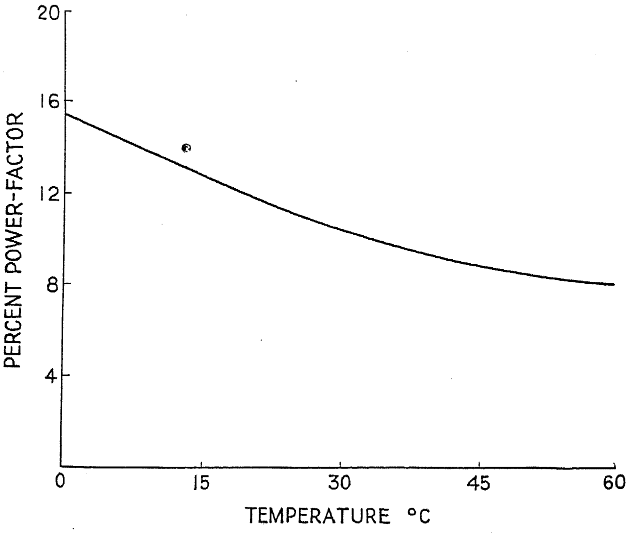

FIG. 5—POWER FACTOR *vs.* TEMPERATURE IN ALUMINUM ELECTROLYTIC CONDENSER FOR RADIO SERVICE

Electrolyte borax and boric acid

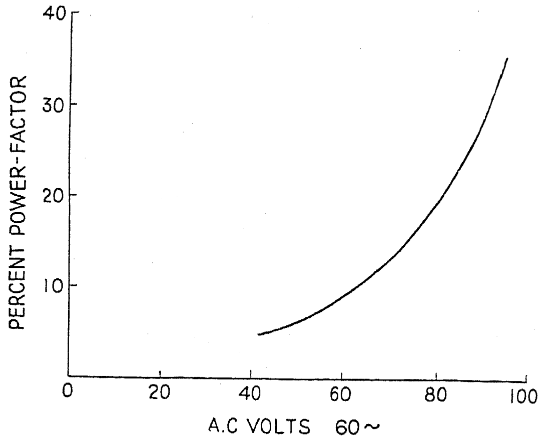

FIG. 6—TANTALUM ELECTROLYTIC CONDENSER ON ALTERNATING VOLTAGE

Electrolyte sulfuric acid (Sp. Gr. 1.275) formed at 100 volts

Temperature increases result in increases in current and a shift in phase angle toward higher power factors as shown in Fig. 9. These changes are small in so far as the wave shape is concerned until comparatively high temperatures are reached. Changing the concentration of the electrolyte does not seem to alter the effect of temperature changes to any great extent except by a further shift toward an increase in power factor when comparatively high concentrations are reached. (Figs. 10 and 11.)

Increases in the frequency of the voltage applied to the condenser are evident in the current wave shape in the same manner that all influences tending to increase the current loading per unit of area of the anodes become evident, namely by increases in the harmonics and a general if not rapid decrease in phase angle. However, the out-of-phase component of current is as usual almost unchanged as seen in Fig. 12.

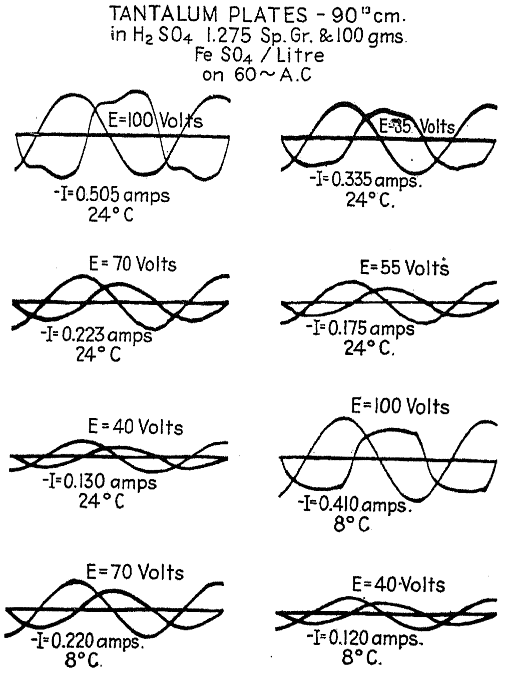

Fig. 7—Current Wave Shape and Applied Voltage

Tantalum condenser

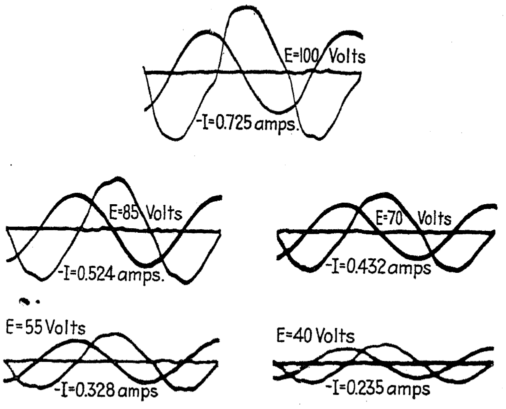

Fig. 8—Current Wave Shape and Applied Voltage

350-sq. cm. aluminum plates on 60 cycles a.c. in 3.5 N. diammonium phosphate, formed at 100 volts, temperature 23 deg. cent.

In general, the current wave shapes of electrolytic condensers in circuits where sine waves of alternating voltage are impressed may possess a large number of harmonics of varying amplitudes. Any changes in operating conditions which tend to increase current density at the anode surface will increase the amplitudes of the harmonics. The harmonics are usually distributed in such a way that a distinct peak in current is evident aside from the out-of-phase condenser component of current which is plainly evident and changes very little, this peak occurring in phase with the impressed voltage.

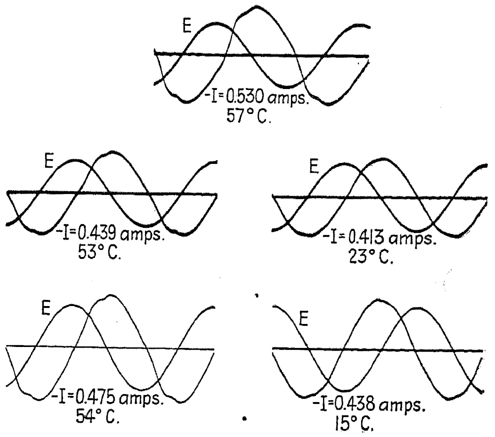

Fig. 9—Current Wave Shape and Temperature of Electrolyte

350-sq. cm. aluminum plates on 60 cycles a.c. in 1.75 N. diammonium phosphate, 80 volts impressed and formed at 100 volts

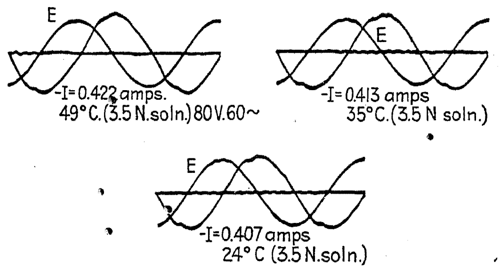

Fig. 10—Effect of Temperature Variations on Current Wave Shape of Electrolytic Condenser of Fig. 9

3.5 N. solution

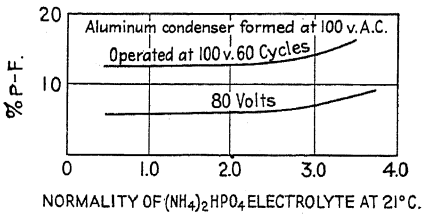

Fig. 11—Power Factor Variations with Changes in Electrolyte Normality in an Aluminum Condenser

Power factors of electrolytic condensers are high for reasons other than leakage and resistance. To ascribe an additional loss to a poor phase angle for the effective dielectric film will account for another part of the total loss. Adhering to three of the original film theories presented, the solid oxide film, the gas-oxide film, and the adsorbed liquid film, the possibilities of the three can be investigated. The solid oxide film theory calls for an extremely thin film of oxide of the anode metal, which may however be shielded to some extent by a gross thickness of film in which there is considerable penetration of the electrolyte. The phase angle of a crystalline film would be much too small to account for the large losses encountered; and the action of an amorphous layer of oxide only a few millimicrons thick is as yet unknown. The gas-oxide theory is open to the same objections from the standpoint of power losses, a thin gas film merely being substituted for the thin oxide film. The adsorbed liquid theory offers more possibilities since polar rotation is undoubtedly present to account for the high dielectric constants, yet it is doubtful that the losses could ever approach the losses found; and in such a condenser variations in electrolyte concentration should have little or no effect upon the phase angle.

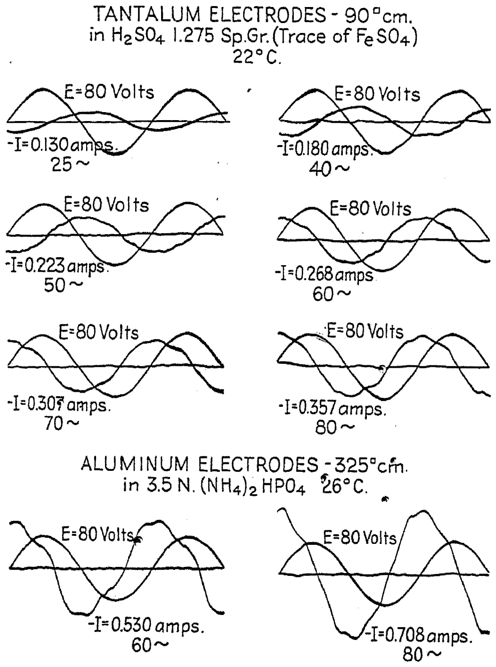

FIG. 12—INFLUENCE OF FREQUENCY ON CURRENT WAVE SHAPE

The most promising field for investigation of the power losses now seems to be the electromotive equilibrium theory. As mentioned above, the electromotive equilibrium theory considered alone is insufficient to explain all the action occurring in electrolytic condensers, but the extent to which polarization capacity enters the question has been investigated in some detail.

An electrolytic cell capable of polarization can be subjected to a flow of current and then discharged through a resistance, acting in some respects as a condenser which has been charged and which has a certain capacity; this is known as the polarization capacity of the cell. Its place is the point of contact between the electrode and the electrolyte, and is quite distinct from the case of a non-conducting layer, for instance an oxide layer as on an aluminum condenser. Polarization capacities have been found to be dependent upon the frequency of the impressed voltage and the magnitude of the voltage or current intensity; and the phase displacement varies from 0 deg. to 90 deg. leading. There are in general three characteristic groups into which electrolytic cells may be divided.

The capacity may be independent of the frequency of the impressed voltage, and the phase displacement is zero or at least very small. The capacity can be considered as the result of an electric double layer as it must exist according to Helmholtz at point of contact of electrode and electrolyte, or perhaps produced by a condensation on the surface of the electrode of the dissolved metal salt according to Warburg. The double layer capacity is especially effective in small concentrations of ions, which practically eliminates it from consideration in electrolytic condensers of the aluminum and tantalum anode type for high current densities and voltages over a few volts.

The polarization may be due to the change in concentration and the diffusion of the dissolved salt. Warburg, M. Wein, and Neumann have shown that the capacity, in case the diffusion predominates, is inversely proportional to the square root of the frequency, with a phase angle of 45 deg. leading. The double layer exists in these cases also, but it becomes entirely insignificant compared to the influence of diffusion if the concentration of ions of the dissolved salt is not too small.

In the case of a salt so little dissociated and of such a concentration that a polarization on account of the change in concentration of the salt under weak currents is imperceptible, nevertheless a polarization can occur, if the speed of dissociation of the non-dissociated part of the salt does not ensue quickly enough compared to the current change to prevent a change in ionic concentration. In this case the capacity is inversely proportional to the frequency, and the phase displacement is 90 deg. This case was found experimentally by M. Wein on palladium heavily charged with hydrogen—and by Kruger with mercury electrodes in solutions of complex mercury salts. This is another case that need not be confused with the action in the ordinary electrolytic condensers.

It therefore appears that the only case of polarization capacity which must be considered is the case of concentration or diffusion capacity. Warburg[8] has developed the theory of concentration cells when a sine wave of alternating current is forced through the cell. Selecting a symmetrical cell with parallel plate electrodes

$$E = \frac{\alpha}{K\omega}\sin\left[\,\omega - \frac{\pi}{2} + \psi\,\right]$$

where K is the polarization capacity. The value of the constant ψ is $\frac{\pi}{4}$ for this case, and

$$K = \frac{c_0\sqrt{kqe^2y}}{2RT\sqrt{\omega f}}$$

In the above equation R is the gas constant, f is the number of ions given by one molecule of the salt, y the total valency of ions from one molecule, and T is the absolute temperature.

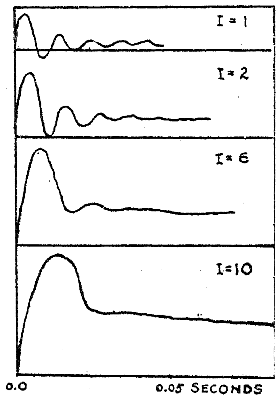

Fig. 13—Degeneration of Oscillatory Charge of Polarization Cell Due to Increase in Current Density

Thus, the current is found to lead the voltage by 45 deg. when a sine wave of voltage is impressed on a concentration cell for low values of current density. The capacity is also observed to be inversely proportional to the square root of the frequency.

When the current density is high the above relation is no longer true, and harmonics are introduced into the current wave when a sine wave of voltage is impressed. However, it is very interesting to note that the effective capacity will not change greatly, and the phase angle of 45 deg. is particularly interesting from the standpoint of power losses in electrolytic condensers.

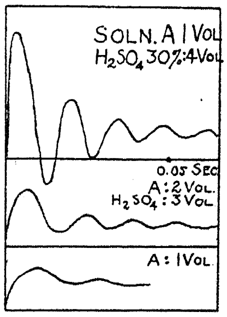

Fig. 14—Degeneration of Oscillatory Charge of Polarization Cell Due to Increased Ion Concentration

Kruger[9] developed equations for the charge of polarized cells in inductive circuits, and obtained experimentally the necessary results to confirm them. Very low voltages were applied to the cells to avoid gas polarization, and it was found that increases in current density would change a charge from an oscillatory charge to an aperiodic charge, Fig. 13. Likewise, increases in the ion concentration of the solution show the same effect, Fig. 14; and one very interesting development is the fact that an oscillatory circuit can be converted very easily to an aperiodic circuit by increasing the inductance in the circuit. It is also observed as a definite characteristic of most electrolytic cells that only a definite number of oscillations occur before the charge degenerates into an aperiodic form which may tail-out indefinitely.

The behavior of an electrolytic condenser made up with nickel anodes in a caustic solution for use in low-voltage filters was very similar to the action found by Kruger for concentration cells. The charging characteristics were strongly suggestive of a condenser in which concentration capacity plays a large part, Fig. 15: the discharge characteristics in inductive circuits also indicate the predominance of concentration capacity with very high initial concentration in the electrolyte. A low-voltage electrolytic condenser of the semi-dry type intended for the same low-voltage filter use showed better condenser characteristics, Fig. 16. The charging characteristics of the semi-dry condenser were found to approach those of a perfect condenser, although the decrease in charging current

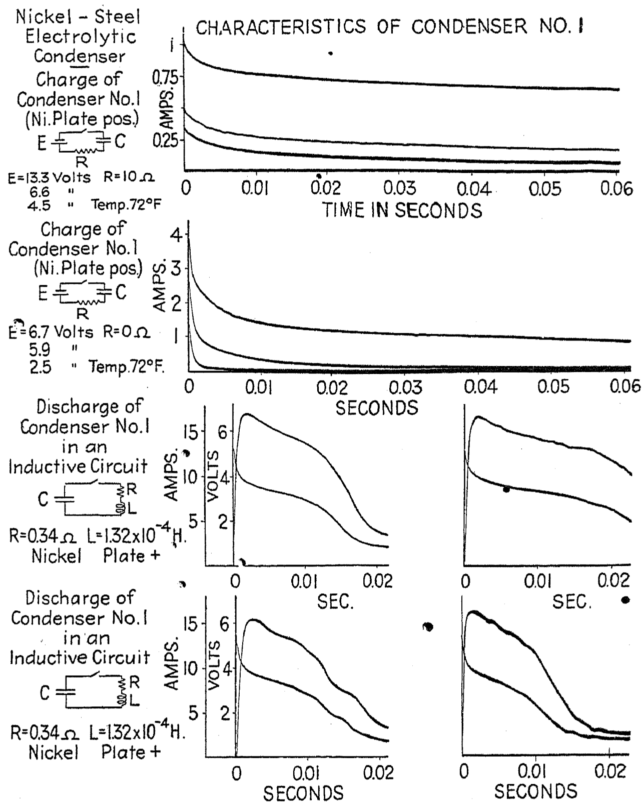

Fig. 15—Low-Voltage Filter Condenser with Large Amount of Concentration Capacity

Charge and discharge characteristics

with time was not rapid enough to satisfy a logarithmic function.

When a high-resistance film is present on the surface of an electrode immersed in an electrolyte, somewhat different results may be expected than from the case of diffusion capacity alone. However, diffusion capacity undoubtedly exists at the point of contact of the film and the electrolyte.

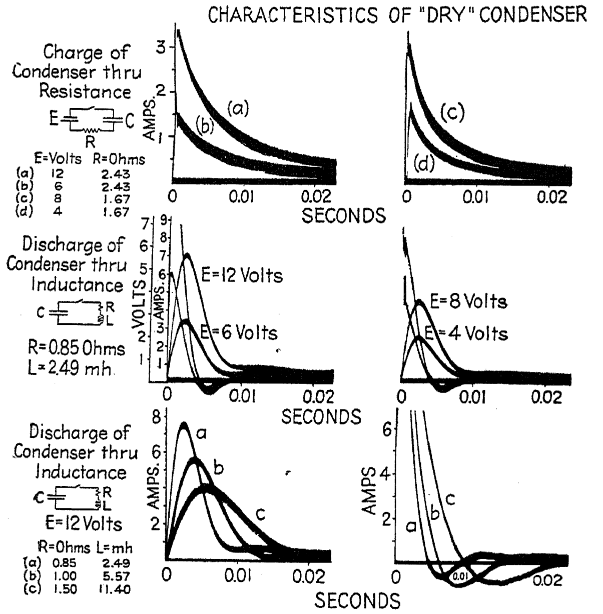

Fig. 16—Semi-Dry Electrolytic Condenser with Pasted Aluminum Electrodes for Low-Voltage Filter Use

Charge and discharge characteristics

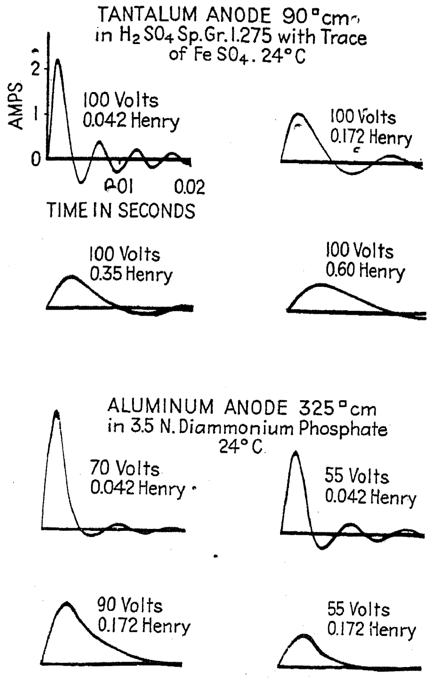

Fig. 17—Oscillatory Discharge of Electrolytic Condensers in Induction Circuits of Very Low Resistance

Ordinary aluminum and tantalum condensers were discharged through inductive circuits and oscillograms of current were obtained, Fig. 17, with strong indications of the presence of diffusion capacity. Tantalum showed the characteristic increase in attenuation of the oscillatory component with increase in inductance, and also was noteworthy in that the period of the oscillatory component became shorter as the cell discharged and the concentration decreased. Aluminum indicated an even greater diffusion capacity than tantalum, and the decrease in attenuation is strikingly evident as the initial concentration is decreased for the same circuit conditions by lowering the initial voltage. Increasing the inductance of the discharge circuit for the aluminum condenser caused a rapid approach to the aperiodic discharge predicted by Kruger[9] for concentration cells.

TABLE I

Volts	Inductance	Capacity μf. per sq. cm.
Tantalum (formed at 110 volts)		
100	0.042 henry	0.201
100	0.172	0.190
100	0.350	0.170
100	0.600	0.161
Aluminum (formed at 110 volts)		
70	0.42	0.085
55	0.42	0.091
90	0.172	0.093
55	0.172	0.087

The effects of diffusion or concentration capacity in ordinary electrolytic condensers are somewhat similar to the effects predicted for simple concentration cells; but there is the fundamental difference in that in most electrolytic condensers the diffusion capacity is in series with a more or less effective dielectric layer.

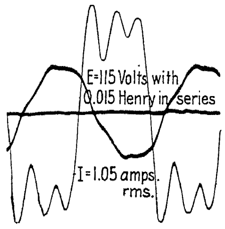

Fig. 18—Aluminum Electrolytic Condenser Formed at 120 Volts in 1.75 N. Diammonium Phosphate in Series with Inductance of 0.015 Henry

Area of aluminum electrodes 350 sq. cm. each, temperature 23 deg. cent., total applied voltage 115-sine wave 60-cycle.

The following capacities calculated from the oscillograms of Fig. 17 by integrating over the first quarter cycle of the discharge to find the coulombs of electricity stored in the condenser for a given voltage, show a decrease in effective capacity for decreasing frequency, Table I.

These variations in effective capacity have been noticed in electrolytic condensers only in freely oscillating circuits and exist to a negligible extent in circuits where alternating voltages are impressed from a source of power supply. Condensers with non-linear current voltage characteristics are difficult to handle as simple capacities without introducing errors.

Harmonics in the current wave can be emphasized in a-c. circuits by placing inductance in series with the condenser, Fig. 18.

In a symmetrical electrolytic condenser on a-c. circuits, in order to prevent heavy leakage currents, it is necessary that the aluminum or tantalum electrodes always have a positive potential with respect to the electrolyte of the condenser. Assuming an equal voltage division between electrodes, the maximum alternating voltage applied to one electrode is equal to half of the total applied voltage. The maximum instantaneous voltage is therefore equal to 0.707 by the total applied voltage, and the d-c. potential of the electrolyte which must be negative with respect to the electrodes at all times is therefore equal to 0.707 by the a-c. voltage on the condenser. In order to insure a minimum of leakage, it is possible to raise the potential of the electrodes with respect to the electrolyte by means of a separate source of direct current, Fig. 19. Referring to Fig. 2, the power factor is improved greatly by operating the condenser on an a-c. voltage well below the voltage of formation; by maintaining the voltage of formation higher than the operating voltage by means of some external source of direct current it should be possible to greatly improve the power factor, providing that other effects do not overweigh the advantages gained. However, when the voltage of the electrolyte was controlled from an external source, the expected results were not obtained, very little if any improvement in power factor being noted. (Figs. 20 and 21.)

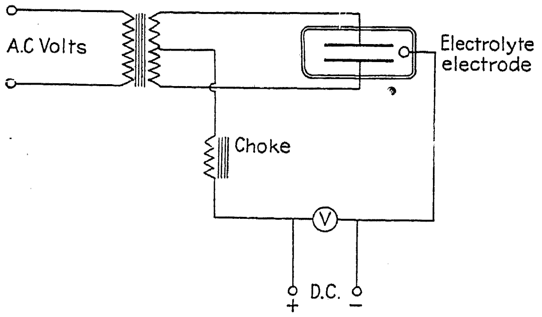

Fig. 19—Circuit for Applying and Measuring Electrolyte Potential in Electrolytic Condensers on A-C. Circuit

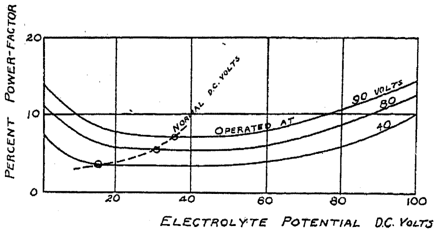

Fig. 20—Power Factor Control by Means of Control of Electrolyte Potential of Electrolytic Condenser

Since there was no improvement in power factor when the electrolyte of a symmetrical condenser was maintained at the proper potential with respect to the electrodes by auxiliary means, the actual potential of the electrolyte was measured with an electro-static voltmeter connected to the midpoint of the transformer supplying the condenser and to the electrolyte. The potentials found for aluminum and for tantalum electrolytic condensers were 50 to 75 per cent of the expected values, Fig. 22. These departures from the theoretical values on the basis of complete rectifier action at the film and a dielectric capacity type of condenser are much too large to admit of experimental error, and furthermore are very consistent in their values.

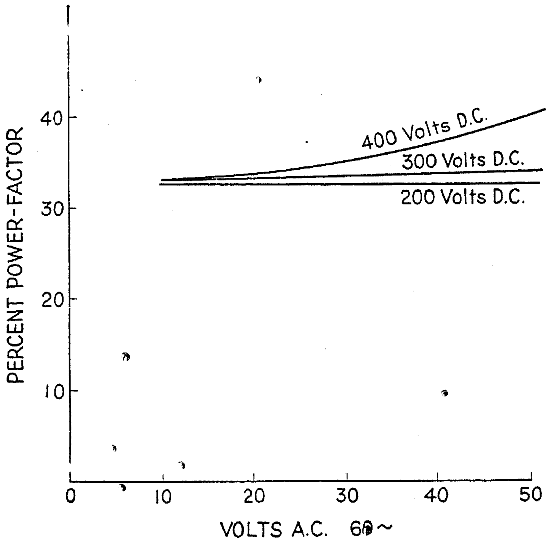

Fig. 21—Variations of Power Factor of 8-Microfarad Filter Condenser with Amplitude of the A-C. Component for Various Values of D-C. Polarization

High power factor due principally to the long electrolyte conduction paths

It was thought at first that these low values might be due to a pulsating electrolyte potential which would reach a maximum value of the theoretical voltage twice every cycle, but subsequent oscillograms of the variation in electrolyte potential disclosed a ripple in the d-c. voltage of approximately one-quarter of one per cent or less, Fig. 23. This is further proof that losses in electrolytic condensers on a-c. circuits should not be ascribed to leakage current when operated at normal voltages.

The maximum value of alternating voltage applied to the rectifying film on the electrodes of electrolytic condensers is therefore 50 to 75 per cent of the theoretical value.

The phase angle of the part of the electrolytic condenser across which this voltage appears is an unknown quantity; but if it is assumed that this effective film is any one of the three mentioned above, a solid dielectric, a gas filled solid, or a liquid filled solid, the phase angle is extremely close to 90 deg. leading.

On the assumption that the power loss in the effective film is negligible, it is possible to locate the voltage vector of the effective film with respect to the current and voltage vectors for the entire condenser, Fig. 24. The condenser current I_c is shown leading the total applied voltage by the phase angle α, and the effective film voltage E_{eff} is approximately in quadrature with the condenser current. In order to complete the voltage triangle, the vector E_{e1} must be added with a phase angle of β with respect to the condenser current. This vector E_{e1} is of interest now, since the power losses are practically all accounted for by its phase angle; it is the voltage drop which appears in the electrolyte of the condenser, and it has just been shown that the voltage drop in the electrolyte is much too small as calculated to account for the magnitude of this vector on the basis of a resistance drop, furthermore the phase angle shows that it is not a pure resistance.

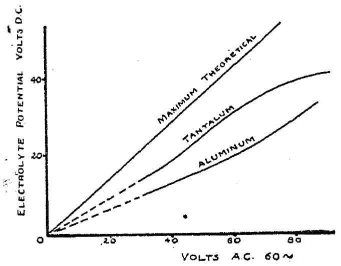

FIG. 22—ELECTROLYTE POTENTIALS OF ALUMINUM AND TANTALUM ELECTROLYTIC CONDENSERS ON A-C. CIRCUITS

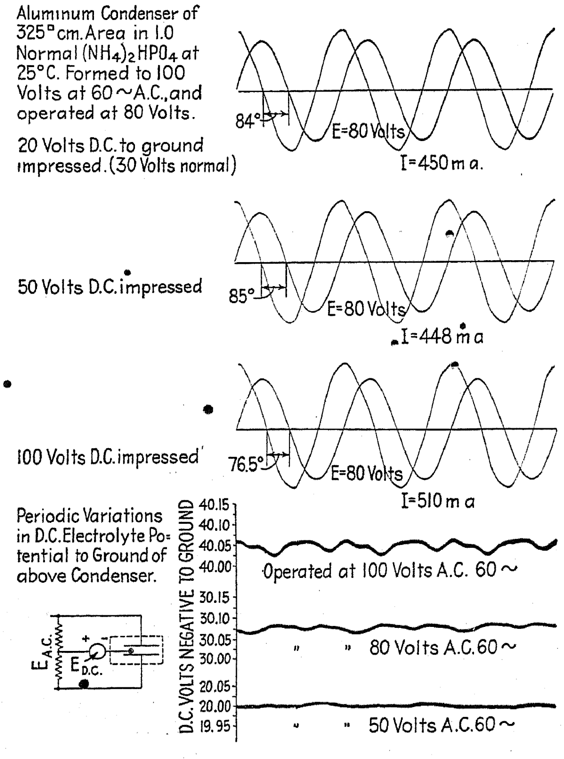

FIG. 23—EFFECT OF WAVE SHAPE ON IMPRESSED ELECTROLYTE POTENTIAL, AND THE NORMAL FLUCTUATIONS IN ELECTROLYTE POTENTIAL

It becomes very quickly apparent upon consideration of the properties of this residual voltage vector after all of the measurable quantities are accounted for, that the electromotive equilibrium theory is justified to some degree. The phenomenon of polarization capacity will account for the leading phase angle of the vector E_{e1}, and when a larger portion of the loss is assigned to the voltage appearing across the effective film, the phase angle of the electrolyte voltage vector increases. Thus, it appears that in the great majority of electrolytic condensers of the formed film type there are two distinct kinds of capacity present, dielectric capacity and polarization capacity.

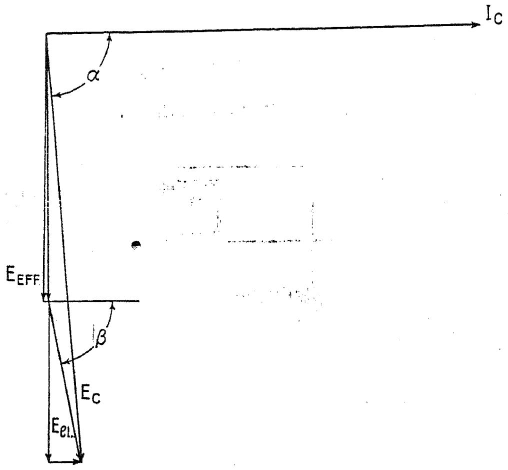

FIG. 24—VOLTAGE DISTRIBUTION IN ELECTROLYTIC CONDENSER ON ALTERNATING CURRENT CIRCUIT

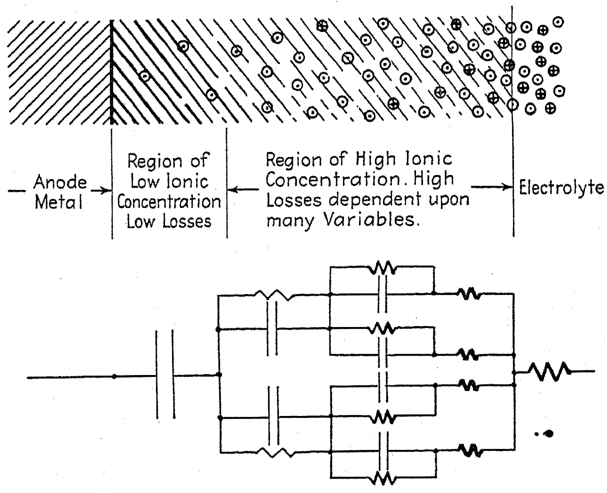
Anode Metal
Region of Low Ionic Concentration Low Losses
Region of High Ionic Concentration. High Losses dependent upon many Variables.
Electrolyte

FIG. 25—SCHEMATIC DRAWING OF CROSS-SECTION OF FILM ON ELECTROLYTIC CONDENSER ANODE

Showing regions of high and low ionic penetration and the equivalent circuit diagram

Selecting a typical example of an aluminum condenser, with a total impressed voltage of 80 volts at 60 cycles, the power factor was found to be 9 per cent. The d-c. potential of the electrolyte was measured at 32 volts, leaving a distribution of 44.8 volts on both electrodes of the symmetrical condenser in series. The remainder of the out-of-phase voltage 34.8 volts, appears as a voltage drop in the electrolyte. Added to this quadrature drop is the in-phase drop of 7.2 volts, giving a total drop in the electrolyte of 36 volts at 20 per cent power factor, or a phase angle of approximately 78 deg. leading. The resistance of the electrolyte will account for less than one volt drop, and that in phase with the current; apparently polarization is occurring in the gross film covering the effective film, or very near to the gross film.

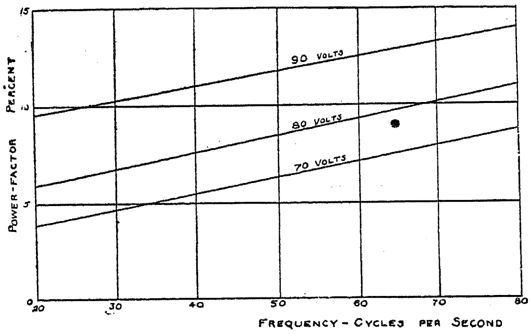

Fig. 26—Effect of Variations in Frequency Upon the Power Factor of Electrolytic Condensers

Aluminum plates 350 sq. cm. in diammonium phosphate 1.0 N. at 23 deg. cent. (formed at 100 volts)

However, polarization capacity of the kind found in this type of electrolytic condenser was found to have a phase angle of 45 deg. leading, which does not agree with the 78 deg. for the above case. Obviously, there are modifying factors and it will appear that polarization capacity as such must be handled differently than from the standpoint of clean electrodes in an electrolyte. The writer has advanced a theory of film penetration to account for the variations in capacity of electrolytic condensers at very low voltages.[2] A diagrammatic cross-section of such a film is shown in Fig. 25, and below it is shown the equivalent circuit. In the equivalent circuit, the in-phase components of the equivalent circuit are shaded lightly or heavily, to indicate low conductivity or high conductivity of the film at that point as the case may be.

Calculations from the measured capacities of such anodes show that the gross film as measured by optical methods may reach thicknesses as great as fifty times that necessary for the requisite capacity. Thus, it is necessary that the film be conducting through at least a part of its thickness, and it can best be described as a semi-permeable membrane which increases in density as the anode is approached.

In such a film structure, the penetration of the film by the electrolyte will be retarded in the case of cations by reason of the direction of the electric field, and in the case of anions the field will accelerate them to even further penetration than under the case of normal diffusion. By way of explanation, the terminology used for the anion and the cation assigns to that ion which is attracted to the cathode the name cation, and vice versa, there being some ambiguity in the terms as used by physicists and electrochemists.

Thus referring again to Fig. 25, a film of insulating material of a thickness comparable with the wavelength of visible light, varying in composition from an extremely dense layer against an anode to a relatively porous outer surface and presumably saturated with water, acquires a space charge which varies in density with the location in the film. The distribution of the electrical field through the film will depend upon the configuration of the space charge and movements in the film, assuming that the film material itself is a relatively good insulator. Thus, the potential gradient will increase as the anode is approached, placing a larger portion of the electrical load upon the region of the film unoccupied by large numbers of anions; and likewise by reason of this high electrical field, subjecting the film to high mechanical forces in compression, tending to increase its density and continuity.

The action of such a film in an electrolytic condenser with pulsating direct current applied is such that the space charge will vary with the impressed voltage, with some dependence upon the rate of diffusion of anions, in such a way that the equivalent electrical circuit shown in Fig. 25 will be disturbed for even small changes in total applied voltage, giving rise of a current wave shape with many harmonics evident as a result of the variable characteristic of the film. The smaller the voltage change, the less will be the variation in the space charge characteristics of the film and the nearer will the current wave shape approach a sine wave for a sine wave of applied voltage.

The region of the film nearest to the anode was described as being relatively free of ions; whether or not it is saturated with the solvent of the electrolyte is open to question, but it is more likely that it is saturated than not since the dielectric constants of the aluminum compounds are relatively low compared to water, and the volt-ampere loading of this region would be excessive in the case of low dielectric constants. This portion of the film is shown in the equivalent diagram as pure capacity with little or no resistance loss, and it is believed that across this region the rectifying action of electrolytic rectifiers occurs. The

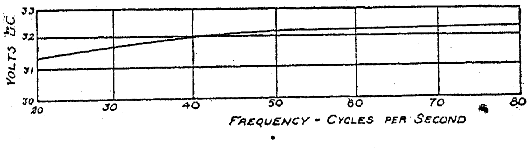

Fig. 27—Effect of Frequency Upon Electrolyte Potential of Symmetrical Electrolytic Condenser

Aluminum

boundary can be fixed definitely on the side adjacent to the anode metal, but on the other side of this rectifying region of low dielectric loss and high electrical resistance, the boundary cannot be fixed so definitely. It moves about with the applied voltage, the thickness of the formed film, the mobilites of the anions, and the temperature of the film and electrolyte. Thus, the leakage current and breakdown voltage of a particular film are dependent upon the above mentioned variables. No definite plane of demarcation between the rectifying region and the adjacent region saturated with ions can be located, the one shading gradually into the other. A definition of the boundary of this rectifying region can be stated as the point at which the maximum voltage drop from the anode is equal to the electrolyte potential when the film is made half of a symmetrical electrolytic condenser in an a-c. circuit.

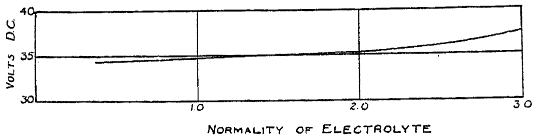

FIG. 28—EFFECT OF ELECTROLYTE CONCENTRATION UPON ELECTROLYTE POTENTIAL OF SYMMETRICAL CONDENSER

Aluminum

The entire film is then seen to form a dielectric sheet similar to the stratified dielectric of Maxwell showing absorption as do all stratified films; and possessing the further interesting feature that the leakage of any one strata is controlled by many factors, and will change periodically with an applied periodic voltage variation, giving rise to a dielectric sheet with a non-linear voltage-current characteristic.

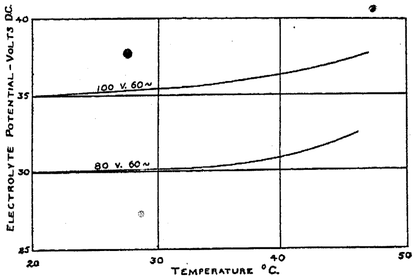

FIG. 29—ELECTROLYTE POTENTIAL OF ELECTROLYTIC CONDENSER AS INFLUENCED BY TEMPERATURE

Aluminum

For the current flow in the resistance paths, the ions in the film constitute the space charge in the film and must also supply the electrical charge to take care of the excess voltage imposed on the rectifying film by reason of the resistance paths. Thus, the ions in the film fulfil two requirements, the one providing electrical conductivity through regions of the film and the other providing an electrical charge of real thickness to supplement the dielectric strain in the parallel strata. The thickness of the ionic space charge influences the characteristics of the condenser, as well as does the mobility of the ions in the film. The increase in effective capacity of the condensers in the above described freely oscillating circuit with increases in natural frequency of the circuit, Table I, can then be attributed to the predominance of the leakage resistance paths over the space charge effect in the cases cited. However, this is the only case where such action has been noted by the writer, the effect of variations in frequency upon effective capacity usually being negligible. This would be expected ordinarily since the phase angle of any stratum approaches 90 deg. leading as the anode metal is approached, and only the outermost strata must have sufficiently poor phase angles to exert any serious influences under increased frequency and the accompanying changes in relative impedance of the respective strata.

Increases in frequency will cause increases in loading of the outer strata, with increases in power factor. The effect of frequency upon the power factor of an aluminum electrolytic condenser is shown in Fig. 26, and the variation in d-c. potential of the electrolyte for the same condenser is shown in Fig. 27.

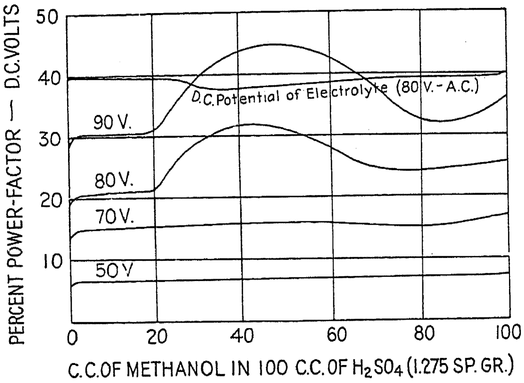

FIG. 30—ELECTROLYTIC CONDENSER POWER FACTORS AS CONTROLLED BY SOLVENT OF THE ELECTROLYTE

Tantalum condenser

Increasing the concentration of the electrolyte should increase the space charge density in the film, with a resultant increase in d-c. potential of the electrolyte, Fig. 28. The condenser illustrated in Fig. 28 is the same condenser as in Fig. 11.

Variations in impressed voltage have a similar effect to both frequency changes and increased concentration for high voltages; and the results are shown in Figs. 2, 6, and 22. Increases in the space charge density are general for increases in voltage, with particularly high increases in the outer strata. Temperature rises produce a similar effect to voltage rises through increased mobilites of anions, Fig. 29.

The temperature drop through the film can be calculated from the known losses and the assumption that the transfer of heat from the film to the electrolyte follows the general rules of heat transfer between liquid and solid surfaces in contact, a value of approximately 7.3 g-cal. per sq. cm. per hour per deg. cent., being acceptable for the case of electrolytic condensers. For the condenser of Fig. 2, with a total area of both electrodes of 700 sq. cm. and a current flow of 400 milliamperes at 100 volts, the power factor is 17 per cent. The loss per hour is 6.8 watt-hr. or 5,860 g-cal. per hour, with a temperature difference between the film and the electrolyte of approximately 1.15 deg. cent.

An attempt was made to change the capacity of a tantalum electrolytic condenser by changing the nature of the solvent for the electrolyte. Methyl alcohol was added to sulfuric acid electrolyte, with some expectation that penetration of the film by the alcohol might occur with a consequent loss in capacity due to the lower dielectric constant of methyl alcohol as compared with water, approximately 30 to 80. Capacity variations were negligible, and the small errors in observation were as large as the observed variations. However, some very curious effects were noticed upon the power factor of the condenser, and a small effect upon the d-c. potential of the electrolyte for the higher operating voltages. The peculiar humps in the curves of Fig. 30 are attributed to the increased loading of the outer strata of the film due to penetration by the alcohol, thereby increasing the voltage drop in the outer strata and thus raising the power factor. The curve of d-c. potential of the electrolyte against alcohol content indicates this action also. Work on electrolyte condensers was discontinued before any further investigations could be made along similar lines.

The author wishes to express his appreciation to Professor Wesley B. Hall of Sheffield Scientific School of Yale University and to Mr. A. E. Knowlton, formerly of Yale for their valuable assistance rendered in the preparation of the material for this paper both while at Yale and subsequently.

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Discussion

FILM CHARACTERISTICS OF ELECTROLYTIC CONDENSERS

(Godsey)

POWER LOSSES IN ELECTROLYTIC CONDENSERS

(Godsey)

H. H. Race: Figs. 25 and 26 of Mr. Godsey's paper interested me particularly because of some recent work in our laboratory which may help to explain power factor—frequency characteristics of electrolytic capacitors even though it was undertaken for an entirely different purpose. In studying the causes of dielectric loss in insulating materials we have made artificial imperfect dielectrics by sealing conducting liquids in fused quartz tubes. We have found that the measured effective capacitance and dielectric loss as functions of frequency of such a composite dielectric can be completely explained in terms of the equivalent circuit shown in Fig. 1 which may be reduced to the simpler circuit shown in Fig. 2.*

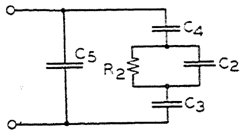

Fig. 1—Equivalent Circuit of Fused Quartz Tubes Containing a Conducting Fluid

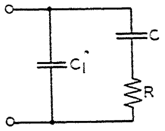

Fig. 2—Simpler-Circuit Which Can be Made Equivalent to Fig. 1 by the Use of Definite Algebraic Relations Between Coefficients

If a dielectric can be represented by the equivalent circuit of Fig. 2 in which the coefficients are assumed to be independent of frequency, then the general form of the effective capacitance and loss curves will be as shown in Fig. 3.

If the coefficients of capacitance and resistance in Mr. Godsey's Fig. 25 can be assumed to be independent of frequency then this complicated combination of series and parallel arrangements of resistance and capacitance can be reduced to an equivalent circuit similar to Fig. 2 above and the changes in effective capacitance and loss with frequency in an electrolytic capacitor should follow Fig. 3. Thus Fig. 26 may be only a special case of this more general form.

Probably the coefficients in Fig. 26 cannot be assumed to be independent of frequency since the ionic concentration at a given point in the oxide film will change with time. Nevertheless, I feel that a study of the dielectric properties of electrolytic capacitors over wide ranges of frequency and temperature should add considerably to existing knowledge of the mechanism involved.

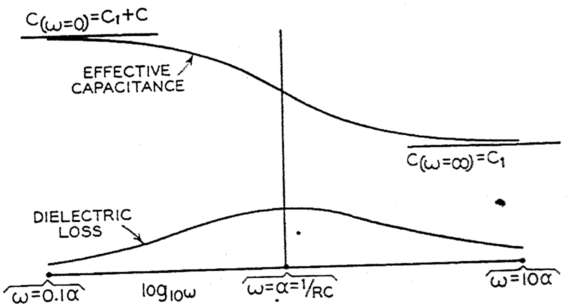

Fig. 3—Variations with Frequency of Effective Capacitance and Dielectric Loss for the Equivalent Circuit Shown in Fig. 2 (Plotted for Case Where $C = C_1$)

J. Slepian: The principal obstacle to the use of electrolytic condensers in the power field has been the high power factor usually obtained in such cells. An attempt to locate and explain the losses occurring in these cells, such as Mr. Godsey has made in this paper, is therefore extremely praiseworthy.

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*The results of this study will be published in the near future.

In work which I carried on over a period of years some twelve years ago, I quickly found that the properties of aluminum cells were extremely complicated and that such cells frequently behave in most unexpected ways. For example, it was generally believed that the halogens were extremely deleterious. I found that the addition of a fractional per cent by weight of a soluble fluoride had an extremely beneficial effect upon the power factor, reducing the losses generally many fold. With the addition of such traces of fluoride I was able to make cells with power factors under 2 per cent.

However, I was never able to make cells with such low power factors which would stay efficient indefinitely. In all cases there was a gradual rise in power factor with continuous operation over a period of years. Starting at 2 per cent the power factor would become 5 or 6 per cent in the course of a year and continue to rise slowly for years afterwards. This was with a superposed d-c. excitation upon the cells. Without the superposed d-c. excitation the rise in power factor was very much more rapid. Investigation showed that apparently changes in the electrolyte played a part in the rise in power factor since, if the plates of an old cell were placed in fresh electrolyte the power factor returned to a low value, although not quite as low as in the original cell. The power factor rose much more rapidly, however, with time in the new electrolyte than in the old, indicating that there had also been a change in the electrode. The change in the electrolyte I assumed to have been the formation of colloidal aluminum hydroxide and I ask Mr. Godsey if he can suggest any reason why the presence of such substance should effect the losses in the cell.

In general I found that in operation on alternating current some of the generalizations made from d-c. studies were not applicable. For example, on direct current the capacity of an aluminum cell seems to be independent of the nature of the electrolyte and to vary inversely with the d-c. voltage, but on alternating current the capacity was different in different electrolytes and varies with the time. Generally also, the capacity was lowest in those cases where the power factor was low. This may seem to be inconsistent with the picture of Fig. 25 which Mr. Godsey has drawn explaining the losses and it would be interesting to have him comment upon that point.

F. W. Godsey, Jr.: The study of artificial imperfect dielectrics referred to by Mr. Race is interesting from the point of view of electrolytic condensers in that the action in a composite dielectric such as Mr. Race describes would closely approach the action in an electrolytic condenser under certain conditions.

However, the circuit of Fig. 25 is only very roughly equivalent and the coefficient of capacitance and resistance can under no circumstances be assumed to be independent of frequency. This is due to the fact that the conducting particles and ions in the outer layers of the film not only constitute leakage resistance paths, but also constitute a considerable portion of the electrical charge of the inner layers of the film. Thus, the capacitance and resistance coefficient are not only variable for different frequencies, but vary through wide limits for different current densities and applied potentials. It is the author's conclusion that it is a hopeless task to attempt to set up a simple equivalent circuit for an electrolytic condenser.

As Mr. Race suggests, it would add considerably to existing knowledge of electrolytic condensers if a study of the dielectric properties were made over wide ranges of frequency and temperature, provided that sufficient precautions were taken to overcome the inherent difficulties due to electrolyte resistance and cell constants.

Mr. Slepian's comments in connection with the addition of halogens to the electrolyte of electrolytic condensers are very interesting. Since most of the previous work on electrolytic condensers was done on direct current, it was soon found that the addition of halogens greatly increased leakage current for a small amount of the halogen ion. However, as I have shown, the leakage current in electrolytic condensers is in general a negligible factor in computing power losses.

Referring to Fig. 25, it will be seen that the addition of any ion to the electrolyte which will serve to penetrate the outer film layers will reduce the value of leakage resistance in the outer layers.

By a proper control of the leakage resistance of these outer film layers, it is possible to either increase the power factor or to reduce the power factor of the condenser. Theoretically, the power factors encountered will be lower when the resistance is zero and when the resistance is infinite. By the addition of halogens, this resistance can be controlled to some extent and it would appear from Fig. 25, that with increased conductivity of the outer film layers, there should also appear an increase in effective capacitance.

The observation that the capacity in alternating current circuits was lowest in cases where the power factor was low can very probably be assigned to the case of newly formed condensers upon which the oxide films are relatively thin and continuous in the outer layers so that there is less penetration of the outer layers by ions. Upon aging, the film grows in thickness and the continuity of the outer layers is in all probability so broken up that ion penetration is much more readily effected and the leakage currents in the outer layers are therefore excessive. It is apparent from an inspection of Fig. 25 that if the leakage resistance in the outer layers is high, the effective capacity will be low.

Mr. Slepian's suggestion that the formation of colloidal aluminun hydroxide in the electrolyte accounts for a large part of the rise in power factor with time of the electrolytic condensers is I believe, substantiated by the references to the importance of the condition of the outer film layers made in the papers under discussion. The placing of an anode with a formed film upon it in an electrolyte in which there is colloidal aluminum hydroxide would result, upon the passage of current, in the collection of this colloidal hydroxide upon the plate. Since the thickness of the oxide film upon the anode is only about one micron for a 300-volt film, a very small deposit of the colloidal hydroxide would very effectively increase the thickness of the outer film layers and would in all probability increase the series resistance of the outer film layers with an attendant increase in power loss. (Two additional papers on electrolytic condensers were presented at the Sixty-First General Meeting of the Electrochemical Society in Baltimore on April 23, 1932. These papers have some bearing upon the comments of Mr. Race and Mr. Slepian, and are identified as preprint 61-27 and preprint 61-28, pages 331 to 357 inclusive.)

Electrical Precipitation

Some Fundamental Theory and Experiments

BY A. W. SIMON*
Non-member

and

L. C. KRON*
Non-member

WHILE numerous articles have been published on electrical precipitation (the Cottrell process), they have been concerned largely with the engineering features of Cottrell treaters or the special features of the application of the process to particular industries. The fundamental theory of the process or the quantitative laws underlying it, on the other hand, have received but scant attention. From the theoretical side we have but two papers which treat the subject with any degree of thoroughness: one, based on the kinetic theory but limited to tube treaters, by Deutsch,[1] and the other, based on the law of mass action, by one of the present authors.[2] From the quantitative experimental side also but little has come to light; we need mention only that in a paper by Schmidt[3] it is set forth that the precipitation efficiency (percentage removal) is an exponential function of the gas volume treated (although it is not stated what kind of an exponential) and also of the "length of discharge electrode;" while in a paper by Horne[4] the relation between precipitation efficiency (percentage removal) and secondary voltage is exhibited in the form of a curve; further that in another paper by Deutsch it is stated that Horne and Anderson have arrived at an empirical formula, connecting the precipitation efficiency η with the length of pass L and the velocity V of the gas, of the form:

$$\eta = 1 - K^{L/V}$$

and, finally, that Anderson[6] gives the equivalent formula:

$$1 - \eta = K^t$$

where t is the time the gas remains in the precipitator.

As a matter of fact, outside of the few points just mentioned, *quantitatively* very little is generally available regarding the fundamental principles involved in electrical precipitation; and many operators today are still entirely in darkness regarding the effect of such fundamental factors as voltage, current, velocity, size of discharge electrode, etc., on the percentage removal, *i. e.*, the "precipitation efficiency".[7] In the present work a number of these relations are, we believe, for the first time, clarified and exhibited.

I. QUANTITATIVE THEORY OF THE COTTRELL PROCESS

From the theoretical point of view it is logical to infer that the Cottrell process can be considered as a reaction between the dust particles on the one hand and the electrically charged gas particles (gas ions) on the other. Such a reaction would presumably follow the law of mass action familiar in chemistry, since after all the chance of a dust particle being precipitated depends on its change of encountering an ion, and therefore should be proportional to the number of ions present, whereas the actual number of dust particles encountering ions should also be proportional to the number of dust particles present. On this basis, if we denote by dq/dt the rate of removal of the dust particles, we have:

$$- dq/dt = kqi \qquad (1)$$

where q is the concentration of dust particles at any instant, i is the concentration of the ions, and k a constant, which depends, however, on the particular gas and fume treated.

Integrating this equation directly we have:

$$q/q_o = e^{-kit} \qquad (2)$$

where q_o represents the original concentration and t the treatment time.

This equation can be interpreted in two ways; either statically or dynamically. The static case would correspond to the experiment where the whole volume of the precipitator is filled with a quiescent gas containing suspended material of concentration q_o and the precipitation process is continued for a length of time t, at the end of which period q is the concentration remaining in suspension; the dynamic case, on the other hand, is where the dust-laden gas is flowing through the precipitator, in which case q_o is the entering and q the emergent concentration, and t the time the gas remains in the precipitator.

The left member of (2) is obviously the fractional part of the entering concentration passing through *unprecipitated*, *i. e.*, emerging from the precipitator, whereas in many cases the fractional part *precipitated* is of interest. If we denote this (fractional) part by η, we have that the left member of (2) is equal to $(1 - \eta)$, whence we can rewrite (2) as:

$$\eta = 1 - e^{-kit} \qquad (3)$$

In a particular case we may assume i to be constant, in which case we can put ki identically equal to a new constant α, whence we have as a final result:

$$\eta = 1 - e^{-\alpha t} \qquad (4)$$

Equation (4) expresses two fundamental laws of electrical precipitation, which we shall now consider in detail.

First, from the definition of η, it follows: *The fractional part of the incoming concentration precipitated is independent of this concentration.* This really justifies the use of the quantity η, that is to say, establishes the

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*Tennessee Coal, Iron & Railroad Company.

[1] For references see end of paper.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

"precipitation efficiency" (percentage removal) as a uniquely determinable quantity (independent of the incoming concentration).

If interest centers not so much on the amount of material precipitated, as on the *cleanliness* of the emerging gas, it is the quantity $(1 - \eta)$, which we will denote by ξ, which is of importance. In this case we can rewrite (4) as

$$\xi = e^{-\alpha t} \tag{5}$$

A second fundamental law which follows directly from eq. (4) is that *the fractional part of the incoming concentration precipitated depends on the length of time the gas remains in the precipitator.*

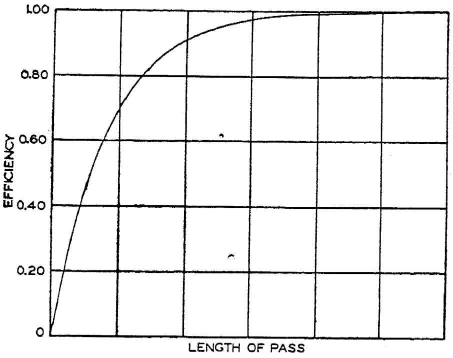


FIG. 1—THEORETICAL PRECIPITATION EFFICIENCY (PERCENTAGE REMOVAL) OF A COTTRELL PRECIPITATOR AS A FUNCTION OF THE LENGTH OF THE PRECIPITATOR

From this law we can deduce directly also how the precipitation efficiency varies with the gas volume V (treated per minute) and the distance D traversed by the gas in the precipitator, *i. e.*, the "length of pass." Obviously, if A represents the effective cross-section of the treater, these quantities are related to each other, and also to the velocity v of the gas in the precipitator by the equation:

$$t = D/v = DA/V \tag{6}$$

whence we can rewrite (4) in the several forms:

$$\eta = 1 - e^{-\alpha D/v} \tag{7}$$

$$\eta = 1 - e^{-\alpha AD/V} \tag{8}$$

$$\eta = 1 - e^{-KD/V} \tag{9}$$

where K is a constant of the precipitator depending, however, on the particular gas and fume treated and on treater conditions (temperature, etc.), but, of course, not on D and V. Eq. (9) expresses some important facts regarding Cottrell treaters now to be considered in greater detail.

In order to exhibit the relations expressed by eq. (9) more clearly, the curve giving the precipitation efficiency as a function of the effective length of the treater is reproduced in Fig. 1; and as a function of the *volume treated per minute* (or its equivalent the velocity of the gas in the treater) in Fig. 2. Anticipating somewhat, it might be stated that most of the quantitative laws involved in electrical precipitation satisfy or approximate either one or the other of these curves, so that they may be regarded as the two fundamental types of characteristic curves of the Cottrell process.

Without, for the moment, going into the mathematical relations, it will be seen from Fig. 1 that the cleaner the gas becomes, that is to say, the more dust is precipitated, the more difficult it becomes to precipitate the remaining dust or to make the gas cleaner, a fact which in practise is found pretty much universally true. In fact the curve of Fig. 1 really expresses a law which holds generally for gas cleaning; for the assumptions made in deducing eq. (9) for Cottrell cleaning hold also *mutatis mutandis* for other types of cleaner. To be specific, it will be noted that the foregoing theory is based on the chance of an encounter between a gas ion and a dust particle; however, it might equally well be, in the case of a wet washer, the chance of a water droplet encountering a dust particle, or in the case of a bag filter, the chance of a fiber ensnaring a dust particle. Therefore, with the proper interpretation of D, this formula can be applied to these cases also.

The above more or less qualitative statement, that the cleaner the gas is the harder it is to make it cleaner, can be restated quantitatively as follows:

Consider the total pass D split up into equal intervals of length l and consider the relation between the dust concentration entering and emerging from the jth of these intervals. In practice each interval might represent a separate Cottrell unit, the gas passing through the separate units in succession (*i. e.*, the units being connected in series). For the purpose in hand it is more convenient to rewrite eq. (9) in the form:

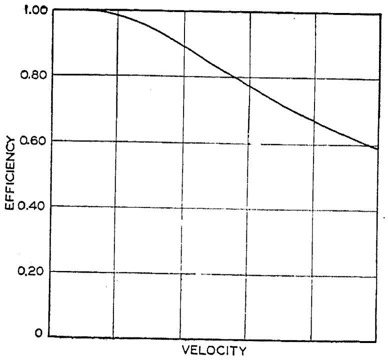


FIG. 2—THEORETICAL PRECIPITATION EFFICIENCY (PERCENTAGE REMOVAL) OF A COTTRELL PRECIPITATOR AS A FUNCTION OF THE GAS VOLUME TREATED PER MINUTE OR THE VELOCITY

$$\eta = 1 - e^{-\beta jl} \tag{10}$$

and then to substitute for $(1 - \eta)$ its value ξ, whence we have finally:

$$\xi = e^{-\beta jl} \tag{11}$$

as the formula to be used. Next if we denote by q_j the concentration of the dust leaving the jth segment of length l, and by f_j the fractional part which q_j is of the concentration q_{j-1} *entering* the interval, we have:

$$f_j = q_j/q_{j-1} = e^{-\beta jl}/e^{-\beta(j-1)l} = e^{-\beta l} = \text{const}_1 \tag{12}$$

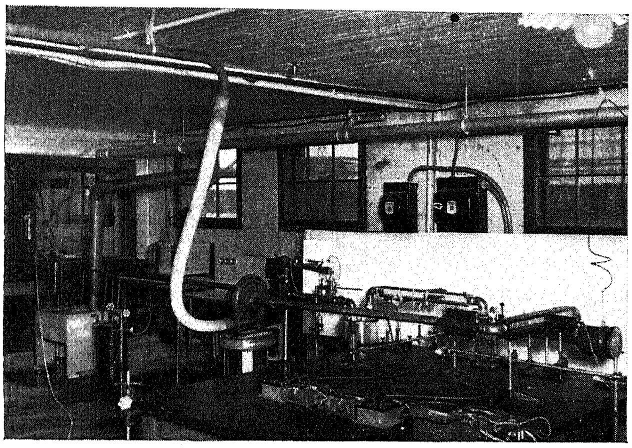
In other words: *the fractional part of the dust passing (unprecipitated) through any pass of a series of identical Cottrell passes is the same for every pass.*

If we denote by p_j the fractional part of the dust precipitated in any pass, we have also, since $p_j = 1 - f_j$:

$$p_j = (1 - f_j) = 1 - \text{const}_1 = \text{const}_2 \qquad \textbf{(13)}$$

that is to say *the fractional part of the dust precipitated in any pass of a series of identical Cottrell passes is the same for every pass.*

In practise this means that if the first pass of a series of passes takes out 70 per cent of the dust entering it, the second pass will take out also 70 per cent of the dust *entering it*, but 70 per cent of the 30 per cent of the original dust leaving the first pass and entering the second; the third pass will take out also 70 per cent of the dust entering it but 70 per cent of only the 9 per cent of the original dust leaving the second pass and entering the third; etc.



View of Set-Up Used in Experimental Study of Electrical Precipitation

From the point of view of the dust concentration *leaving* successive passes, the relations become still simpler, the first pass *transmits*, for example, in the above case 30 per cent of the dust entering it; the second pass 30 per cent of the dust entering it but 30 per cent of 30 per cent of the original dust; the third pass transmits 30 per cent of 30 per cent of 30 per cent of the original (entering) dust, etc.

Gathering these results in a table we have:

Pass	In	Down	Through
1.....	100.0%	...70% of 100.0% = 70.0%	...30% of 100.0% = 30.0%
2.....	30.0%	...70% of 30.0% = 21.0%	...30% of 30.0% = 9.0%
3.....	9.0%	...70% of 9.0% = 6.3%	...30% of 9.0% = 2.7%
4.....	2.7%	...70% of 2.7% = 1.89%	..30% of 2.7% = .81%

From the point of view of costs, after the first 70 per cent has been removed, it costs just as much to take out the next 21 per cent as it did the first 70 per cent, and then as much again to take out the next 9 per cent as it did the first 70 per cent, and so on. Of course to most engineers this is a familiar case, since there are numerous other devices which obey the same economic law.

Let us now turn to an examination of Fig. 2. As already stated, this curve exhibits the relation between the precipitation efficiency and the gas volume treated (per unit time). Since the latter is proportional to the velocity of the gas in the treater Fig. 2 also exhibits the relation between the precipitation efficiency and the *velocity* of the gas in the treater.

From this curve it is readily seen that the rating of a Cottrell treater in terms of the gas volume alone is meaningless, since the rating depends on the desired efficiency, which in turn depends on the dust content of the original gas and the desired cleanliness of the treated gas. That is to say, if a customer is satisfied with dirtier gas, the rating of the treater can be increased, and *vice versa.*

A further series of deductions which can be made from the above fundamental formulas, namely eqs. (7), (8), and (9), concerns the effect of the *manner of connection* of Cottrell units on the resulting precipitation efficiency. In practise, of course, for well-known reasons, a Cottrell treater is not made in the form of a single large unit, but of a number of smaller units, and in installing these the question naturally arises whether there is any advantage (as regards precipitation efficiency) in connecting them in series over connecting them in parallel; or whether perhaps a series-parallel connection is better, etc. In particular, for example, if six units are available they can be connected symmetrically in four different ways as shown in Fig. 3 and designated *A*, *B*, *C*, *D*.

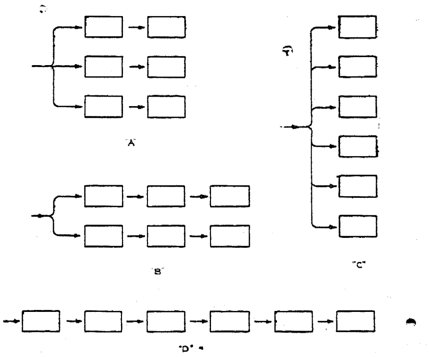


Fig. 3—Four Methods of Connecting Symmetrically Six Cottrell Passes as Regards Gas Flow

In general, if there are available a total number N of passes, such that $N = mn$, where m and n are any two integral factors of N, then it is obvious that the installation can be connected so that it consists of m units in parallel where each unit consists of n passes in series, or n units in parallel, where each unit consists of m passes in series.

In order to answer the question raised above, we have only to note that the fundamental variable which determines the cleaning efficiency is the treatment time t, *i. e.*, the length of time the gas remains in the precipitator. Since this quantity is equal to D/v, where D is the total distance traversed by the gas in the electric field and v is the velocity, which in turn is given by V/A, where V is the total volume treated per minute and A the total effective area of the installation, the cleaning efficiency for any two arrangements is the same provided D/v is the same for the two cases.

For the case then that we have m units in parallel each consisting of n passes in series, we have, provided we let l represent the length of each pass and a its effective area:

$$t = D/v = nl/(V/ma) = N\,(al/V), \qquad (14)$$

that is to say, the resulting cleaning efficiency of the installation depends only on the total number of passes (and the total volume treated) and not on their manner of connection. For the alternative case where we have n units in parallel each consisting of m passes in series we have:

$$t = D/v = ml/(V/na) = N\,(al/V), \qquad (15)$$

as before.

On the basis of cleaning efficiency alone then the four schemes of connection shown in Fig. 3 would be exactly equivalent.

It will be seen that the schemes of Fig. 3 all represent *symmetrical* schemes of connection; it might be asked what would happen for a non-symmetrical scheme such as shown in Fig. 4. Without going into details, it might be answered that for this case, if the units are left wide open, the resulting efficiency will be less for the case of Fig. 4 than for the case of Fig. 3; but by distributing properly the gas flow the scheme of Fig. 4 can be made to give exactly the same efficiency as the scheme of Fig. 3.

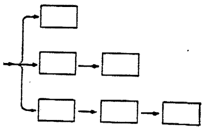


FIG. 4—NON-SYMMETRICAL CONNECTION OF SIX COTTRELL PASSES

In particular, if 1/6 of the gas is passed through the single pass, 2/6 through the double pass and 3/6 through the triple pass (by properly adjusting suitable valves), then the resulting efficiency theoretically will be the same for Fig. 4 as for Fig. 3.

Here again what we have said for Cottrell cleaners ought to hold also for wet washers or bag filter installations, or any other type of cleaner which satisfies our fundamental assumptions.

II. EXPERIMENTAL STUDY OF THE COTTRELL PROCESS

Unfortunately the theory developed above does not give us the precipitation efficiency in terms of all the fundamental variables involved in the Cottrell process; that is to say, we have had to predicate always that certain conditions remained constant; for example, the curve of Fig. 1 would hold for a particular gas and fume in a particular treater, whereas if the nature of the gas, nature of the fume, spacing of discharge electrodes, operating voltage, etc., were changed, our formulas give us no inkling of how the precipitation "constant," that is, the quantity α in eq. (4) would change. Strictly speaking, the theory developed above indicates how the precipitation efficiency varies with the treatment time, *all other conditions being constant.* For this reason, in the experimental portion of this work, it is necessary to follow a slightly different procedure than would correspond to a direct verification of the theoretical formula.

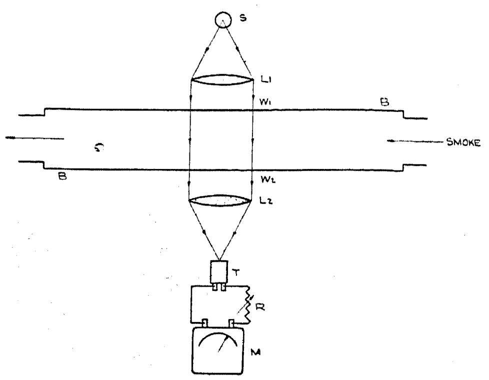


FIG. 5—SCHEMATIC DIAGRAM OF LABORATORY SMOKE CONCENTRATION METER

To begin with we shall have to list those variables which are known by experience to enter in, and determine experimentally how they affect the precipitation efficiency. We shall also verify the validity of our theoretical formulas for the conditions under which they are assumed to hold.

The selection of these variables requires some knowledge of the Cottrell process. We might list: the (secondary) operating voltage, the (secondary) current, the gas velocity, the length of the treater, the size of the discharge electrode, the spacing of the discharge electrodes, spacing of the discharge electrode to collecting electrode, the nature of the fume treated, the nature of the gas carrying the fume (last including temperature and pressure).

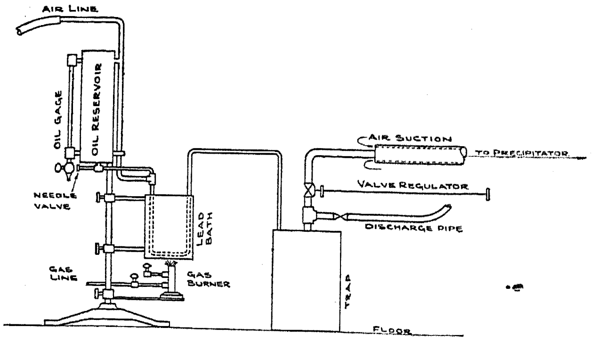


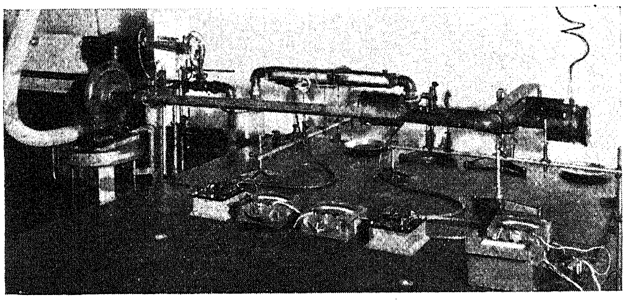
FIG. 6—SCHEMATIC DIAGRAM OF SMOKE GENERATOR USED IN THE EXPERIMENTS

Of course it is not necessarily true that all these variables are independent or fundamental; that is to say, the size of the discharge electrode or the voltage might enter in only as they affect the current flow; and it might be possible to express the whole process in terms of a few fundamental variables such as the space current, the electric field, the velocity, and the mobility of the fume particles in the particular gas.

However, since the effect of the fundamental variables might be difficult of direct verification, the procedure in the present study has been to determine the effect of the secondary variables (voltage, current, etc.) directly on the precipitation efficiency and also to determine the interrelation between the secondary variables; for example, the effect of the voltage on the precipitation efficiency is determined directly, and, also, for example, the relation between the voltage and current is determined independently. With this prelude we can proceed to the experiments.

Development of the Method of Measuring the Concentration of Suspended Material in Smokes, Fumes and Mists

Possibly the reason so little has been done heretofore in the experimental study of the Cottrell process was the lack of a suitable means of measuring quickly and accurately the concentration of the suspended material entering and leaving the experimental precipitators.



Close-Up of Experimental Precipitation and Smoke Recorder

Heretofore, the only method available for this purpose was the rather tedious and inaccurate filter method, in which a measured volume of the gas is drawn off through a filter and the resulting deposit weighed. Theoretically this process sounds simple enough, but in practise it is attended with many difficulties tending to produce inaccuracy, furthermore, the method is very tedious, and finally, the result is only an *average* value over a considerable time interval. Where many experiments have to be carried out, as was the goal in our present study, the filter method is too slow even if it were sufficiently accurate.

We set ourselves the problem of developing a rapid (in fact, instantaneous) and accurate method of determining the *concentration* of suspended material in gases. In the following we shall refer to the resulting instrument as the *smoke concentration meter,* now to be described.

The development of the instrument and a study of the principles underlying it was a considerable task in itself, and since the fundamental principles involved and a form of the instrument have been described elsewhere,[8] in the present work we shall confine ourselves to a brief description of the method and apparatus used.

Briefly the instrument depends on the fact that when a beam of light is passed a fixed distance through a gas containing suspended material, the fractional part of the light transmitted depends on the concentration of the suspended material. In particular it can be shown that if L_o is the intensity of the light *entering* the smoke-laden column of gas, L the intensity *emerging,* x the length of the column, and m the concentration of suspended material, (expressed, of course, in weight of material per unit volume) then we have the relation:

$$\log (L_o/L) = kmx$$

where k is a constant which depends, however, on the nature of the material (only on the grain size in the case of *opaque* material, but also on the wave length of light used and on the optical properties of the suspended material in the case of semi-opaque material such as acid and oil mists).

In the case of electric precipitation studies one is interested usually not so much in the actual concentrations (in grams per cubic meter or grains per cubic foot) as in the *ratio* of concentrations, as appear, for example, in our fundamental formula (2). It is then not necessary to make an absolute calibration of the meters, but more convenient to choose an arbitrary unit, which in our particular case we chose as that concentration for which over a two-inch path, 100 log (L_o/L) is equal to unity. It will be seen that this is a very convenient unit to operate with where this type of meter is used, since in order to get the concentration (in terms of this arbitrary unit), it is necessary only to subtract log L from log L_o and multiply the difference by 100. In fact, in our particular case we kept L_o at a constant value and prepared a chart which gave us the concentration in terms of the observed L directly.

The particular form of meter used is shown diagrammatically in Fig. 5. In this, B represents a long rectangular box, with two glass sides W_1, W_2, through which passes lengthwise the smoke-laden gas to be measured. The box is made long so that the gas (and smoke) will distribute itself uniformly between the windows W_1, W_2. Perpendicularly to the axis of this box and on an optical bench is set up an optical system consisting of a light source S, a lens L_1 which renders the light from the light source parallel, after which it passes as a parallel beam through the window W_1, the smoke-laden gas, and the exit window W_2, whence a second lens L_2 reconcentrates it on a light sensitive element T (a thermopile) which in turn sends a current I (proportional to the amount of light L falling on it) through the milliammeter M.

Careful experimental investigation showed that the current developed by the thermopile T was rigorously proportional to the amount of light L falling on it, so that we can put:

$$\log (I_o/I) = \log (L_o/L) = kmx \qquad (17)$$

The setting corresponding to L_o, namely I_o, is made (with no smoke in the apparatus) by adjusting a series resistance R in the thermopile circuit. As above stated I_o is adjusted to a constant value and to facilitate the work the concentration (in arbitrary units) is read directly from a previously prepared chart.

Two such meters are used; one on the inlet and the other on the outlet side of the precipitator.

The "Smoke" Generator

Prerequisite for experiments of this type is a constant and controllable source of smoke of constant characteristics. The development of this element of our set-up was also no small task. Furthermore, it was desired to produce a smoke which would not deposit on the windows of the smoke meters and thus coat them over, since otherwise the meters would read inaccurately. As might be inferred, the production of the required smoke generator was successful only after considerable experimentation with various substances and design of generators.

Ironically, while it is fairly easy to produce smoke, it is by no means an easy task to produce a *constant* source of smoke of *constant* characteristics, *i. e.*, to produce a smoke both the quantity and quality of which are controlled, and furthermore one which will not coat the windows of the measuring apparatus.

After much experimentation the scheme finally adopted was a modified form of the one originally developed by Tolman[9] and his colleagues in their experimental study of smokes. The "smoke" strictly speaking, mist-generating apparatus is shown in Fig. 6. The material "smokefied" was lubricating oil just as in Tolman's work. The principle of operation is that oil is dropped at a regulated rate into a copper U-tube immersed in a lead bath maintained at a temperature of 480 deg. cent. by means of a Bunsen burner and thus vaporized, while a stream of air is blown simultaneously through the U-tube, this air stream recondenses the oil in the form of a fine mist and serves to carry it out of the apparatus.

The amount of oil and air fed into the apparatus is carefully controlled and duplicated by means of needle valves fitted with graduated dials. The function of the various parts of the smoke generator will be clear from the accompanying figure.

Inserted between the generator and the electric precipitator, however, is a galvanized iron tank which serves the double purpose of an equalizer or mixer and a trap for the larger droplets. This tank is provided with two valves by means of which the amount of smoke fed into the precipitator is regulated, the unused portion being allowed to escape to the outer air.

By duplicating the air pressure (read on a manometer) and the oil feed, as well as the temperature of the bath, smoke of uniform quality and quantity is produced, the exact concentration required being regulated by means of the gate valves on the mixer tank.

Furthermore, our last requirement of a smoke which will not coat over the windows is realized, because in practise the windows take on a transparent film of oil of constant thickness, the excess simply draining off. The absorption by this film, of course, is corrected for by making the I_o setting with the oil film present.

The Experimental Electric Precipitator

The precipitator used in the experiments to be described below is shown diagrammatically in Fig. 7.

It consists of a horizontal iron pipe, through which the smoke-laden gas flows as shown by the arrows, with a discharge electrode in the form of a wire extending along the axis of the pipe and supported by means of insulators (made of Pyrex cane) bent in the form shown. The wire is adjusted to center by sighting through four sets of peep sights and adjusting the insulators. The high tension current is supplied by the lead-in, which enters through a Pyrex bushing (made of Pyrex capillary tubing). The effective treater length is from the center of the inlet pipe to the center of the outlet pipe. If desired, the length of discharge electrode can be fixed exactly by slipping a tube over the wire, thus limiting the distance over which corona forms.

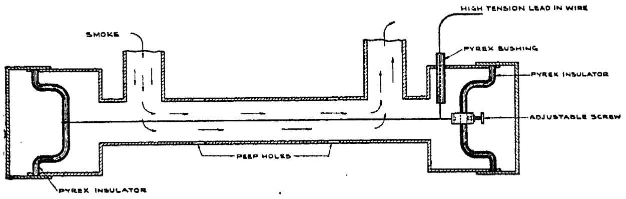


FIG. 7—SCHEMATIC DIAGRAM OF THE ELECTRIC (COTTRELL) PRECIPITATOR USED IN THE EXPERIMENTS

The precipitator is set on a slight incline so that the precipitated oil runs out of the precipitator pipe and into the end bells from which it is removed from time to time.

The High-Tension Generator

It would be desirable to use high-tension direct current for the experiments; however, since this is difficult to generate, we chose pulsating alternating current rectified by means of a kenotron. While in commercial installations mechanical rectification is employed, the mechanical rectifier with its attendant sparking complicates the phenomena by introducing considerable high frequency into the circuit. However, a more serious drawback is that with mechanical rectification a considerable and variable fraction as high as half of the secondary voltage is across the mechanical rectifier, and this fact complicates the voltage measurement. In the case of the kenotron it may be assumed that there is a negligible drop across the kenotron during the transmitting cycle, so that the secondary voltage can be calculated from the applied primary voltage or in most cases the phenomena can be expressed directly in terms of the primary voltage, to which the secondary voltage is simply proportional.

The secondary current was measured by means of a direct current milliammeter. It would have been better, of course, to use double-wave rectification, however, it is not likely that the phenomena would be essentially different, and we chose single wave rectification for the sake of economy.

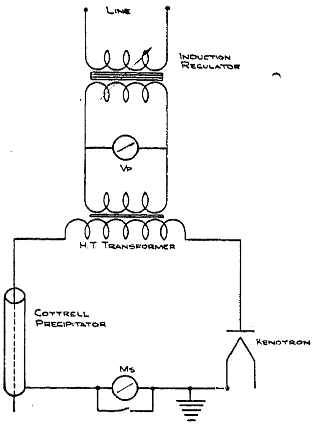


FIG. 8—DIAGRAM OF HIGH-TENSION CIRCUIT EMPLOYED IN THE ELECTRICAL PRECIPITATION EXPERIMENTS

A wiring diagram of the high tension circuit is shown in Fig. 8. The kenotron was rated at 30,000 volts, 100 milliamperes, although only about 1 milliampere was used in the experiments. The voltage applied to the primary of the high-tension transformer was regulated with an induction regulator.

The amount of gas passing through the precipitator was determined by means of an orifice meter, the differential across the orifice being measured by means of a draft gage.

The set-up consisted of (1) the smoke generator, (2) the equalizer trap, (3) the orifice meter, (4) the inlet concentration meter, (5) the precipitator, (6) the outlet concentration meter, (7) an exhauster to draw the air through the system, (8) an outlet to the stack for the waste gas; all in series.

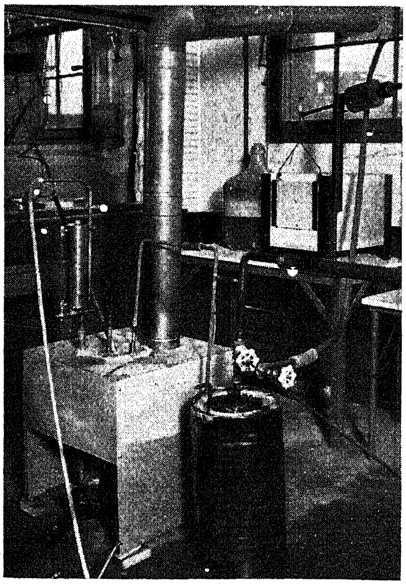
In setting up the apparatus extended use was made of the laboratory supports and supplies furnished by the Gaertner Scientific Corporation of Chicago, whom we wish to thank for the excellency and usefulness of this apparatus. In particular, for example, the concentration meters were made by mounting light source, lenses, etc., on Gaertner optical benches; the boxes of the meters, the precipitator, the orifice meters, piping, etc., were mounted on Gaertner universal laboratory supports, which were found extremely useful and convenient.

III. EXPERIMENTS ON ELECTRIC PRECIPITATION PROPER

The use of an oil mist in these experiments in place of a dry fume has three great advantages which need to be emphasized:

First, a practical difficulty encountered in precipitating dry dust, metallurgical fume, etc., is that after the material reaches the collecting electrode it does not necessarily stay there but may be blown off by the gas stream or thrown off by the electric (coulomb) force exerted on it by the charge on the collecting electrode (it is of course a well-known fact in electrostatics that an element of area of a charged surface is subjected to a repulsive force numerically equal to $4\pi\sigma$, where σ is the surface density of charge). In such cases the precipitation proper is masked by this phenomenon. By the use of a liquid such as oil, it is assured that all the material which reaches the collecting electrode is held fast there, that is to say, it is assured that one is dealing *purely* with electric precipitation.

Secondly, in the precipitation of certain dry (dielectric) materials, as soon as these deposit on the collecting electrode, the latter exhibits corona also due to the local overstressing of the field by the dielectric material. This phenomenon, known in precipitation parlance as "back corona," is obviated in our case by the use of a liquid, in particular by the use of an oil.



THE SMOKE GENERATOR

Thirdly, when dry fume is precipitated, as the process goes on, a layer of material is built up on the collecting and also on the discharge electrode, these layers, even if they do not exhibit back corona, progressively change the electrical conditions. This is obviated also by the use of oil, for the latter drains continuously away and leaves a certain equilibrium thickness of film on both current developed by the thermopile T' was rigorously proportional to the amount of light L falling on it, so that we can put:

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The setting corresponding to L_o, namely I_o, is made (with no smoke in the apparatus) by adjusting a series resistance R in the thermopile circuit. As above stated I_o is adjusted to a constant value and to facilitate the work the concentration (in arbitrary units) is read directly from a previously prepared chart.

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Inserted between the generator and the electric precipitator, however, is a galvanized iron tank which serves the double purpose of an equalizer or mixer and a trap for the larger droplets. This tank is provided with two valves by means of which the amount of smoke fed into the precipitator is regulated, the unused portion being allowed to escape to the outer air.

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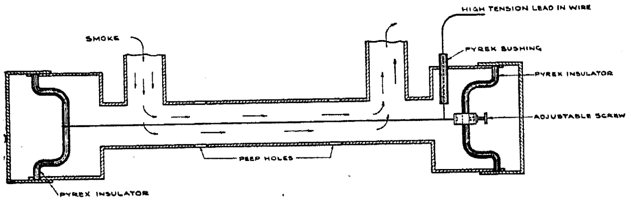

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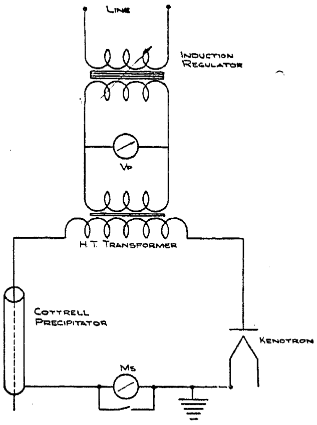


FIG. 8—DIAGRAM OF HIGH-TENSION CIRCUIT EMPLOYED IN THE ELECTRICAL PRECIPITATION EXPERIMENTS

A wiring diagram of the high tension circuit is shown in Fig. 8. The kenotron was rated at 30,000 volts, 100 milliamperes, although only about 1 milliampere was used in the experiments. The voltage applied to the primary of the high-tension transformer was regulated with an induction regulator.

The amount of gas passing through the precipitator was determined by means of an orifice meter, the differential across the orifice being measured by means of a draft gage.

The set-up consisted of (1) the smoke generator, (2) the equalizer trap, (3) the orifice meter, (4) the inlet concentration meter, (5) the precipitator, (6) the outlet concentration meter, (7) an exhauster to draw the air through the system, (8) an outlet to the stack for the waste gas; all in series.

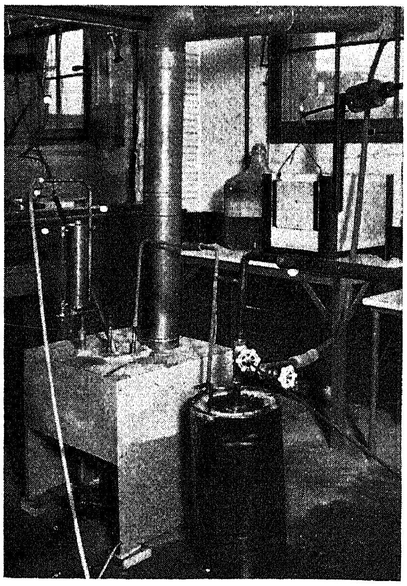
In setting up the apparatus extended use was made of the laboratory supports and supplies furnished by the Gaertner Scientific Corporation of Chicago, whom we wish to thank for the excellency and usefulness of this apparatus. In particular, for example, the concentration meters were made by mounting light source, lenses, etc., on Gaertner optical benches; the boxes of the meters, the precipitator, the orifice meters, piping, etc., were mounted on Gaertner universal laboratory supports, which were found extremely useful and convenient.

III. EXPERIMENTS ON ELECTRIC PRECIPITATION PROPER

The use of an oil mist in these experiments in place of a dry fume has three great advantages which need to be emphasized:

First, a practical difficulty encountered in precipitating dry dust, metallurgical fume, etc., is that after the material reaches the collecting electrode it does not necessarily stay there but may be blown off by the gas stream or thrown off by the electric (coulomb) force exerted on it by the charge on the collecting electrode (it is of course a well-known fact in electrostatics that an element of area of a charged surface is subjected to a repulsive force numerically equal to $4\pi\sigma$, where σ is the surface density of charge). In such cases the precipitation proper is masked by this phenomenon. By the use of a liquid such as oil, it is assured that all the material which reaches the collecting electrode is held fast there, that is to say, it is assured that one is dealing *purely* with electric precipitation.

Secondly, in the precipitation of certain dry (dielectric) materials, as soon as these deposit on the collecting electrode, the latter exhibits corona also due to the local overstressing of the field by the dielectric material. This phenomenon, known in precipitation parlance as "back corona," is obviated in our case by the use of a liquid, in particular by the use of an oil.



THE SMOKE GENERATOR

Thirdly, when dry fume is precipitated, as the process goes on, a layer of material is built up on the collecting and also on the discharge electrode, these layers, even if they do not exhibit back corona, progressively change the electrical conditions. This is obviated also by the use of oil, for the latter drains continuously away and leaves a certain equilibrium thickness of film on both electrodes, which film then remains constant. The high degree to which results could be duplicated in the experiments described below attest to the effectiveness of this step. Also, of course, the use of oil obviates having to clean out the apparatus mechanically, since the accumulated oil drains out by itself.

1. Relation between the outlet and the inlet concentrator. According to our theory above, in particular eq. (2), the ratio of the outlet to the inlet concentration, other factors being constant, should be constant; that is to say, if the inlet concentration is varied, the outlet concentration should vary in proportion.

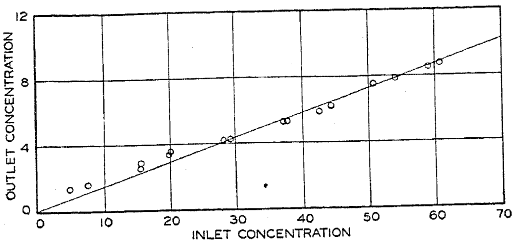


FIG. 9—RELATION OF OUTLET CONCENTRATION TO INLET CONCENTRATION IN THE EXPERIMENTAL PRECIPITATOR

In Fig. 9 the experimental results obtained in this connection have been plotted. They show that over a considerable range proportionality indeed exists between the outlet and the inlet concentration. This fact greatly simplifies further experimentation, since except in special investigations the subsequent experiments can all be carried out at a single conveniently chosen concentration.

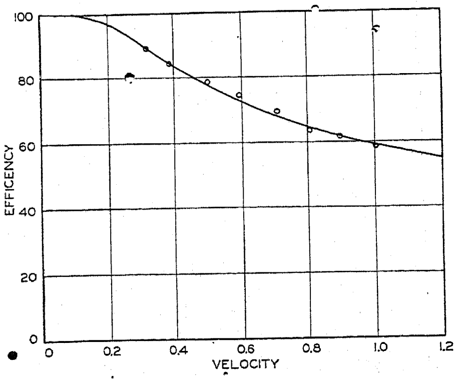


FIG. 10—EXPERIMENTAL EFFICIENCY-VELOCITY CURVE. COMPARE WITH THE THEORETICAL CURVE OF FIG. 2

In particular, also, this experiment establishes the uniqueness of the quantities ξ and $(1 - \xi)$ or η; and justifies the use of the term "precipitation efficiency," since this is now experimentally proved to be independent of the concentration.

2. Relation between the precipitation efficiency and the treatment time. Since the treatment time is difficult of direct measurement and in practise is directly proportional to the length of pass and inversely proportional to the velocity, we shall discuss the phenomenon in terms of the latter two quantities, (see below).

3. Relation between the precipitation efficiency and the length of pass. While strictly speaking, in order to determine the relation between the precipitation efficiency and the length of pass, the length of the precipitator tube itself would have to be varied, in a *tube* treater the effective length of the treater can be varied also by varying the effective length of the discharge electrode. This can be done with the apparatus of Fig. 7 by slipping a tube over a definite section of the discharge electrode, thus cutting down the corona. For convenience, however, in place of a tube, slotted aluminum cylinders (large size aluminum wire) is slipped over the discharge electrode in sections. With a sufficiently large wire covering the discharge electrode the corona can be limited to the uncovered part of the latter. We found, however, that with the particular size of wire we used, there was still some corona coming off the aluminum sections, so that the results have to be interpreted in a slightly modified way, for the precipitator really consists of two precipitators with different sizes of discharge electrodes in series. In order to evaluate this case it is necessary to proceed as follows:

Consider (Fig. 11) a precipitator tube of total length D, of which a section of discharge electrode of length x is of one diameter, while the remainder of length $(D - x)$ is of another. Let q_o denote the concentration entering the first section; q_1, that leaving the first and entering the second; and q_2 that leaving the second. Then we should have the relations:

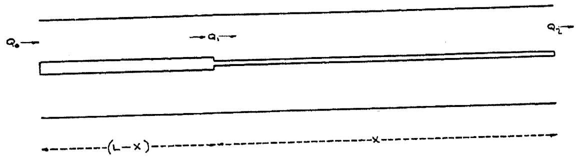


FIG. 11—METHOD OF LIMITING EFFECTIVE LENGTH OF DISCHARGE ELECTRODE

$$q_1/q_o = e^{-\alpha_1(D-x)} \quad \text{and} \quad q_2/q_1 = e^{-\alpha_2 x} \tag{18}$$

from which by direct multiplication we obtain:

$$q_2/q_o = e^{-\alpha_1(D-x)-\alpha_2 x} \tag{19}$$

or taking logarithms:

$$\log (q_2/q_o) = - \alpha_1 D - (\alpha_2 - \alpha_1) x \tag{20}$$

Since all quantities except x on the right hand side of (20) are constant we can put:

$$\log (q_2/q_o) = c_1 - c_2 x$$

In other words for a precipitator of this type, provided the inlet concentration is held constant, the logarithms of the observed $(1 - \eta)$'s when plotted against the corresponding exposed lengths of discharge electrode should fall on a straight line. The slope of this line furthermore represents logarithm of the ratio of the concentration leaving a section of unit length of the precipitator to that entering it.

The experiment is then to hold the inlet concentration constant and vary the exposed length of discharge electrode by slipping sections of (slotted) larger wire over it. In particular, 4-in. lengths of aluminum wire 0.125 in. in diameter were slipped over a discharge electrode 0.023 in. in diameter.

The experimental results obtained are exhibited in Figs. 12 and 13. In the first of these the logarithm of $(1 - \eta)$ is plotted against the exposed length of discharge electrode. It is seen that except for very short lengths of discharge electrode, the logarithm of $(1 - \eta)$ under these conditions is indeed a linear function of the length of discharge electrode as demanded by eq. (**21**), which it will be noted is a direct consequence of the fundamental law expressed by eq. (**2**). In order to exhibit these results in a way more in accordance with Fig. 1, in Fig. 13 the observed efficiencies have been plotted directly against the exposed length of discharge electrode, the circles representing the experimental results and the full line the theoretical exponential curve which best fits the data. It will be seen that the agreement is very close except for very short lengths of discharge electrode, hence we can conclude that the *precipitation efficiency is an exponential function of the length of pass* as demanded by eqs. (**7**), (**8**) and (**9**).

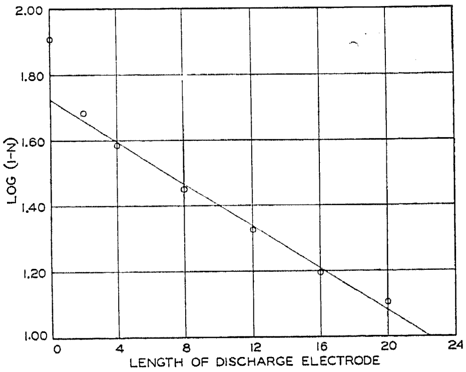


FIG. 12—TEST OF WHETHER THE EXPERIMENTAL EFFICIENCY-LENGTH OF PASS CURVE IS AN EXPONENTIAL

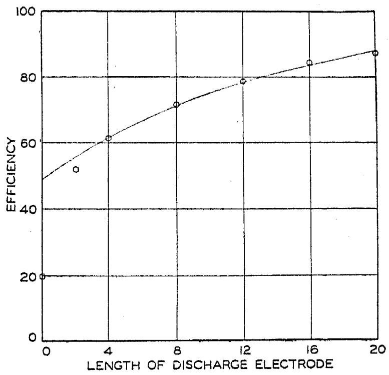


FIG. 13—PRECIPITATION EFFICIENCY AS A FUNCTION OF LENGTH OF PASS (LENGTH OF DISCHARGE ELECTRODE) IN EXPERIMENTAL PRECIPITATOR

Circles denote experimental points; full line the theoretical exponential curve

4. Relation between precipitation efficiency and the velocity of the gas in the treater. Since the exact velocity is not of great interest but merely the law of variation of the efficiency with the velocity, for convenience in the curves in place of the velocity the square roots of the pressure differentials across an orifice plate inserted ahead of the experimental precipitator have been plotted. The square root of this differential is of course proportional to the velocity. According to theory the efficiency-velocity curve should have the same general form as the curve exhibited in Fig. 2. In Fig. 10 the experimental values obtained have been entered as the small circles, while the full drawn curve represents the theoretical exponential curve which best fits the data. It is seen that the agreement here between theory and experiment is very perfect.

5. Relation between precipitation efficiency and the secondary current. It will be noted that our fundamental relation, namely eq. (2), involves the quantity i, which was defined as the concentration of the ions. This quantity can be considered under a given set of conditions (ionic velocity) as proportional to the space current in the precipitator. If the latter does not vary from point to point *lengthwise* of the precipitator we may take i proportional to the (total) secondary current, in which case the precipitation efficiency would be also an exponential function of the secondary current. Unfortunately, however, it is not possible directly to vary the secondary current in a precipitator without simultaneously varying the voltage, hence we must be satisfied with the determination of the relation between the precipitation efficiency and the secondary current *with the voltage also varying*. The exponential curve obtained in this connection is exhibited in Fig. 14. It will be seen that it does resemble an exponential (compare Fig. 1), although careful analysis shows that it is not a pure curve of this type.

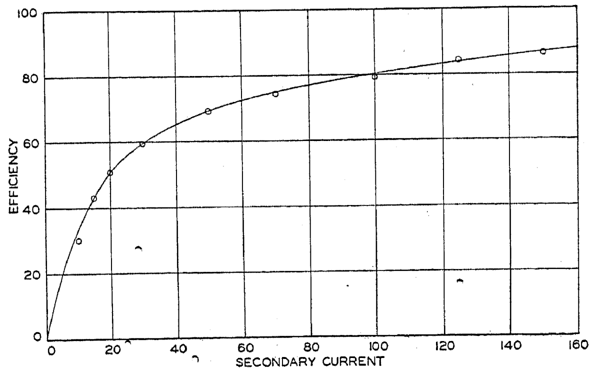


FIG. 14—PRECIPITATION EFFICIENCY AS A FUNCTION OF THE SECONDARY CURRENT (CURRENT VARIED BY CHANGING APPLIED VOLTAGE)

It was noted, however, in taking these points that the secondary current itself, and to some extent the secondary voltage, also, was affected by the *concentration* of the smoke entering the precipitator; it was noted, for example, that the admission of smoke to the precipitator always resulted in a drop in the secondary current, and that this drop increased with the concentration of the smoke admitted. This led to a special study of the secondary *current* (smoke) *concentration* relation now to be described.

6. *Relation between the secondary current and the (inlet) concentration of smoke.* The phenomenon mentioned just above was studied quantitatively. For this purpose the inlet concentration of the smoke was varied in definite steps and the corresponding secondary currents (primary voltage being constant) were noted.

The experimental results are exhibited in Fig. 15. Here again the experimental points are entered as small circles while the theoretical exponential curve which best fits the data is drawn as the full line. It is seen that the fit is fairly good, but still not perfectly exact, pointing to some disturbing factor.

The explanation of this phenomenon is simple. The admission of smoke results in the formation in place of the very mobile gas ions present in the clear air, of the much less mobile and slower moving charged oil drops with a corresponding decrease in the speed of transfer of charge between electrodes, *i. e.*, reduction in current. Paradoxically then in practise a precipitator actually takes less current when under load than it does when idle; however, this is explained by the fact that when no suspended material is present the energy is wasted in carrying gas ions across the intervening space between electrodes against the frictional resistance of the gas.

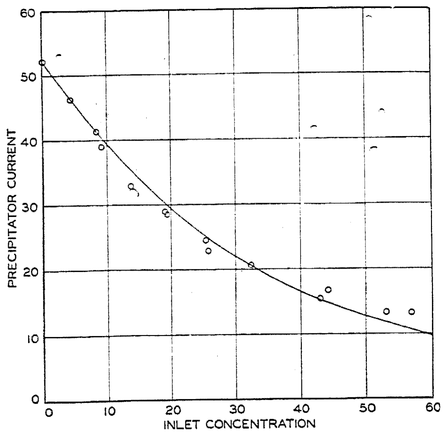


FIG. 15—SECONDARY CURRENT AS A FUNCTION OF THE CONCENTRATION OF THE SMOKE ENTERING THE PRECIPITATOR

Small circles represent experimental points; the full line a theoretical exponential curve

Incidentally, of course, the question also arises whether the secondary voltage is modified by this phenomenon. We tested also this point by connecting a Braun electrostatic voltmeter across the precipitator, and it was noted that there was an *increase* in secondary voltage simultaneously with the *decrease* in the secondary current as might be expected.

7. *Relation between secondary current and primary voltage (corona characteristic).* The relation between the current and voltage for the case of concentric cylinders for air is of course well known. However, in view of the results of the previous sections it becomes of interest to examine how the corona characteristic of such a system is modified by the presence of a dispersoid. Accordingly, characteristic curves of the particular precipitator were obtained with clean air and smoke-laden air in the precipitator.

The results are exhibited in Fig. 16. In this figure the upper curve 1 represents the corona characteristic for clean air or gas; the other three the same for various definite concentrations of smoke- or mist-laden air. The latter were obtained by keeping the inlet concentration constant for each curve but assigning to it the successive values of 55, 25, and 8 arbitrary units (of concentration). These lower curves then represent the corona characteristics for a precipitator under various working conditions. Of course, in view of the results of the previous section, the amount by which the current is depressed will depend on the concentration of the smoke entering the precipitator. It is of importance to note that the arcover point is not affected appreciably.

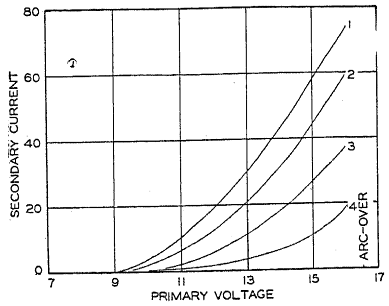


FIG. 16—CORONA CHARACTERISTIC OF CLEAR AND SMOKE-LADEN AIR

Curve 1 for clear air, lower curves for successively increasing concentrations of smoke

8. *Relation between precipitation efficiency and primary voltage.* Unfortunately, as in the case of the current curves, the effect of the voltage (or more fundamentally the electric field within the precipitator) cannot be determined directly, since a change of the voltage involves a change of the current also as already noted in a previous section. It is of interest nevertheless to determine how the precipitation efficiency varies directly with the applied voltage.

The results of the experiment are exhibited in Fig. 17. It will be noted that the curve also partakes of the general nature of an exponential. It will be seen that no precipitation occurs until the voltage reaches about 9.3 volts. This is explained by the fact that for the particular wire used the corona current does not begin (Fig. 16) until this voltage is reached. The curve then rises almost linearly, but eventually bends over much like the curve of Fig. 1, and ends abruptly at the arcover voltage. This curve is very similar to that obtained by Horne. However, it must be emphasized in connection with this curve that the relation is not a fundamental relation because the current is also allowed to vary (compare section 11).

9. Relation between precipitation efficiency and power input. Since it is not possible to vary current and voltage independently in a given precipitator, it is of interest to study the precipitation efficiency as a function of the power input. For this purpose, while this procedure is not rigorously correct, we formed the product of the primary voltage and the secondary current and took the result as proportional to the power input.

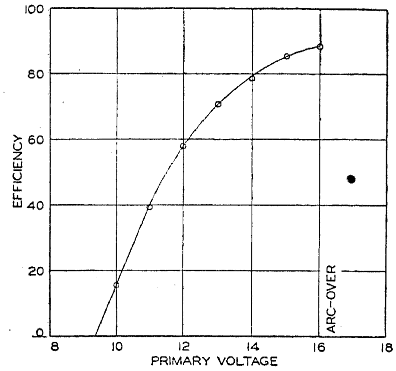


Fig. 17—Precipitation Efficiency as a Function of the Applied Primary Voltage (Current Also Varying)

The experimental results are plotted in Fig. 18. The curve is very similar to the efficiency-current curve. It will be seen that here also as in the case of the efficiency-pass curve, the cleaner the gas becomes the harder it is to clean it further, *i. e.*, the more energy it takes to remove the residual.

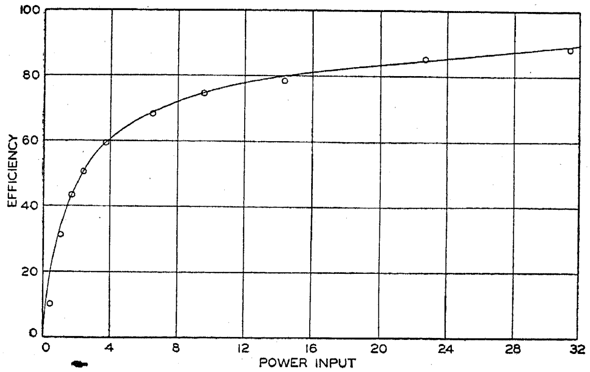


Fig. 18—Precipitation Efficiency as a Function of Power Input into the Precipitator

10. Effect of diameter of discharge electrode on the efficiency-voltage relation. Of great interest in connection with the design of a Cottrell treater is the effect of the diameter of the discharge electrode on the precipitation efficiency. Firstly, of course, it is well-known how the wire diameter affects the corona characteristic, in particular, that with larger wire sizes the same current is obtained at a higher voltage than with a smaller wire. The first effect is then to shift the corona characteristic, that is to change the operating voltage. Secondly, it is also well known that the size of the discharge electrode affects the magnitude of the electric field at any point within the treater tube; in particular for the same applied voltage the field in the vicinity of the collecting electrode and in the greater portion of the treater tube will be increased by using larger wires. Both effects combined tend to increase the electric field within the treater (ignoring, of course, the zone immediately around the discharge electrode). Hence by changing the wire size we have a method of changing the electric field within the treater; in particular of answering the question whether a high field and low space current or the reverse is desirable. In this and the following sections this point is studied in detail.

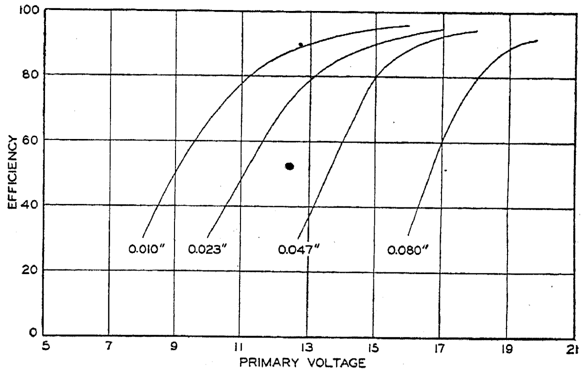


Fig. 19—Efficiency-Voltage Curves for Various Diameters of Discharge Electrode

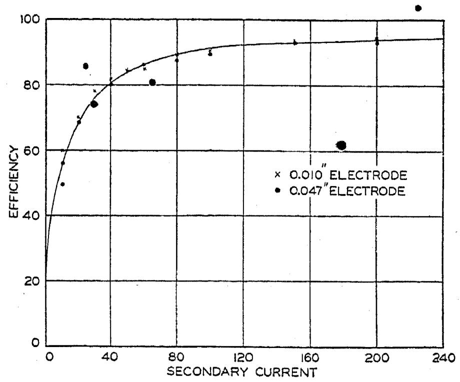


Fig. 20—Efficiency-Current Curves for Two Diameters of Discharge Electrode

Firstly, let us take up the effect on the efficiency-voltage curve as exhibited in Fig. 17. The experimental results for four wire sizes, 0.010-in., 0.023-in., 0.047-in. and 0.081-in. are exhibited in Fig. 19. It will be seen that all of these curves are of the same general shape and reach approximately the same peak value of precipitation efficiency. Hence from this angle *there is apparently little to be gained by changing the wire size, the result is only to shift the phenomenon to a different voltage range.* This indicates that the voltage *insofar as it determines the electric field within the treater volume is of little importance*—a highly interesting result—all the more so when it is considered that the voltage did not enter into our fundamental deductions. It would seem then that the voltage enters in only as it determines the current flow, and we shall next study the phenomenon from this angle.

11. Effect of size of discharge electrode on the efficiency-current relation. The rather remarkable result obtained in the previous section suggests plotting the efficiency for various wire diameters in terms of the (secondary) current flowing in the treater. The experimental results for two wire sizes, namely 0.010-in. and 0.017-in. are plotted in Fig. 20. They show that *the current alone is the determining factor;* that is to say, if the current flow is duplicated (with different sizes of discharge electrode) the efficiency is duplicated—a very remarkable and important result, particularly as regards the theoretical interpretation of the Cottrell process. It tends to support the theory advanced by Hauer[10] and Deutsch.[11] that the charging of the smoke particles has nothing to do with the strength of the electric field, *i. e.*, is not an influence effect, but is due solely to the charges imparted to the smoke particles by the kinetic agitation of the gas molecules. Put in another way, the whole trick of electric precipitation consists merely in charging the dust particles. The only effect of the electric field is subsequently to sweep the charged particles out of the space between electrodes, and for this purpose its actual magnitude between limits, of course, is not of great importance. The result also supports our theoretical deductions, since our fundamental assumption was that the whole process could be treated on the basis of chance encounters between ions and smoke particles, and that this chance of encounter was not influenced by the electric field.

IV. SUMMARY

On the basis that the chance of a dust particle being precipitated depends on its chance of encountering an ion, a formula is deduced giving the precipitation efficiency (percentage removal) as a function of the length of time the gas remains in the treater. As a corollary the precipitation efficiency is deduced as a function of the length of the treater and the gas velocity. As further corollaries the resultant precipitation efficiency of a number of equal Cottrell passes (or separate treating units) is deduced to be independent of their manner of connection as regards gas flow.

An optical method of measuring the concentration of suspended material in smokes and mists, a method of generating smoke of constant and controllable quality and quantity as well as a laboratory Cottrell precipitator for studying experimentally the fundamental laws involved in electrical precipitation are described.

The relation between the concentration leaving a Cottrell precipitator, other conditions being constant, is shown experimentally to be a constant fraction of the concentration entering the precipitator. The precipitation efficiency (percentage removal) as a function of the length of pass, the gas velocity, the electric current, the voltage, and the power input is determined experimentally. It is demonstrated that these curves partake of the nature of exponentials. The current flowing in the precipitator is shown to be a function of the incoming concentration, decreasing with it according to an exponential law. Corona characteristics (current-voltage curves) of a smoke-laden gas are determined and shown to be different from that obtained in clean air or gas. It is shown that the voltage insofar as it determines the electric field within the treater has little effect on the cleaning efficiency, this holds also for the size of discharge electrode. The voltage and size of discharge electrode are only of importance in so far as they determine the electric current, which is the fundamental variable. With different sizes of discharge electrode it is necessary to duplicate only the current in order to duplicate the precipitation efficiency. From these results it is indicated that the charging of the dust or smoke particles, not the sweeping of the particles out of the field is fundamental in determining the efficacy of the process, or, the performance of a particular treater under a given set of conditions.

V. CONCLUSION

While the present theoretical and experimental study is more extensive perhaps than any that has been made before, it is still far from complete; and it is hoped that the experiments described and experimental method developed will stimulate further research in this field.

In conclusion we wish to thank Mr. G. G. Crawford, at whose suggestion this work was undertaken, for his interest and inspiration, and Mr. H. C. Ryding for the generous support he has given us in this work. We also wish to acknowledge our indebtedness to Mr. H. Raymond for many contributions in the design and construction of the apparatus used in these experiments.

References

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2. *Iron and Steel Eng.*, **6**, p. 143, (1929).
3. TRANS. A.I.E.E., **41**, p. 805, (1922).
4. TRANS. A.I.E.E., **41**, p. 811, (1922).
5. *Zt. tech. Physik*, **6**, p. 433, (1925).
6. *Trans. Am. Inst. Chem. Eng.*, **15**, p. 85, (1925).
7. In the following we shall use the term "precipitation efficiency" in a special technical sense, namely as meaning the fractional part or percentage of incoming material removed, *i. e.*, precipitated, in the treater. Both theory and experiment justify the use of this term, since the percentage removed or "precipitation efficiency" is independent of the incoming concentration.
8. See paper read before the American Iron & Steel Institute Meeting of October 24, 1930; also "Blast Furnace & Steel Plant," November 15, 1930.
9. *Jl. Am. Chem. Soc.*, **41**, p. 307, (1919).
10. *Ann. d. Phys.* **61**, p. 303, (1920).
11. *Loc. cit.*[1]

Discussion

J. C. Hale: The collected data covering work done under the auspices of Research Corporation, Western Precipitation Company, Lurgi Apparatebau-Gesellschaft, Siemens-Schuckertwerke and Lodge-Cottrell, Ltd. have been freely interchanged and it is from all these that I am drawing the following information.

As the authors have pointed out, there has not been a great amount of data presented through the engineering societies on the quantitative laws governing precipitation. One reason is that few outside of those in the development and engineering departments of the companies engaged in the application of the processes in the various industries are interested in this phase of the problems of gas cleaning. Of the large volume of data which has been collected and exchanged by the various investigators, usually only that portion has been published which has a direct bearing on some particular application.

Referring to the authors' mathematical analysis, equations (**4**), (**5**), (**7**), (**8**) and (**9**) are, as the authors state, various forms of the same expression. Consider equation (**4**); e is the base of the naperian logarithm and α is a constant. I may, therefore, select a new constant K equal $e^{-\alpha}$ and rewrite this equation as

$$n = 1 - K^t \text{ or } 1 - n = K^t$$

With this simple substitution, we have the identical equation developed and reported by Anderson and his co-workers and presented in his 1919 report of tests made during that year at an experimental plant at Santa Cruz.

Equivalent formulas have been arrived at and published by European investigators, notably Heinrick and Deutsch, Engineers of the Precipitation Companies in Germany. This equation has been a most valuable contribution to the art of precipitation and it has been very extensively used. It must not, however, be regarded as inexorable. The fact that the "constant" of this equation is not always constant has served first, to stimulate the attention given by engineers in an attempt properly to evaluate it and second, to retard the publication of the results so far obtained until these results are more complete. It might be mentioned that K has been shown to vary considerably with the gas velocity. Maximum and minimum determinations of K under different conditions have shown well established variations of the order of 2 to 1. In the hands of one thoroughly familiar with its limitations, the formula is nevertheless a valuable working tool.

The mathematical analysis leading to equation (**15**) shows that identical efficiencies will obtain whether precipitator units be connected in series or in parallel or in series parallel with respect to gas flow provided the gas is properly divided in the cases of unsymmetrical arrangements. This conclusion serves, however, as a good example of the pitfalls to which one is led by the application of scientific principles to a field which is at least as much an art as it is a science. For example, in a problem requiring a very low friction loss, equation (**15**) points to the use of a large number of units in parallel. It can be shown mathematically by a somewhat involved calculation, that the combined efficiency of parallel units decreases rapidly as the gas distribution becomes unequal. Equal distribution of gas through too many parallel units is most difficult and the selection of a proper number of units to be paralleled is, therefore, not a matter of efficiency calculated on the basis of any formula, but a matter of experience with regard to distribution. Similarly, to go to the other extreme, we might be led to a design with a large number of units connected in series, to eliminate the difficulties of equalizing gas distribution. This, however, would involve a high gas velocity and material precipitated in the first of a series of units might be mechanically swept along the surface of the collecting electrode of that unit, picked up by the gas stream and carried to the following units and ultimately to the outlet main. This contingency is not considered in any formula for efficiency, but serves to indicate one of the many dangers to which scientific formulas may lead unless they are made subservient to the many requirements of different conditions met with in practise,—again pointing to precipitation as an art more than a science.

As the authors point out, the constant which appears in the formula applying to smoke concentration meters is dependent upon the grain size of the smoke particles. In spite of this statement, however, efficiency is determined in all of the tests without regard to this factor. That it is not an insignificant factor is indicated in the paper by Tolman and others, referred to by the authors. It is there stated that the light beam strength varies with the concentration of dust between certain limiting formulas in which, for small particles, the particle diameter appears directly as the cube and for large particles inversely as the first power. The Tyndall beam instrument gives, fundamentally, a measurement of visibility which is proportional to concentration only if the size of particles is a constant. A user of a precipitation installation will seldom be satisfied with an efficiency determination based upon visibility, as he requires the removal of the suspended matter from the gas stream and not merely its conversion into larger particles.

This characteristic of the concentration meter used will account for some of the errors which I believe exist in the authors' results as given in sections 8, 9, 10 and 11.

Admittedly, the filter method of measurement of concentrations gives integrated values over a period of time and is a slow, tedious process. It is our experience that it is the most accurate method and the only one acceptable as a true measurement not requiring calibration and independent of variables, such as size of particles, which influence other apparatus. Even this method has, of course, certain practical limitations in its commercial application, especially where there may be very sudden changes in gas velocity.

The smoke generator used is certainly a most ingenious and carefully worked out device. I most heartily agree with the authors' comments on the difficulty of producing a controlled quantity and quality of smoke for experimental use. That the smoke produced may vary as to size of particle is pointed out by Tolman in the paper previously mentioned.

In the absence of conclusive information as to the relative merits of half wave and full wave rectification, of locating the vacuum tube in the grounded side of the circuit or in the high potential side, and of using a mechanical rectifier, I see no reasonable objection to the use of any of these methods. The important point is that the same equipment and the same methods of measurement of electrical values be used throughout.

There are three of the authors' conclusions which lead to a difficulty if they are expressed mathematically and then compared.

Equation (3) may be written $\eta = f(i)$.

The statements in section 6 and the curve Fig. 15 may be expressed $i = f_1(c)$.

By combining these we may write $\eta = f_2(c)$.

The authors' statement in italics at the bottom of page 451 may be expressed mathematically as "η is not any function of c."

The incompatibility of these equations indicates the necessity of some explanation or qualifications of the authors' conclusions upon which they are based.

I do not believe that any general conclusion should be drawn or simple formulas of relationship established between efficiency and current voltage or power input as given by the authors in Figs. 14, 17 and 18. While these are characteristic curves applicable to some problems, there are marked exceptions which are both too numerous and too conclusive to be ignored. Many well confirmed tests on commercial installations prove that under some conditions increasing the current and power does not give any increase in efficiency.

The conclusions based on the data represented by Figs. 19 and 20 are misleading in that two important factors entering into the results exert an entirely different effect on the results obtained by the experimental equipment used and results obtainable by equipment of commercial dimensions. These factors are the limitations of the smoke concentration meter already referred to and the relatively small diameter of the pipe used.

The authors state that "the whole trick of electrical precipitation consists in merely charging the dust particles." This reduces the whole problem to the simple question of selecting an electrode assembly to give the maximum corona discharge, which is certainly a question capable of accurate scientific solution. We know, however, based upon a very wide experience in dealing with a variety of problems, that it is out of the question to draw any such an ideally simple conclusion. We have learned the futility of attempting to disregard the wide variations in other conditions. It is the existence of an almost infinite number of combinations of these variable conditions which so far removes the problem from the range of an exact science as to lead those most intimately connected with it to prefer the designation "art of electrical precipitation."

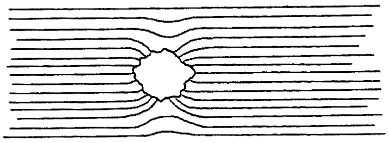

Fig. 1

Electrical precipitation has been under study and development both here and abroad for over twenty years, and from my nineteen years experience I feel that in justice to those scientists and engineers, who have during that time devoted so much research and effort to the art, the part of the authors' conclusion that refers to the possible scope of the work which has been done heretofore should be sincerely questioned.

J. J. Torok: In making some studies on precipitation some time ago I reviewed a treatise on this subject by Ladenburg which was published in the *Annalen der Physik*, 1930, Vol. 4, page 863. Herein Ladenburg presents a very interesting picture of the process of electrostatic precipitation. I believe that it would be worth while to review this at this time for it attacks the problem from a different standpoint than that taken by Messrs. Simon and Kron.

Ladenburg considers the smoke or dust as a minute particle floating in the atmosphere. This particle he assumes to have various diameters. The irregular shapes of such particles can effectively be represented as spherical. In an electrostatic field through which a large number of ions are moving the behavior of this particle will be as follows:

First assume that the particle has a dielectric constant greater than one. Under this condition the electrostatic field around the particle will be distorted. Lines of force now tend to concentrate into the particle as shown in Fig. 1 of this discussion. The ions which are moving in this field will follow very closely the direction of the lines of force of the electrostatic field, consequently they will strike the dust particles where they will be retained. Thus the dust particle acquires a charge. This charge increases until the field around the particle is distorted by the charge so that instead of the lines of force converging on the particle they now swing around it and the other ions in passing through the field will not strike the particle. In other words, the charges acquired by the particle in a particular field will increase until they repel the oncoming charges and thus it will come to an equilibrium. It can readily be seen from Fig. 2 that the larger the particle the greater the distortion it will cause in the field and consequently the larger charge it will acquire. The forces acting upon the particle, however, are proportional to the charges it acquires. Consequently, the larger particles which have acquired the greater charge will be removed from the field first. The smaller particles which have acquired less charge will require more time to be forced across the field and as a result their time of precipitation is somewhat longer.

If we assume that the dust particle has a dielectric constant equal to air there is no concentration of the lines of force into the particle, thus only the ions in whose path the dust particle lies will be entrapped upon the dust particle. This perhaps will require a little longer period of time for the particles to acquire a charge which will distort the field so that no further ions will be entrapped upon it. It is to be remembered also that under this condition the charges entrapped on the particle are less than if the particle had a higher dielectric constant.

As the size of the particle diminishes, the charge acquired by it becomes so small that the time of precipitation becomes very long. However, these smaller particles are precipitated in a period of time much less than required from a theoretical standpoint. This discrepancy is accounted for by the electric wind effect. The turbulent motion set up within the precipitator by the corona causes the air to move rapidly from one electrode to another. In this turbulent action the small particles are from time to time brought very close to the electrode and with the small motion it can obtain through its electrostatic forces it will be pulled to the plate and be deposited there.

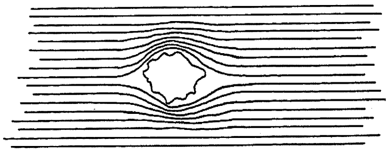

Fig. 2

From this analysis we find that in precipitators the larger particles are removed first and as time progresses the smaller particles will be taken out.

Commutation Considered as a Switching Phenomenon

BY R. E. HELLMUND* and L. R. LUDWIG†
Fellow, A.I.E.E. Associate, A.I.E.E.

THE usual commutation theories deal with the reversal of the current in the commutating coil and with means for bringing about such reversal either along a straight line or a curve. The principal object in such studies is to accomplish commutation without excessive current densities under the brush. It is recognized that frequently it is impossible to realize this, but little attention has been paid to the fact that unless such ideal conditions are obtained we actually have to deal with a switching phenomenon and that the problem should be analyzed with this in mind. The necessity for introducing the concept of switching is important inasmuch as ideal conditions can be obtained in but very few practical cases. This paper gives a brief analysis of the various considerations entering into the switching phenomena at both the leading and the trailing edges of the brush and indicates how commutator and brush burning is affected by aiding or hindering the switching operations.

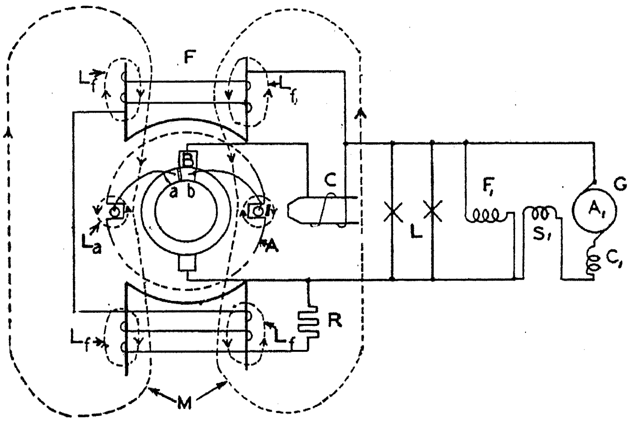

FIG. 1.

Let us assume that the circuit of armature coil 1 in Fig. 1 is about to be short-circuited by brush B and that at this moment a voltage e is induced in the coil by its rotation in the commutating field, or possibly by the stator field of an a-c. machine. It is evident that sparking in the minute point or edge of the brush B which first touches the segment a will be largely influenced by the speed with which the circulating current in the coil can build up as compared with the rate of increase of the area common to the brush and the segment a. This in turn means that the speed of the commutator will have an influence as well as the rate of increase of the circulating current. The latter is determined by the voltage e and the self-inductance and resistance of the circuit of coil 1, if the coil carries no flux common to other closed circuits. This condition would exist if brush B does not short-circuit any other armature coil and if the field winding F is open-circuited, and under the further assumption that there are no secondary damping currents in the stator structure. Under these conditions, the entire main flux F and the leakage fluxes L_a cause a rather high self-inductance in the armature coil 1, and consequently a slow building up of the circulating current. Fig. 2 represents a circuit equivalent to the armature coil 1 with the assumptions made so far.

The conditions change materially if the shunt winding F is connected across the armature circuit as shown in Fig. 1, as well as across certain load circuits L and various circuits of the generator G. We now have within the armature coil a leakage reactance flux L_s and a mutual inductance M between the armature coil and the field circuit. The latter in turn has a leakage flux L_f, and it is closed partly through the armature circuit having resistance and inductance, and partly through several other circuits also having a resistance or inductance, or both; in other words, we have equivalent circuits somewhat as illustrated in Fig. 3. The rate of change in the circulating current will now depend upon all these factors, and therefore any change of constants in the field circuit or any of the other circuits may have a certain influence upon the commutating conditions. Similarly, any damping currents which can be induced in the solid portion of the stator yoke, or in coil shield or damper windings surrounding the poles, may have an influence upon commutation. The various resistances and reactances shown in the secondary circuit of Fig. 3 are replaced by a single inductance X_2 and a single resistance R_2 in Fig. 4, which while not fully equivalent to Fig. 1 somewhat simplifies the problem.

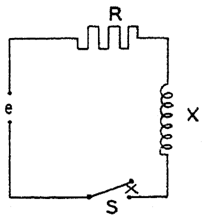

FIG. 2

All the various constants previously mentioned will also have an influence when the circuit of the coil 1 is opened while a circulating current is still flowing in it. There will be one difference, however, in that the high leakage and self-inductances, which are helpful in the case of closing the circuit, will be harmful in the opening because they considerably increase the energy to be dissipated in the spark which forms at the trailing edge of the brush. Mutual inductance, which aids in opening the circuit as will be explained, is disadvantageous when it is closed because it allows a rapid building up of current. The effect of inductance on the opening operation may be made clear by following the switching operation in detail first for a machine with open stator circuits, wound with one coil per slot, and a brush spanning one commutator bar, so that no account need be taken of mutual inductance between a commutating coil and any other closed winding. A circulating current is assumed to be flowing when coil 1 passes from under the brush.

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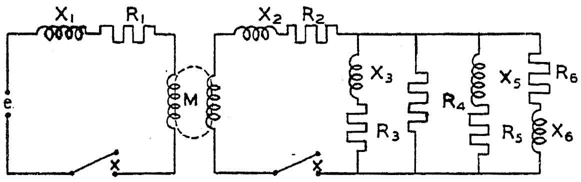

Fig. 3

When a circuit containing resistance and self-inductance such as shown in Fig. 2 is opened by the separation of contacts (the brush and the bar in this case), an arc or spark will be formed between the electrodes, because at the last point of contact the temperature may become very high and "explosions" take place at this point, giving rise to sufficient energy density to establish the cathode spot of an arc on the negative electrode.[1] As the contacts separate further the arc becomes longer and finally becomes unstable and vanishes. Any calculation of the decrease to zero of the circulating current due to commutator bars passing from under the brush must consider this short arc or spark formed. The arc characteristic, *i. e.*, the arc voltage as a function of the current through it, depends on the arc length. The latter in turn is varying and depends on the way in which the arc is lengthened as a function of time, or, in other words, on the speed of the commutator.

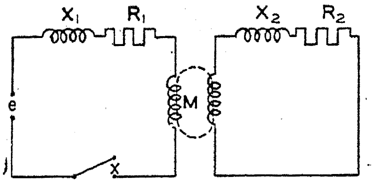

Fig. 4

If the transient arc characteristic is accurately known for a particular condition, the energy dissipated in the arc can be calculated.[2] This energy may be divided into two parts; the energy stored electromagnetically in the circuit, and the energy supplied to the circuit during the time duration of the arc. In the case of the commutating coil, the latter energy may be supplied by the voltage induced in the coil by its rotation in the commutating field, or in an a-c. machine it can also be induced by transformer action between the stator field and the coil. Usually the total arc energy will lie between the values of once and twice the energy stored in the magnetic circuit. This stored energy varies directly with the self-inductance coefficient L and the square of the current to be broken, and hence the arc energy also depends upon these factors. The length of time during which the arc or spark is present also increases with an increased coefficient of self-inductance. Since a trailing edge spark is harmful in so far as it burns the commutator and brushes, it is desirable to keep the energy to be dissipated in the spark as low as possible. This can be done by lowering the self-inductance, reducing the current to be interrupted, and also by the proper utilization of mutual inductance, which will be discussed next.

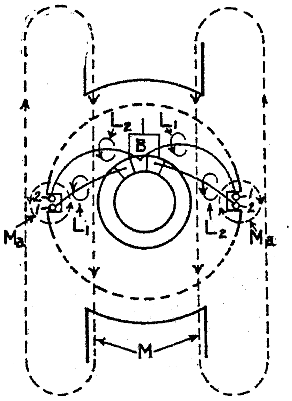

Fig. 5

As previously indicated, circuits having mutual inductance with a commutating coil are established by the field or stator circuit being closed through the armature and other circuits, or by damping currents in the stator structure. Other circuits having mutual inductance with the commutating coils usually exist if the winding is chorded, if there is more than one coil per slot, or if a wide brush is used; or, in fact, whenever the sides of at least two coils which are being commutated during a common time interval lie in the same slot. In this case the mutual inductance between coils must be considered. For example, if the brush is wide as shown in Fig. 5 and short-circuits the two armature coils 1 and 2, coil 1, which is about to be open-circuited, still has leakage fluxes L_1, but it also has mutual fluxes M with the field winding as previously described, and mutual fluxes M_a with the armature coil 2, the latter in turn having leakage fluxes L_2 of its own. This may better be

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[1] Slepian, *Temperature of a Contact*, Jour. of A.I.E.E., 1926, p. 930.

[2] Rudenberg, "Elektrische Schaltvorgange," p. 188. (Julius Springer, 1923.)

shown by the simplified although not fully equivalent circuit indicated in Fig. 6. If such a circuit containing mutual inductance in addition to self-inductance and resistance is opened with a switch, the decrease of the current in the primary circuit may again be calculated if the transient arc characteristic curve is known. It is found that in this case all of the stored energy need not be dissipated in the switch. The decrease of current in the primary circuit will induce voltages in the secondary circuits such as to cause their currents to change. This means that part of the energy stored in the primary circuit may be transferred to the secondary circuits and dissipated as I^2R loss in them or held in storage in the secondary circuits. Consequently, the energy dissipated in the spark may be, and usually is, reduced by introducing mutual inductions. The magnitude of the reduction depends on the degree of mutual coupling between the circuits, and upon the circuit constants, such as resistance and self-inductance, of all the circuits having mutual inductance with the commutating coil. It follows, therefore, that as conditions in the stator circuits change, the commutating condition may also change, although such factors as load current, speed, and commutating fields, which are usually considered as the only ones determining the commutating conditions, remain unchanged. For similar reasons, different conditions of mutual inductance between the various armature coils will have an appreciable influence. The conditions of the mutual flux M_a and leakage fluxes L_1 and L_2 in Fig. 6 will of course be appreciably different when the coils are located in separate slots and when the coils are located in the same slot. This difference leads to the frequently observed condition that certain commutator segments show more burning than others. These considerations also explain why frequently a split-throw of the armature coils improves commutation as a result of its effect upon the relation between leakage and mutual inductances of the coils. Without the split-throw a good transfer of energy from coil to coil in the same slot may take place, but the last coil in the slot is not well coupled to the first coil in the next slot and hence considerable energy must be dissipated when the circuit of the last coil in a slot is broken. The split-throw reduces this maximum dissipation of energy in the spark because at least one coil side of the coil under commutation is always closely coupled with another coil side, and, assuming that the brush is wide enough to short-circuit the adjacent coil, better transfer of energy can take place. The beneficial effect thus obtained consists not only in decreasing the maximum sparking but also in bringing about more uniform sparking on the different commutator bars, which in turn reduces irregular wear and differences in the heating of the bars; irregular heating of the bars if occurring to any marked extent is likely to cause differences in expansion of the bars and therefore mechanical irregularities in the commutator surface, with consequent further accumulative harmful effects on commutation.

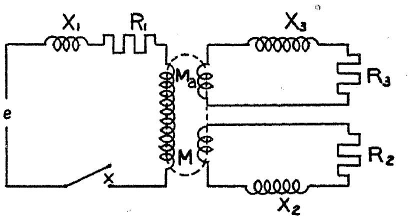

Fig. 6

In all of the previous discussions on the constants of the circuits which may influence commutation, only resistance and reactance have been mentioned. However, with the high frequencies of commutating currents, it is quite possible that at times certain capacity effects in the various circuits may exert an influence upon commutation conditions.

Having established the new concept of switching and the fact that the various machine and external circuits may have an influence upon commutation, we have found a means for explaining many other practical experiences which cannot be explained by the conventional commutation theorites. It can be readily seen, for instance, that there may be a difference in commutation between shunt, series, or separately excited machines, between laminated or solid field structures, etc., although the speed, current, and commutating field are the same. It follows that in the case of a-c. commutator machines the commutation may be affected by the size and type of transformer to which the machine is connected and by various other factors. Interesting results may be obtained with two series-connected motors on a motor car, especially in a-c. motors having no damping effects in their field structure. If in such a case the commutating impulses transformed into the fields of the two motors are exactly in opposition, no damping current can flow in the fields. Such opposition of voltages will occur if the relative position of the two commutators is such that the brush edge in one motor lines up with the mica insulation while in the other motor it happens to be in the center of the bar. If the relative commutator position changes so that the brushes in both motors will have the same position with regard to the mica insulation, the commutating impulses reflected in the fields will be in phase and damping current current can flow, thus causing conditions entirely different from those previously mentioned. With two such motors on a motor car having slightly different speeds caused by a difference in the wheel diameters, periodic variations of commutation will take place. With the two motors connected in parallel across the same transformer, similar variations may take place. Even a change in design from series to parallel connected fields in the motor may influence commutation. With such machines as phase-advancers or regulating machines connected into the secondary of induction motors, the switching theory readily explains differences in commutation which have been observed by the authors for different connections of such machines to various external circuits.

The introduction of the switching concept into the commutating problem is, of course, not necessary when ideal commutating conditions exist. There are also cases which approach the ideal sufficiently close to make the consideration of the switching concept unnecessary in practise. Again, where the commutating conditions are not ideal but where the brush short-circuits several armature coils, their mutual effect may be so predominating that the characteristics of the field and external circuits may have little influence. However, in general the commutating phenomenon cannot be considered as completely analyzed unless the possibility of the switching phenomenon as influenced by the various circuits has been given due consideration.

Discussion

Ernst Weber: I cannot quite agree with the authors in that the commutation problem has been given little attention as a switching phenomenon. On the contrary, the complete fundamental differential equations of commutation in a general case were established as early as 1908 especially by R. Rüdenberg (*E.T.Z.* 1908, page 65) following previous discussions on the problem which go back to 1905 when G. Mie and E. Arnold derived a mathematical condition for good commutation which since then often has been the center of much discussion. The complete differential equations take into account all the inductive couplings of the armature coils at the point to be short-circuited, and under short-circuit conditions. In addition the inducing effects of the external fields as produced by the main poles and by the commutating poles are taken care of. The one reason why no further progress has been made in this mathematical investigation is that it is not possible in general to integrate the underlying systems of differential equations.

An excellent brief summary of the theory of commutation may be found in German Manual *Starkstromtechnik*, Vol. I, U. 494 (published by W. Ernst u. Sohn, Berlin 1930) treated by C. Trettin. The system of differential equations, when the brush covers for example four segments and thus short-circuits three armature coils, is given by:

$$\begin{aligned} i_1 r_1 - i_2 r_2 - L\frac{du}{dt} - M_{12}\frac{dv}{dt} - M_{13}\frac{dw}{dt} &= e' \\ i_2 r_2 - i_3 r_3 - M_{12}\frac{du}{dt} - L\frac{dv}{dt} - M_{23}\frac{dw}{dt} &= e'' \qquad (1) \\ i_3 r_3 - i_1 r_1 - M_{13}\frac{du}{dt} - M_{23}\frac{dv}{dt} - L\frac{dw}{dt} &= e''', \end{aligned}$$

where i_ν denotes the currents in the segments; u, v, w the respective currents in the armature coils under short-circuit condition, r_ν the resistances of the closed coil-circuits, L the self, and $M_{\nu u}$ the mutual inductances of the coils, and e', e'', e''' the electromotive forces induced by the fields in the commutation zone. If all the parameters involved were constants it would be easy to solve for the currents and to deduce theoretically their shape under any condition of external uncompensated fields. However, all the parameters are variable, such as the inductances due to saturation, eddy currents and the changing positions of the coils relative to the field system; the resistances due to the varying contact area of brush and segment, the dependence of brush resistance on temperature, contact pressure, and current density. The equivalent circuits given in Figs. 3, 4 and 6 of the paper appear, from this recognized point of view, as very rough approximations. The variation of resistance for example extends over a range from several thousands of ohms to infinity.

In view of this extreme complexity we may finally state that commutation is one of the most intricate problems of scientific engineering. To arrive at some result, it was but natural, that either restrictions in great number were imposed on the illustrative system (1) in order to fit it to the mathematical means available for a solution, or estimations were made as to the influence of the various terms when straight line commutation was set as the aim. Several of the conclusions arrived at by authors from energy relations can thus be found in the literature. In a recent book published by J. Springer, Berlin, 1929, L. Dreyfus treats the whole problem of commutation on a mathematical, rigorous, but not very convenient way. The Siemens Schuckert-Werke, (Mr. C. Trettin), Berlin, as well as L. Dreyfus, hold several patents to use damper windings in slots or as wedges in order to influence the inductance at the instants of short-circuiting and breaking the short circuit of the coils, by these means causing the electromagnetic energy to be dissipated in heat in several parallel circuits.

The variations of the inductances with time are very small contrasted with those of the resistances. Assuming that over small time intervals both parameters may be taken as constants, the transient will be of the type of an exponential curve, the time constant of which is given by $T = L'/R'$, if L' is the effective inductance for the considered interval of time. At the first instant, R' is very large, the time constant very small, the rate of change of the current large and therefore a tendency to spark. However, since the time constant increases rapidly to a finite value, the danger is not pronounced. Conversely for the breaking of the contacts, since R'', the respective resistance at the trailing edge of the brush, is rapidly increasing, the time constant will decrease at the same rate to almost zero, so that the tendency to spark at the trailing edge is obvious and unavoidable. This physical picture is a first approximation and not at all the single criterion; however, it serves to show the importance of the variation of the resistance.

In conclusion may I emphasize the paramount influence of the brush material. Practise shows again and again that cases of bad commutation, considered as hopeless by theoreticians, are satisfactorily solved by engineers who patiently try various kinds of brushes until they succeed in finding the "adequate brush" for that special case. Investigations are under way to throw some light on the action of contact resistance and its dependence on the brush material, but we are still lacking complete data on this subject.

Thomas T. Hambleton: The consideration of commutation from the standpoint of switching may not be entirely new as most designers of commutating machines must recognize commutation as a high-speed switching of armature coils. The consideration of the possible influence of external circuits is entirely new to me. Of course it is desirable to view any complex problem from as many angles as possible, and it is in this respect that the paper should prove to be of value.

I fail to see where the paper presents any new knowledge that will enable designers to produce better commutating d-c. machines. I would therefore ask the following questions:

1. Has the consideration of commutation as a switching operation resulted in a method of more accurately pre-determining the commutation performance of d-c. machines?
2. Has it indicated any change in design proportions which has produced better commutation?
3. Have any specific cases been observed where commutation in d-c. machines has proved to be affected by external circuits; field circuit, damping circuits or external load circuits?
4. Is it not a fact that the considerations set forth in this paper apply almost entirely to a-c. commutating machines?
5. Has it been possible to compensate in the design of machine for the effect of external circuits on commutation?

G. W. Penney: The usual theory of commutation is based on the assumption that the brush contact drop is the same for all bars and that every other part can be built exactly as designed, so that the current reversal can be a straight line function or some other function which would be uniform for every coil. We are all familiar with the fact that the theory developed from these simple assumptions does not explain many of the troubles encountered in practise. For example, in the case of a large 250-volt generator running at constant speed and load there is commonly one coil per slot, so that all coils are theoretically alike and therefore for any given load and speed there should be a value of interpole flux which would exactly reverse the coil current in the period when a bar is under the brush, thereby giving black commutation. And yet we know that there are many cases where even for steady load conditions there is no value of commutating pole flux which will give black commutation. If the sparking is observed with a stroboscope it will often be found that a given value of interpole flux will give sparkless commutation on certain bars, but sparking at other bars. If the interpole flux is changed slightly, the sparking will disappear at the given bars, but sparks will appear at other bars, so that when observed with the unaided eye, the sparking will appear to be constant for a considerable range of commutating pole flux.

When we consider that a relative movement of one-ten-thousandth of an inch between one bar and the adjacent bars may seriously disturb the uniformity of contact drop, or that a slight deposit of carbon on one bar may have an equally serious result, it seems unreasonable to expect to maintain a uniform contact drop at all times.

The method of analysis described in this paper when combined with the characteristics of the brush under quickly changing current density in a portion of the brush face gives a means of analyzing the performance of the coil during commutation when ideal conditions do not exist.

The ability of a machine to commutate satisfactorily when slight disturbances occur either in the contact, at the commutator or in the rest of the circuit, should be a much more valuable index of its performance in service than the conventional analysis based on perfect conditions.

C. G. Veinott: The general principles, given in the paper, of course, apply equally well to lap or wave windings but the statement that with "a brush spanning one commutator bar . . . no account need be taken of mutual inductance" does not. This is true with a lap winding but is not true for a two-circuit wave winding of four poles or more because the number of commutator bars per pole is not an integral number. The discussion to follow does not refute any of the facts presented in this paper but only supplements the latter with information on wave windings not covered in the paper.

Some time ago I studied the commutating conditions in a wave winding and obtained, by graphical integration, the curve of the reversal of current in the commutating coil against time. I came to the conclusion that, in a wave winding of 4 or more poles, when only two brush arms are used, the commutating conditions are different under the two brush arms. This is due to differences in the coupling conditions of the coils undergoing commutation. As the authors have brought out that differences in the coupling conditions affect commutation, I only need to show how the coupling conditions under the two brush arms differ.

To be specific, consider a 4-pole wave-wound, retrogressive armature having 33 slots and 33 commutator bars with the armature winding chorded ¼ of a slot. Assume two brushes spaced 90 deg. apart, each brush spanning ¾ bars. Calling the brushes positive and negative solely for the purposes of identification and not to indicate actual d-c. polarity, consider the period when the positive brush just starts to span bars No. 8 and No. 9. This condition is represented in Fig. 1. Let us ignore, in this analysis, the mutual inductive effects of the field circuit and only consider other armature coils since effects due to each other circuits would only be superimposed upon the effects we are considering. (In Fig. 1 the coils represent single coil sides, the numbers the slot in which the coil side lies, and the letter *b* or *t*, whether the coil side is in the bottom or in the top of the slot.) We observe this significant fact: the coils short-circuited by the positive brush *begin* commutation without being coupled with any other coils undergoing commutation. At this point the negative brush is merely riding upon bar No. 33.

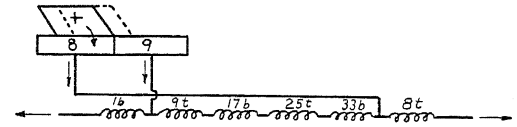

Fig. 1

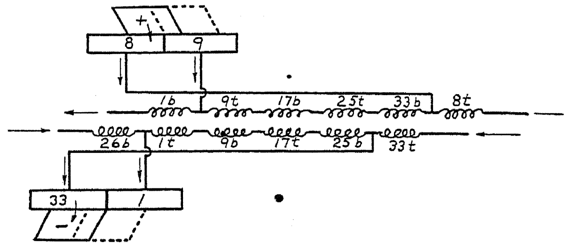

Fig. 2

In Fig. 2, we see the positive brush finishing its commutation. Observe that during the last ¾ths, including the *finish* of the period of commutation, three of the four coil sides short-circuited by the positive brush are mutually coupled with three of the four coil sides short-circuited by the negative brush at that time. It can be seen further that the coils short-circuited by the negative brush *start* commutation with three coil sides coupled.

In Fig. 3 we see the coils short-circuited by the negative brush *finishing* commutation with no coupling. At this time the positive brush is riding upon bar No. 9.

This brings out very clearly the fact that the commutation may be quite different under one brush than under the other.

If the brushes span one bar, somewhat the same conditions obtain except that the coils under the positive brush start commutation loosely coupled (one coil side coupled) and finish with close coupling (3 coil sides coupled). The coils under the negative brush start commutation closely coupled and finish commutation loosely coupled.

Considering commutation as a switching phenomenon, it follows that, when the coils under the positive brush finish commutation, if an arc is formed, all of the stored energy need not be dissipated in the arc to burn the commutator since much of this energy can be transferred through mutual inductive effects. When the coils under the negative brush finish commutation, if an arc is formed, they can transfer but little, if any, energy to other coils undergoing the process of commutation. Of course, there is always the possibility of transferring some of this energy to the armature working circuit, but this transfer of energy must be relatively small.

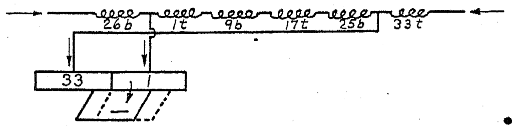

Fig. 3

F. W. McCloskey: It has become increasingly difficult of late years to predict the commutation characteristics of d-c. machines in which the loads applied are of a rapidly fluctuating nature.

Undoubtedly this is due in part, at least, to the fact which is brought out in this paper, that the customary method of figuring commutation does not embrace all the factors involved.

Street-car motors are subjected to this type of load, particularly in notching up the controller. Observations of such motors have shown that with peak currents of, say, twice the one-hour rating obtained in notching, the sparking may be in the neighborhood of twice as severe as when the same load is applied under steady conditions. It is therefore evident that stand tests in which steady loads are applied cannot be relied upon entirely as a criterion of the commutation results to be expected in service.

As pointed out in the present paper, this is one of the conditions which the usual theories of commutation do not take into account. In the design of motors for this type of service, particularly in these days of rapid acceleration of street cars, careful consideration must be given to this angle of commutation.

Thomas Linville: Messrs. Hellmund and Ludwig have presented their ideas without having carried them beyond pure theory. Therefore, in a practical sense the paper does not help solve the designers' difficulties with commutation. What must be hoped for is a commutation analysis reaching definite conclusions based on consideration of all the factors involved making it impossible to derive conflicting results by rational argument. Such results are possible now because the problem is so large and complex that restricted standpoints often form the basis for analyses, only a part of the factors being considered. The result is that many different opinions are held making confusion and disagreement.

Rising from these observations to the plane of theoretical thought in order to discuss the paper on its own level it seems that commutation has always been considered (in most of our minds and broadly speaking) as a switching phenomenon. The object has been to accomplish the switching; *i. e.*, to make and break the contact between the brush and each segment, with no current flowing. It makes little difference whether we start, knowing that the current will flow and appear as sparks if certain conditions exist, or with the premise that the current is already flowing. The objective in each case, is to prevent sparking and the analysis is likely to be the same. The possible difference is that we cannot definitely treat a sparking case mathematically as such.

In the commutation of a machine, the brush in making and breaking the contact with each segment increases and decreases the contact area at a definite rate. If in decreasing the area, for instance, the circuit conditions tend to diminish the current at a slower rate, the current density in the contact increases. This is equivalent to saying that the job of "forcing" the current change is thrown on the brush. Carbon brushes have a fairly definite limit to their ability in this connection. As the area diminishes the circuit resistance increases diminishing the current faster. If too much is demanded of the brush (if this variable component of the circuit resistance must increase too rapidly to reduce the current in the definitely prescribed time keeping the density within the limits of stability) then the brush arcs.

In my own work I have endeavored for some time to calculate, entirely by circuit analysis, the variable current density in the contact between the brush and any segment of both simple and complex machines. The development of a general method giving definite and practical conclusions now seems possible.

It is based, of course, upon the assumption of stable brush characteristics. Hence, sparking is not considered. Interesting results are promised since, for poorly commutating machines, the solution shows high current density occurring in some cases on the leading edge of the brush, more often on the trailing edge, and sometimes on both. These high current densities indicate that sparking is likely,—that it actually will occur if the density is sufficiently high. When it is determined at what density (for average conditions) the contact drop characteristic of a brush becomes unstable, or glowing and arcing begin, I expect we shall be able to say, by the calculation, when a machine will spark and when it will not. Furthermore since the circuit conditions depend upon the design dimensions it looks possible to answer by calculation many of the questions now perplexing the designers.

In one sense it does not seem that commutation should be called a "switching phenomenon" or by any name suggestive of a restricted point of view. Rather, it seems preferable to consider it always broadly as a circuit problem in which boundary conditions must be met in a fixed interval of time. Truly in the contents of the paper Messrs. Hellmund and Ludwig have considered it thus.

One point should be called to attention. In practical machines where several coils are commutating simultaneously the mutual inductances between the armature coils and the main pole windings, when taken together, have the effect of canceling one another with the result that no current is induced in the main pole windings. Hence, the external load circuits have little effect upon commutation except in particular cases. The fact that they do is a true idea but, possibly, overemphasized. The most important disturbers of good commutation are still those fluxes locally in and about the commutating slots; fluxes established by each armature coil which are not mutual with any other circuit. As suggested in the paper reducing these to improve the contact breaking conditions makes the contact making conditions worse but the balance is now, and surely always will be, in favor of the former.

R. M. Baker: The authors have exposed in this paper the very interesting observed fact that the commutation of a machine is indirectly affected by the nature of the connected load. As described in the paper, the effect is due to the change in the possible damping of the main pole windings. The results of some measurements of armature coil inductance taken with and without the field coils short-circuited; and also the maximum possible inductance of the armature coil with no damping follow:

Inductance with field coils short-circuited separately 7.28×10^{-6} henrys.

Inductance with field coils open 10.6×10^{-6} henrys.

Maximum possible inductance (calculated) 31.0×10^{-6} henrys.

The machine on which these measurements were made was a 20-hp. d-c. shunt generator with frame made up from heavy laminations (0.05 inches thick), and the measurements were made at 700 cycles.

From the above data, it can be seen that for this particular machine, a change from perfect field damping to no field damping can increase the inductance of an armature coil by about 30 per cent. In a machine having a very perfectly laminated field structure, the increase would be greater, whereas if a machine has a solid yoke, the increase would surely be less. We should expect, therefore, to find the phenomena described by the authors principally in those machines having a perfectly laminated magnetic circuit.

R. E. Hellmund: Referring to the discussion by Mr. Weber, I cannot agree that the subject matter of our paper is anticipated by the work which he mentioned and the mathematical treatment given by him. It is true that the commutating phenomenon has been referred to as a switching phenomenon for a great many years. It is also true that the effect of mutual inductance between various short-circuited coils has been taken into account in a great many previous theories, including the one referred to by Mr. Weber. However, as far as I am aware, all of this previous work deals exclusively with the reversal of the current in the armature coil while it passes under the brush. All of such work is referred to in our paper as the conventional commutation theory. In contrast to these conventional theories dealing with phenomena under the brush, our paper is intended to deal only with the phenomenon at the two edges of the brush, and, more particularly, with the arcing and its effect at the trailing edge under commutating conditions which result in arcing. The theories referred to by Mr. Weber relate to our paper only in so far as they have an influence upon conditions which may be found to exist at the edges of the brush as a result of what happens during the passage of the coil under the brush.

Referring now to both Mr. Weber's and Mr. Hambleton's discussion, I can only again emphasize what is stated at the end of the paper; namely, that the material given in the paper is of practical importance only in those cases in which ideal conditions by means of the proper commutating fields can not be obtained. Since this is more frequently the case under the various operating conditions of a-c. machines than of d-c. machines, it follows directly that the considerations are of greater importance in connection with a-c. problems.

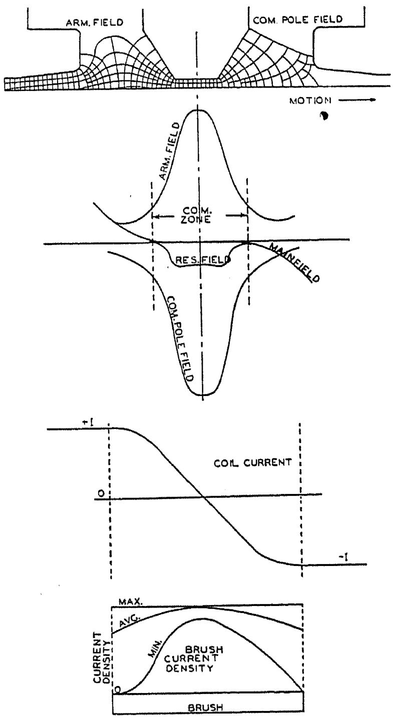

FIG. 4

Some discussion relating to Mr. Hambleton's first and second questions will be given later in connection with some figures. With regard to his third question, some cases have been observed where the commutation of a d-c. machine under actual operating conditions was different from that obtained with the test method used at times, in which the armature is short-circuited and the field excited through a high resistance just sufficient to give the desired current in the armature. In this case the damping currents in the field are of course lower than under normal operating conditions, and this may partly or entirely account for the difference noted in commutation. On the other hand, this case is further complicated by the fact that the commutating field is somewhat different under the two conditions of operation. The considerations given in the paper may also be of importance in certain machines having a third-brush or other special d-c. arrangements. In these and other cases the considerations in the paper may indicate the desirability of using the split throw in the armature, closely interlinked damper windings, etc. Aside from the fact that in most normal conditions of d-c. commutation the commutating fields can be adjusted reasonably well, there are other factors favoring the usual d-c. machines. In this connection the tests described by Mr. Baker in his discussion, which were made after this paper was written, are most interesting and somewhat surprising. They show that even in a machine with heavy laminations the damping effects of the stator may be appreciable, which in turn indicates that in machines having solid frame portions the normal damping effects may have a very favorable and predominating influence. Mr. Hambleton's fourth and fifth questions are answered by the previous remarks, as far as d-c. machines are concerned.

Referring now to a-c. machines, various examples can be cited in which the considerations of the paper have been of utmost practical importance and where a definite influence of the machine and external circuits have been noted. In several instances the change to a split throw has made sufficient difference to change practical failure to successful operation. This was especially important when for reasons other than the motor design proper the motors were built with more flux per pole than is desirable from a motor design point of view. Another practical example is that of the periodic sparking described in the paper. With certain low-frequency exciter machines for induction motors, the results when such machines were operated in connection with the induction motors were entirely different from results obtained on the test floor with resistance load. Here again the difference was very marked. These examples are also interesting because greater difficulties were experienced with 25-cycle motors than with 60-cycle motors, which is the direct opposite of what designers of a-c. commutating machinery are accustomed to. The reason for this was that in these cases the leakage reactance between the short-circuited armature coil and other circuits is made up largely of differential leakage, which in turn is proportional to the total main flux per pole. The latter was larger in the 25-cycle than in the 60-cycle machines and as a result led to less favorable commutating conditions in the 25-cycle machines.

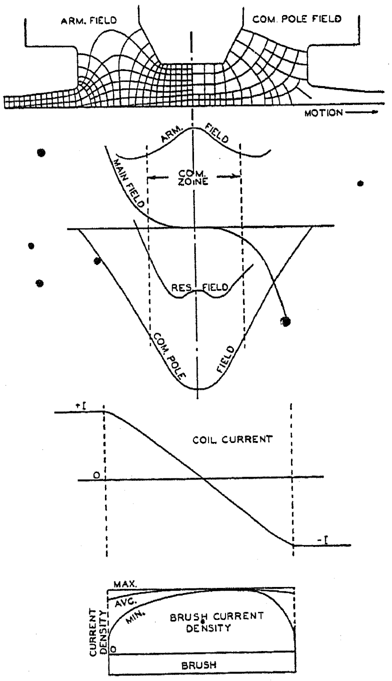

FIG. 5

The influence of external circuits in a-c. machines is also of greater importance not only because due to the highly laminated stator structure there are no damping effects in the stator core, but also because there frequently is, especially in motors for reversing operation and with consequent tilting of the carbons in their holders, only one armature coil short-circuited by the brush and consequently none of the other armature coils is able to supply any damping effect.

With reference to the question as to whether the switching operation at the edges of the brush in d-c. machines can be avoided, a brief discussion of the following figures may be of interest and may cover certain points raised in the discussions by

Messrs. Weber, Hambleton, and Penney. Ideal conditions are obtained if at the moment an armature coil is short-circuited its voltage is zero, and consequently zero current in the edge of the brush at that instant. Similarly, sparking and switching conditions at the trailing edge are impossible if the current in the edge of the brush is zero when the circuit is opened. This condition is fulfilled in Fig. 4. The armature and commutating pole fields have been carefully charted,* and it will be seen that the resultant field is zero both at the entering and trailing edge. The current in the commutating coils is shown below and has a characteristic which is sometimes referred to as "sinusoidal commutation." The bottom figures show the current density in the brush, and the minimum curve indicates that zero values apply at the time the coil enters and leaves the commutating zone. These results were obtained with a very small gap after much careful scheming with the shape of the interpole and with an armature without chording and one coil per slot.

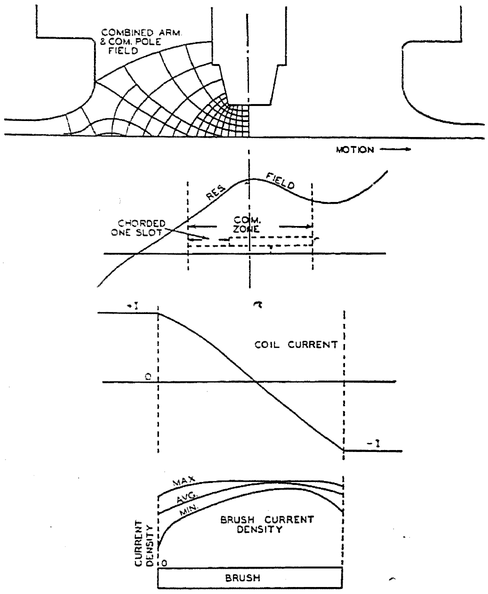

FIG. 6

Fig. 5 shows the same machine with the conventional large air-gap. The conditions are not nearly so favorable inasmuch as the resultant field induces a voltage in the commutating coil at the ends of the zone and there are certain current densities at both the entering and trailing edges. Fig. 6 shows a machine with chording of one slot and results are still less favorable; in fact, it was found impossible with chorded armature to obtain by changes in the interpole the ideal conditions shown in Fig. 4. In cases where two or more coils per slot are used, switching conditions are altogether unavoidable because, due to the influence of the main poles, the commutating field as shown in Fig. 7 is usually unsymmetrical. This means that the field cannot be correct for both coils. It is too weak for the commutation of the first coil and therefore commutation is incomplete at *b* when the first coil leaves the brush. However, on account of the close coupling with the other coil in the same slot, the energy can be readily transferred into the other coil, as indicated by the upswing of the current in the second coil. The curve for the second coil shows that because the commutating field in the range of its commutation is too strong, a reverse current is built up as indicated at *d* which has to be interrupted when the circuit opens. These figures, except Fig. 4, indicate clearly that there actually is a switching operation at the edges of the brush in most practical d-c. cases. This, however, is not of very great practical consequence in most d-c. cases for the various reasons previously indicated, although it may at times be largely responsible for the sparking which cannot be entirely eliminated.

L. R. Ludwig: In one of the discussions a question was raised regarding the feasibility of some special commutating arrangements. In answer to this, it may be stated that many such arrangements lead to irregular wear of the commutator parts, which would result in sparking regardless of any merits from an electrical point of view. There are means which would not be open to this difficulty and they have been and are being carefully considered. If any notable results are obtained, this work will be made the subject of later papers.

The possibility of bad commutation resulting from pulsations in the main field flux has been carefully investigated by us several times. In d-c. machines this effect is of minor importance due to the large air-gap and the damping effect of the stator structure. In a-c. machines it has been found that such pulsations may introduce high short-circuit voltages into the commutating coil, and consequently this point should always receive careful attention. It is the usual practise to construct a-c. machines, particularly those having relatively weak fields, with skewed slots in the rotor. Under this condition careful investigation has failed to reveal any commutating difficulties due to pulsations of the main field fluxes.

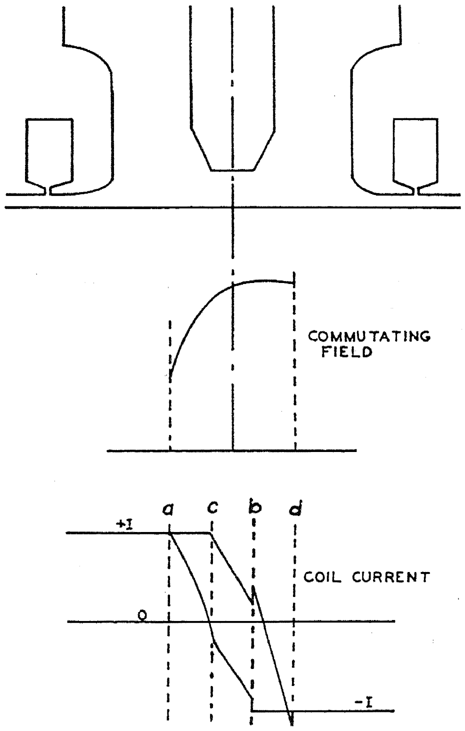

FIG. 7

The discussion given by Mr. Veinott is a valuable study of the special problems involved in a wave winding, and it again makes clear the necessity for a complete analysis of the electrical conditions existing within a commutating coil.

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*We are indebted to C. S. Newell for carrying out this work.

Equalizing Currents in the Armature of a D-C. Machine

BY ROBERT M. BAKER*

Associate, A.I.E.E.

***Synopsis.**—This paper contains an analysis which shows how the equalizing currents in the armature coils of a d-c. machine act to correct the unbalance causing these currents to flow. It is shown that even with very small inequalities of main pole air gaps (plus or minus 7 per cent of an average main pole gap), the equalizing current in an armature coil may reach, several times each revolution of the armature, a peak value as high as 25 per cent of the full load coil current value. The analysis is unique in that it is based upon experimentally determined curves of equalizing current variation in an actual machine.*

INTRODUCTION

WHEN the parallel type winding was first used on the armatures of large d-c. machines it was almost impossible to obtain sufficiently good magnetic balance of the main field circuits to permit satisfactory operation. This was due to the fact that the voltage of any circuit of a parallel or lap-wound armature winding is generated entirely by the flux of two adjacent main field poles. Unequal air gaps, or unequal reluctances in the iron circuits, result in unequal flux in the various main poles and consequently cause unequal voltages to be generated in the parallel circuits of the armature. This results not only in heavy circulating current in the armature winding, but also in an increased load on the carbon brushes.

It is obvious, therefore, that some device was needed to balance the flux of the main poles automatically and overcome the trouble resulting from the unavoidable irregularities of manufacture. Thus it was that B. G. Lamme in 1896 conceived the idea of connecting together in the armature winding of a d-c. machine, those points which should normally be at the same potential. He reasoned that with these polyphase connections, currents would circulate between the various parts of the armature winding to equalize the flux of the different main poles, just as a number of synchronous machines operating in parallel tend to equalize their fluxes by causing exciting currents to flow in the armature circuits. This was the beginning of the equalizing connections or cross connections as they are used today.

Equalizing connections have proved themselves capable of taking care of considerable unbalance in the main fields, and have made it possible to build large d-c. machines to operate satisfactorily with lap-wound armatures of twenty or more circuits. Their performance has in fact been so satisfactory that very little time has been given to a study of the details of this performance. Purely theoretical analyses are valuable in giving one a clear mental picture of the process by which the equalization is accomplished; but they are always necessarily based upon simplifying and often inaccurate assumptions and are therefore of little quantitative interest. The analysis given in this paper is based upon the experimentally determined variation of equalizing currents in the armature coils and shows how these known currents act to overcome the magnetic unbalance causing them. The value of such a treatment lies in the fact that it will give the reader some idea of the magnitude and nature of equalizing currents arising from a certain unbalance in the main field circuits and will make the whole subject more tangible by showing records of the equalizing currents in the armature winding of an actual machine. It will also suggest a method for studying the subject in more detail.

THE MACHINE

The machine used in this study is a 25-hp., 4-pole, 220-volt, 1,500-r.p.m., d-c. generator. The armature is full pitch lap wound and has 40 slots, one coil per slot, ten turns per coil. It is 100 per cent cross connected on the front. That is, each commutator bar is in turn connected to another commutator bar just two pole-pitches ahead. Since the armature has 40 slots and a full pitch winding, it is entirely symmetrical. The average main pole air gap for this machine is only 0.074 inches.

ANALYSIS OF TEST RESULTS

To obtain oscillographic records of equalizing currents in armature coils it was necessary to insert a small resistance in series with the coil or coils to be studied, and to record the voltage drop across this resistance. The value of resistance chosen was only 25 per cent of the resistance of a single armature coil, and since equalizing currents are limited almost entirely by armature coil inductance, the insertion of this small resistance caused no distortion. All records were taken with the commutator brushes lifted, so that the only current in the armature coils was the equalizing current.

The first record taken was of the equalizing currents in two diametrically opposite armature coils with all four field coils normally excited in series. The only unbalance in the magnetic circuit was due to the air gaps under the two lower poles being slightly less than those under the two upper poles. The record of the coil currents resulting from this unbalance is shown in Fig. 1. It is seen that the equalizing connections paralleling these two coils are sufficiently low in resistance

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Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

that the two coils carry at each instant, exactly the same current. Although it is not shown on the oscillogram, the currents in these two coils are in opposite directions. Thus it is equivalent to a circulating current between the two coils set up by the difference of their generated voltages. Their generated voltages result not only from the cutting of flux, but also from their mutual interlinkages with other coils which are carrying current. An inspection of Fig. 2 will show that from symmetry, the equalizing current variation in any armature coil should repeat, except for a reversal of sign, twice each revolution. The fact that this condition was not obtained in Fig. 1 is explained by a dissymmetry of rotor iron. That is, the air-gap under any main field pole varies slightly as the armature is rotated.

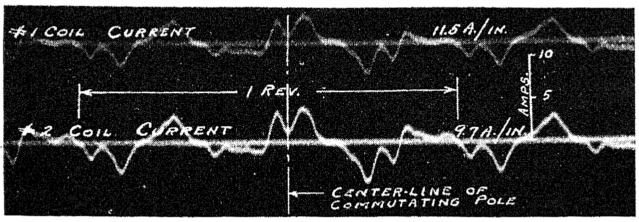

Fig. 1—Equalizing Current in Two Diametrically Opposite Armature Coils. (See Fig. 2)

Using the experimentally determined curve of equalizing current given in Fig. 1, an analysis will be made to show how equalizing currents correct the unbalance causing them to flow.

To eliminate the effect of dissymmetry of rotor iron, the curve of equalizing current is averaged so as to repeat, with signs changed, every two pole-pitches, and is plotted as the solid line curve of Fig. 3. It can be assumed that this is the current variation in the conductors of the top coil side of slot *A*, for after this assumption is made, the relation between the direction of the current and the direction of the resulting m.m.f. can be determined by the substitution of a known magnetic unbalance with a known polarity such as the partial excitation of a single field coil. From an inspection of Fig. 3, it is obvious that the bottom coil side of slot *A* is, at any instant, carrying a current of the same magnitude, but opposite in direction to that carried by the upper coil side just one pole-pitch earlier. Thus it is possible at once to plot the dotted curve of Fig. 3 showing the variation of conductor current in the lower side of slot *A*. The average conductor current in a slot multiplied by the number of conductors gives the total slot current as shown in Fig. 4.

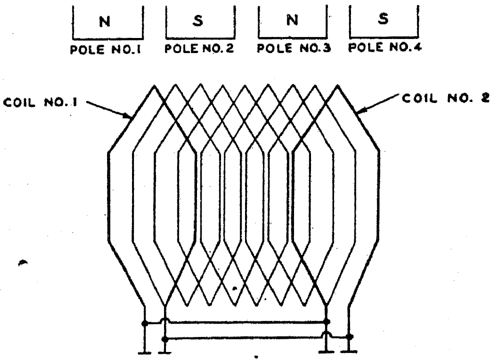

Fig. 2—Development of Armature Winding Showing Coils Nos. 1 and 2

For simplicity, only one pair of equalizing connections is shown

Since the value of current in a slot is dependent only upon the position of the slot relative to the main field poles, the curve of slot current will just as well represent a current sheet around the armature, fixed in space and therefore setting up a constant magnetic field relative to the main field poles. The m.m.f. resulting from this current sheet is represented by the heavy line curve of Fig. 4, which is simply the integral of the slot current curve. The scale of the m.m.f. curve was determined from the current scale and the number of slots per unit length of armature periphery, and is plotted in per cent of the normal excitation of a single main field pole.

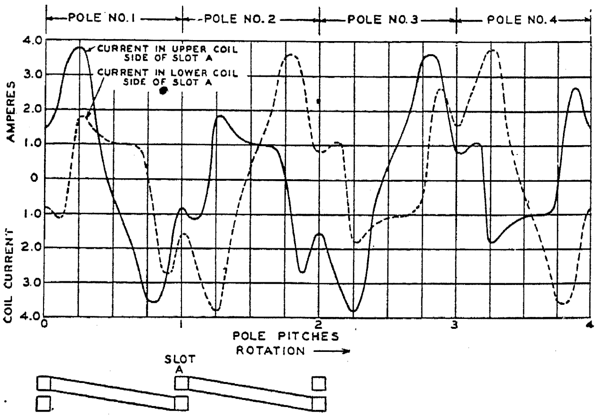

Fig. 3—Conductor Current in Upper and Lower Coil Side of Slot *A* Plotted Against the Position of Slot *A* Relative to the Main Field Poles

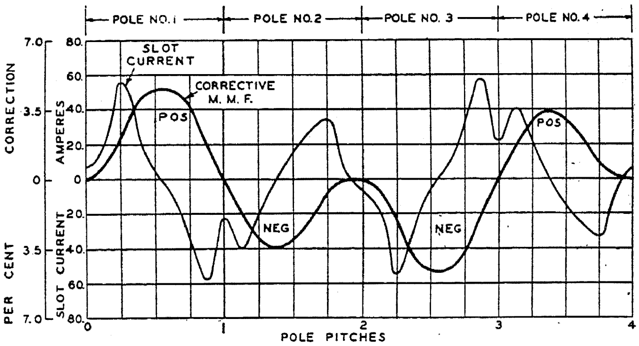

Fig. 4—Slot Current and Corrective M.M.F.

Referring to Fig. 5, it can be seen that the correction is proper. The corrective m.m.f. is in such a direction as to aid the flux of poles Nos. 3 and 4 and to oppose the flux of poles Nos. 1 and 2. An oscillogram of generated coil voltage showed that the equalization was practically perfect.

In a way the four-pole machine is unique. For instance, it has already been observed that the curve of equalizing current in an armature coil must repeat with the signs changed every two pole-pitches or twice each revolution. From the foregoing analysis it follows that the curve of corrective m.m.f. must also repeat twice each revolution. That is, the correction of two opposite poles must be equal and opposite. This is not at all surprising, for the unbalance must occur between opposite poles before it is effective in setting up equalizing currents. The unbalance of adjacent poles may be very great, and yet if opposite poles are equal in strength, the flux in all four poles will be equal.

The oscillogram of Fig. 6 was taken with only pole No. 2 of the machine excited, and shows records of generated coil voltage and coil current of one armature coil. It is interesting to notice the completeness of the equalization. Although only one field coil was excited, the flux in all poles is of the same amount and in the proper direction. The excitation of the pole was about 30 per cent normal. An analysis of the coil current curve showed that the equalization was accomplished by a corrective m.m.f. of 15 per cent sending flux from pole No. 4 into the armature, and a corrective m.m.f. of 15 per cent opposing the original 30 per cent excitation of pole No. 2. The correction under poles Nos. 1 and 3 was practically nil.

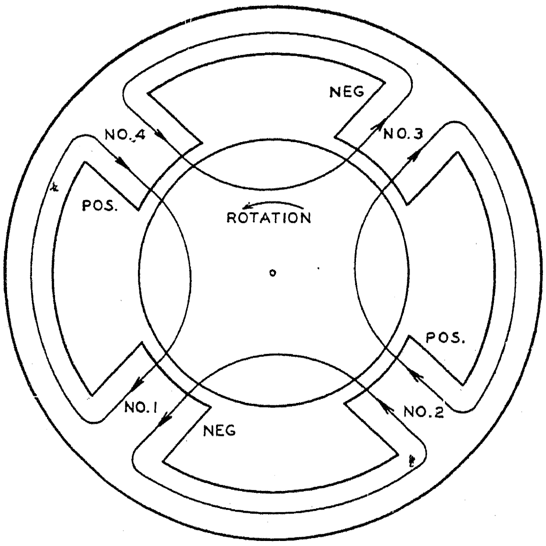

Fig. 5—Schematic Diagram of Magnetic Circuit Showing Normal Flux Distribution

A similar record taken on the machine with poles Nos. 1 and 2 excited to 30 per cent normal value is shown in Fig. 7. In this case the equalization required a corrective m.m.f. under each of the poles. Under pole No. 1 it was 15 per cent from pole to armature; under pole No. 2 it was 15 per cent from armature to pole; under pole No. 3 it was 15 per cent from armature to pole; and under pole No. 4 it was 15 per cent from pole to armature. With two poles originally excited, the final flux density under the poles was just twice that obtained from the original excitation of one pole.

It might be said that the analysis set forth in this paper was started almost with the answer. No attempt was made to solve the very difficult problem of theoretically determining the equalizing current set up in the armature coils by a certain unbalance in the main field circuit. The main purpose of the writer was to show how the oscillograph might be used in a study of the behavior of equalizing connections.

The process by which equalizing currents are set up in an armature is fairly well known, but the complications arising from the mutual inductance between armature coils and the existence of many overlapping circuits are so great that an accurate analytical solution of the problem seems practically impossible. It is firmly believed that a complete analysis similar to the one given in this paper, but one made on a machine with a large number of poles, would furnish enough empirical data that an accurate quantitative analytical solution would not be necessary. The number and size of the equalizing connections could be changed and if necessary, the analysis could be extended to two or three sizes or types of machines, so that the results could be interpolated for intermediate points. If later it does appear desirable to make a complete theoretical analysis of the problem, the analyzed data obtained on the actual machines would be of great value as a check on proposed theories.

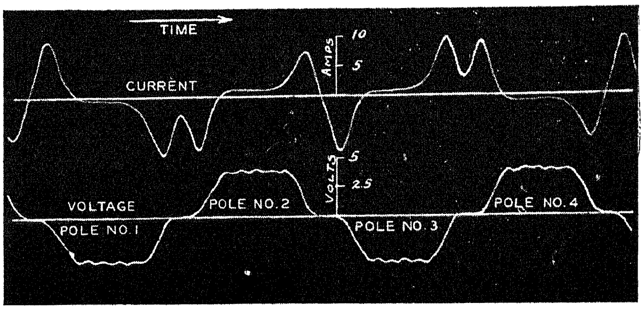

Fig. 6—Current and Generated Voltage in Armature Coil, with Pole No. 2 Excited 30 Per Cent Normal

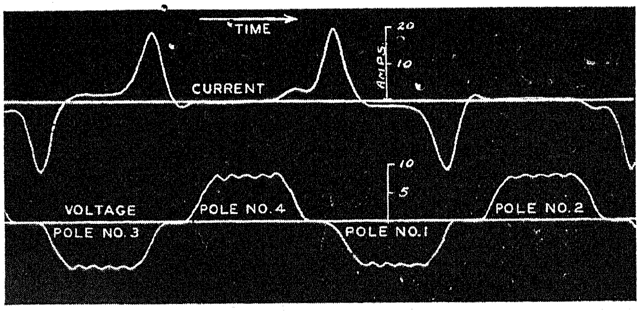

Fig. 7—Current and Generated Voltage in Armature Coil with Poles Nos. 1 and 2 Excited 30 Per Cent Normal

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Discussion

R. E. Hellmund: Such a study as that covered by Mr. Baker relating to equalizing connections on commutating machinery is very desirable because, in spite of the fact that equalizing connections have been used for a great many years, the design engineer is still continuously confronted with questions relating to their application. While equalizing connections are of course always used with parallel-wound armatures, it is frequently difficult to decide to what extent they should be used and which of the various possible arrangements at the front or rear end of the armature leads to the most economical and reliable design.

Equalizing connections fulfil two functions: that of equalizing the fields of the various poles, as covered in Mr. Baker's paper, and, in addition, a function which is not as generally recognized. This second function relates to the transfer of current from one brush arm to another of the same polarity whenever the brush, on account of mechanical inaccuracies of the commutator, temporarily jumps and fails to make contact with the commutator. This is exceedingly important in practise because, as is probably recognized by all designers of large and high-speed commutating machinery, by far the largest part of their commutating difficulties are due to mechanical inaccuracies or so-called "high bars" of the commutator. It can readily be seen that when a brush leaves a commutator bar which is directly connected by an equalizing connection to another bar of the same polarity, the current can temporarily transfer through this equalizing connection to such other bar, and unless such transfer is opposed by high self-induction of the equalizing connection, no sparking will result at the brush leaving the commutator. If, on the other hand, there is no equalizing connection to such bar, the transfer of current has to take place through a part of the armature winding. This may apply either if the cross-connections are at the rear of the armature or at the commutator end but limited to a small number of bars. Any transfer of current taking place would then be opposed by the self-inductive effect of part of the armature winding, and the self-inductive voltage thus obtained is likely to maintain a spark between the commutator and the jumping brush. It is for this reason that best results are usually obtained with 100 per cent cross-connections at the commutator, or, still better, with the so-called "involute winding."

Unfortunately the merits of the more expensive and often difficult arrangements for cross-connections cannot be readily determined from ordinary comparative tests because both functions previously mentioned are affected by inaccuracies of manufacture which are difficult to duplicate. The function relating to the equalizing of the field is affected by inaccuracies in various portions of the air-gap which cannot be measured easily. The function relating to the transfer of brush current is affected by inaccuracies of the commutator as small as one ten-thousandth of an inch, which are also difficult to determine, and which moreover are subject to change during the tests due to different expansion and contraction with changes in temperature. It is for this reason that the only hope of clarifying the subject rests in systematic studies in which a number of variables is eliminated, such as carried on by Mr. Baker.

Thomas T. Hambleton: The introduction to the paper recognizes the fact that armature equalizer connections are a matter of ancient history and that their action is thoroughly understood by all designers.

It seems appropriate from the historical standpoint to mention a coincidence which is not infrequently encountered in the development of an art or science. It occasionally happens that two or more individuals independently and concurrently arrive at the same solution of a problem.

Such a coincidence appears to have occurred in connection with armature equalization. The old records of the General Electric Company contain drawings made in 1895 showing involute formed equalizers mounted at the end of commutators and connected directly to the segments, thus providing complete equalization at the most advantageous point as regards another function of the equalizers which Mr. Baker has not mentioned.

Besides providing paths for the circulation of magnetizing currents which equalize field fluxes, equalizer connections also carry between parallel brush groups interchange currents caused by irregularities in the mechanical condition or the contact surface condition of the commutator.

An equalizer located at the commutator end of the machine has minimum impedance for these currents; when the equalizer is located at the back end of the armature these currents must at least twice traverse the armature slots in addition to the equalizer, thereby encountering the high impedance of the iron-embedded conductors.

It is evident that a type of equalization considered perfect today was used on some very early machines.

R. E. Hellmund: I agree with Mr. Hambleton in his statement that 100 per cent cross-connections at the front end are desirable as far as obtaining the best result from the equalizing connections is concerned. The only reason for not applying cross-connections in this manner is that with smaller machines and less severe conditions they may not be justified economically. Again, in other cases, especially under the crowded conditions found in railway motors for instance, difficulties from an insulation and structural point of view are often encountered and the use of the very desirable involute winding is at times practically impossible.

Thomas Linville: Equalizing connections and the currents which flow in them have been taken for granted, their function being clearly understood by all designing engineers. However, their magnitude and the extent to which they increase the losses and thereby diminish the capacity of a machine have not been widely known. These data are presented in the paper. It seems that the value of the connections has been so well demonstrated that they must be considered a fundamental part of a machine. Since we cannot divorce ourselves from them, as we know in general through years of experience what good and harmful effects they produce, more detailed studies might seem of little importance. Contrary to this I believe it essential that as much as possible be learned, particularly since we occasionally want to build a machine with purposely unbalanced magnetic fields and want to know how effectively the equalizing currents will function. Again because of the existence of these currents in commutating coils, and because the brushes always form parallel equalizing paths, it is important to know exactly how they affect commutation. Also a great deal of information concerning the action of equalizing currents in some types of a-c. machines is needed. Finally for the currents to be effective in the desired way they must be largely reactive currents. Hence, we want to know how low the resistance of the connections must be and how complete the equalization will be with a given resistance. The fact that the currents are determined practically by reactance alone is what makes their calculation difficult. Much experience, test data, and well-established convictions answer these wants but the need for still more definite data is great. Therefore, I think Mr. Baker's work is worth while.

It is encouraging to see such excellent results obtained with the oscillograph because its use is essential in making experimental studies and in proving theories. In some cases, with big machines, real difficulties are met due to stray fields, etc., causing distorted oscillograms when extreme care is not used.

Calculation of No-Load Damper Winding Loss
in Synchronous Machines

BY E. I. POLLARD*
Non-member

Synopsis.—Pulsations in the air-gap induction due to the slotted stator construction produce pole-face loss. When a damper winding is present the pulsations link the damper bars producing circulating currents causing an additional loss, which is designated by the term "no-load damper winding loss." The magnitude of this loss is found to vary from practically zero to as much as eight times the pole-face loss, or to as much as half of the total no-load core loss. A method of calculating the no-load damper loss is developed based on theoretical considerations. As far as the writer is aware no theoretical work has been previously published on this subject.

DEVELOPMENT OF THE FORMULA

IN synchronous machines, the non-uniform nature of the air-gap due to the presence of alternate teeth and slots on the stator results in a non-uniform distribution of flux density at the surface of the rotor, the maxima of density being opposite the teeth and the minima opposite the slots. Flux maps of the air-gap field for a slotted stator have been published by several writers.[1,2] The relation between the actual and average densities from point to point over a tooth pitch along the air-gap, derived from flux maps, is shown by Wieseman.[1] The induction at the rotor side of the gap may be resolved into an average induction and a superimposed pulsation or ripple. The latter is the object of interest from the standpoint of damper loss. In producing the ripple, the teeth may be regarded as north poles, the flux returning to consequent or imaginary south poles occupying positions in the slots as indicated in Fig. 1.

In most machines, the damper slots are but partially closed, so that the reluctance of the opening above the slot is large compared with the reluctance of the path through the iron around the slot. The pulsation flux from a north to a south pole will, therefore, link the damper slots which interrupt the otherwise smooth surface of the pole-face. Any two bars form a closed circuit linking the enclosed pulsation flux. The position of maximum interlinkage occurs when the central point between the bars is opposite the center of either a stator tooth or slot. The frequency of the flux pulsation with respect to a point on the rotor surface is equal to the number of stator slots per pair of poles multiplied by fundamental or line frequency. Therefore, the rate of change of interlinkage of pulsation flux with a pair of damper bars is quite large, inducing a considerable voltage around the closed circuit formed by the two bars. Assuming that the circuit is open, it is possible to calculate the voltage induced around the circuit, the total inductive reactance, resistance, and impedance of the circuit, and from these calculate the current which flows when the circuit is closed. However, since the effect of the resistance upon the impedance is quite small, it is preferable to assume that the reactance and impedance are equal and calculate the current directly on the basis that the magnitude of the current is determined by zero interlinkage of flux with the circuit. In other words, the flux produced by the flow of current is equal to and completely cancels the flux enclosed by the circuit before it is closed. This damping action leads to the suggestion that the circuit formed by each pair of adjacent damper bars be defined as a "damping circuit." On this basis each damper bar becomes a part of two damping circuits, the return paths of these circuits being the two bars adjacent to the one considered, as indicated in Fig. 1. The total current in the bar is, therefore, the resultant of the currents flowing in the two damping circuits of which the bar is a part. The current in the damping circuit is obtained by use of the relation:

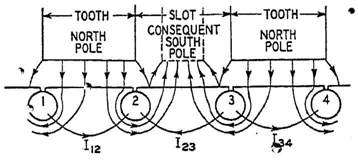

FIG. 1

damping circuit m.m.f.

$$= \frac{\text{total enclosed pulsation flux}}{3.19 \times \text{total permeance of damping circuit}}$$

or

$$I_c = \Phi / \sqrt{2}\ 3.19\ \lambda \qquad (1)$$

The total enclosed pulsation flux must be construed to include the mutual flux set up in the damping circuit considered by currents in all other damping circuits, as well as the flux set up in it by the non-uniform air-gap permeance. That is:

$$\Phi = \Phi_p + \Phi_m \qquad (2)$$

The total permeance of the damping circuit may be separated into the air-gap permeance and the permeance of the slots forming the circuit:

$$\lambda = \lambda_g + 2\ \lambda_s \qquad (3)$$

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1. For references see Bibliography.

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maps for various values of $b_r/K_g\,g$ shows that λ_g is independent of $b_r/K_g\,g$ while Fig. 4 shows that λ_g does depend upon $\tau_r/K_g g$ and in fact may be taken as approximately equal to $\tau_r/K_g g$.

$$\lambda_g = \tau_r/K_g\,g.$$

By calculating λ_g in this manner, the non-uniformity of the air-gap permeance due to the stator teeth and slots is neglected. More complete analysis would show that for a given combination of slot and gap dimensions and bar pitch, λ_g is equal to $\tau_r/K_g g$ plus a superimposed periodic function of time. Referring to Fig. 1, λ_g for bars 1 and 2 is larger than for bars 2 and 3, so that the current I_{12} is less than I_{23}. But since we are interested chiefly in the resultant current in bar 2, which is the same as if the average value of λ_g had been used for both circuits, the final result is unaffected. In Fig. 1 by omitting bars 1 and 3 the condition of $\tau_r/\tau_s = 1.0$ is obtained. In this case, since a damper pitch extends over a complete stator slot pitch, the periodic component of λ_g is zero and, of course, does not affect results. For intermediate ratios of τ_r/τ_s, the resultant bar current is altered somewhat by the periodic component of λ_g, but for the sake of simplicity this will not be considered.

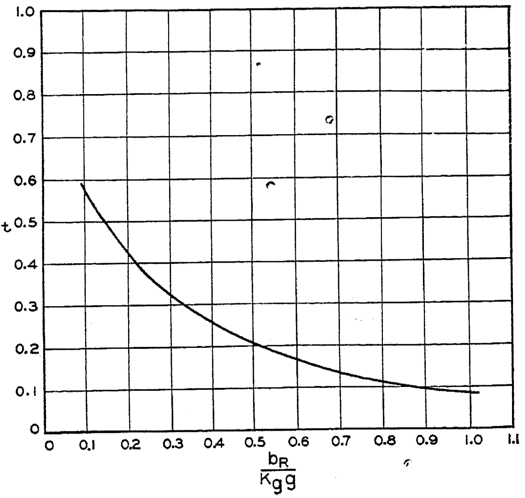

FIG. 5—DAMPER TOOTH-TIP PERMEANCE λ_t

4. *Calculation of increase of resistance, K_f.* Various investigators[4] have calculated the ratio of the a-c. to d-c. loss in a solid conductor placed in an open rectangular slot, based on the assumption that the flux produce by current in the conductor crosses the slot in straigh parallel lines. Since partially closed rather than ope damper slots are almost universally used in moder synchronous machinery the effect on K_f of partiall closing the slot is to increase the ratio of a-c. to d-c loss, K_f. In the absence of a rigorous treatment of thi subject, K_f may be calculated by the open-slot formula remembering that it is optimistic.

COMPARISON OF CALCULATIONS WITH TESTS

Rigorous experimental verification of the variou factors appearing in equation (**10**) is not available. A rough check is obtained by comparing the total calculated no-load loss consisting of core, tooth, pole-fac and damper losses with the tested core loss on a numbe of machines. This is shown in Table I.

The tooth, core, and pole-face losses were calculate by the usual design methods. Although the pole-fac loss itself is actually reduced somewhat in most case by the presence of the damper winding, correction wa not made for this reduction, which partially compensates the optimistic method of calculating K_f. Equation (**10**) was used to calculate the damper loss. Error in methods of calculating the tooth, core, and pole-fac losses, and variations in manufacturing processes and i the materials themselves tend to prevent consistency o the results of such a check. In choosing the example for Table I, machines having relatively large pole-fac loss were preferred in order to minimize as much a possible the effect of errors in the calculation of th core and tooth losses. These machines were all designe and built before the factors controlling damper loss wer known. As a result, some very unfavorable combinations were accidentally used. Example No. 12 is outstanding in this respect, the damper loss being ove eight times the pole-face loss. The designer of thi machine estimated the total core loss to be abou thirty kilowatts, and the test results could not b explained. These remarks also apply to exampl Nos. 6, 7, and 10. In each case the high damper lo was due to a combination of unfavorable conditions: τ_r/τ_s = about 0.5, bar resistance is high and the tot number of bars is large. In examples Nos. 1, 3, 8,

TABLE I

	Poles	Kva.	Calculated core and tooth loss	Calculated pole face loss	Calculated damper loss	Total calculated loss	Test loss
1	8	3,000	17.1	5.1	0.38	22.6	18
2	8	2,920	17.3	5.3	4.5	27.1	31
3	14	5,270	28.3	10.0	0.40	38.7	40
4	16	692	3.32	1.86	3.04	8.22	7.5
5	24	3,750	30.5	15.8	13.5	59.8	57.5
6	26	1,500	7.8	5.4	12.7	25.9	26.0
7	26	1,500	6.39	1.9	3.74	12.03	15
8	40	1,125	8.8	5.5	2.4	16.7	17.7
9	48	1,750	10.5	3.5	0.33	14.3	13.6
10	56	1,250	14.6	9.8	17.2	41.6	36.7
11	58	2,000	11.9	9.4	0.49	21.8	23.6
12	88	5,000	24.1	3.3	27.6	55.0	54.0

and 11, favorable combinations were accidentally chosen when these machines were designed, and the test core losses were considered unaccountably low. Table I emphasizes the extremely variable nature of the no-load damper loss, and indicates the value of some method by which the magnitude of this loss may be estimated.

CONCLUSIONS

Several ways by which the damper loss may be minimized are suggested by equation (**10**). In general, any change made in a design with the object of reducing pole-face loss also decreases the magnitude of the damper loss, unless the ratio τ_r/τ_s is thereby made less favorable. However, such changes usually affect the fundamental proportions of the machine; air-gap length, number of slots, etc., which are frequently determined not by losses but by other characteristics. It is, therefore, preferable to look for better methods of controlling this loss. Equation (**10**) shows that the loss is proportional to the number of bars on the machine and to the bar resistance, suggesting that it is desirable to keep these quantities small.

Perhaps the most powerful method of reducing the loss is by adjustment of the ratio τ_r/τ_s. When $\tau_r/\tau_s = 1.0$ the terms $1-\cos\phi$ and $1-\cos 2\phi$ in equation (**10**) are equal to zero, causing the damper loss to vanish. The loss is quite small down to $\tau_r/\tau_s = 0.85$. However, when $\tau_r/\tau_s =$ about 0.5 the loss is usually excessive, and this condition should be avoided if possible. In cases where a low value of telephone interference factor is imperative, the use of nearly equal rotor and stator slot pitches should be applied with caution since some unpublished work of Mr. S. L. Henderson* indicates that this condition frequently results in a higher telephone interference factor than where τ_r/τ_s is about 0.65. With this exception, this method of reducing damper loss may be applied without impairing other characteristics.

Increasing the rotor slot pitch increases λ_g, and assists in reducing loss, being a secondary effect accompanying reduction in the number of bars. Increasing the term λ_s by deeply embedding the damper bars in the pole-face also assists in reducing loss.

In conclusion the author wishes to express his indebtedness for helpful criticism and suggestions to Messrs. J. F. Calvert, L. A. Kilgore and S. Beckwith,* with whom he conferred from time to time during his investigation of this problem, and to Messrs. M. W. Smith and C. M. Laffoon* for criticism of the manuscript.

NOMENCLATURE

a = area of damper bar (sq. in.)

B = average air-gap induction (kilolines per sq. in.)

h_b = depth of damper bar (in.)

h_r = depth of rotor slot tip above bar

f = stator tooth frequency = number of slots per pair of poles × machine frequency

g = air-gap length (in.)

I_c = current in damping circuit (amperes)

I_r = resultant current in damper bar

K_g = air-gap coefficient

K_f = ratio of a-c. to d-c. loss in damper bar

K_p = double amplitude of air-gap induction pulsation in terms of maximum induction, at rotor surface

K_w = magnitude of field form harmonic in terms of K_p (Fig. 3)

l = length of damper bar (in.)

n = total number of damper bars on machine

P = no-load damper loss (kw.)

τ_r = rotor slot pitch (in.)

τ_s = stator slot pitch (in.)

R = d-c. resistance of damper bar (ohms)

b_s = stator slot width (in.)

b_r = width of rotor slot opening above bar (in.)

b_b = width of damper bar (in.)

λ_a = permeance of rotor slot above bar

λ_b = permeance of that portion of the rotor slot which is occupied by the bar

λ_g = air-gap portion of damping circuit permeance

λ_t = rotor tooth-tip permeance (Fig. 5)

Φ_p = maximum amount of pulsation flux due to air-gap permeance variation, which is linked by a damping circuit

Φ_m = mutual pulsation flux linked by a damping circuit due to currents in other circuits

ϕ = phase angle between currents in adjacent damping circuits = $\tau_r/\tau_s \times 360$ deg.

ρ = resistivity of bar material (microhms per cu. in.)

Subscripts 1 and 2 refer to fundamental and second harmonic respectively.

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2. "Pole Face Losses," F. W. Carter, *Journal I.E.E.*, 1916.
3. *Discussion of Papers on Graphical Determination of Magnetic Fields*, J. F. Calvert, Figs. 13 and 14, TRANS. A.I.E.E., 1927, p. 151.
4. *Eddy Current Losses in Large Slot Wound Conductors*, A. B. Field, TRANS. A.I.E.E., 1905.

Discussion

C. C. Shutt: Mr. Pollard's paper gives the designer of synchronous machines another useful tool for the predetermination of their performance. The test results of example 12, Table I, demonstrated clearly the necessity for the investigation which he undertook. The particular machine of example 12 in Table I happened to be a motor. The damper bar material had a resistance 15 times that of copper and the slot ratio was conclusive to high damper winding losses.

In synchronous motor design the use of relatively high resistance damper bars is becoming increasingly common as a means of reducing the starting kva. and improving the starting power factor. From equation (**10**) of the paper, it is apparent that the loss increases directly with the effective resistance of the damper bars. This makes it particularly necessary to check the design of synchronous motors from the standpoint of ratio of rotor to stator slot pitches.

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On a new line of synchronous machines, which was recently developed, this feature was checked before making the drawings. The number of stator and rotor slots per pole was determined by other considerations. The pitch of the slots in the pole punchings was set so as to avoid improper ratios from the standpoint of no-load damper loss. This necessitated unequal pitches of the pole face slots in several of the poles. The tests of no-load loss on this line of machine compare favorably with the calculated values.

R. E. Hellmund: The practical importance of damper winding losses has, of course, been appreciated by designers for some time. I recall cases where what seemed to be a rather insignificant increase in the ratio of the secondary bars of a squirrel-cage motor to the number of primary slots increased the no-load losses of the motor from 50 to 100 per cent. Furthermore, it is easily seen that if the top of the teeth of the rotor are smaller than the slot openings of the stator, it will result either in large flux fluctuations in the rotor teeth and consequent core losses, or else in large losses from the corrective current in the squirrel-cage bars. As a matter of fact, in the example of the induction motor previously referred to, the losses caused by the flux changes in the teeth, if allowed to exist by removing the winding, were of about the same order as the losses caused by the corrective current with the winding in place. Although there was previously a qualitative appreciation of the factors touched upon in Mr. Pollard's paper, his quantitative analysis of the problem is of great value.

As in most other cases of design, there are in this case several reasons why the use of the proportions giving ideal results in one respect, such as the minimum damper winding losses covered in the paper, cannot be applied. This in turn makes it desirable to determine just how great a departure from the ideal is advisable in order to favor other factors of importance. A large number of secondary bars, for instance, is usually desirable on account of the favorable effect upon the power factor and other performance features of induction motors. Again, it is pointed out in the paper that while a slot ratio of 1:1 gives minimum damper winding losses, it is likely to lead to telephone interference. This is due to the fact that higher harmonic currents different from those calculated in Mr. Pollard's paper are set up in the primary and secondary of synchronous and induction motors for the following reason:

With a definite voltage impressed upon these motors, the various currents have to adjust themselves so as to set up fields inducing a counter e.m.f. equal and opposite to the impressed e.m.f. Inasmuch as the field forms of the machines are usually not sufficiently ideal to accomplish this with sinusoidal magnetizing current, a flow of certain higher harmonic currents is necessary. These higher harmonic currents are usually of large amplitude with a slot ratio of 1:1 or equal slot pitch, and therefore are likely to cause telephone interference. The losses caused by these harmonics and not covered in the paper are usually of no practical importance. (In the usual open-circuit, no-load test of synchronous machines, such losses are of course not present at all.) This problem can be attacked along the lines suggested in a paper which I presented before the A.I.E.E.* and an article published in the *Electrical World* of 1906 on the subject "Magnetizing Currents in Polyphase Induction Motors."

The method given by Mr. Pollard should prove an excellent guide in choosing a secondary slot pitch appreciably different from the primary without reaching the point where the secondary damper winding losses assume undesirable proportions.

Ernst Weber: The formula (1) of the paper for the total damper winding loss contains two factors kw_1 and kw_2 which are very important since they represent the relative magnitudes of the fundamental and second harmonic of the pulsation wave and are involved in equation (**10**) as the square. It is important, therefore, to know their values with some accuracy. The author determined kw_1 and kw_2 from flux maps taking into account the effect of the relative widths of slot and tooth upon the field form. Obviously this method can be very inaccurate.

In 1928, I published a paper on the air-gap coefficient in machines* in which I gave a mathematical expression for the pulsation curve as

$$B_p = \overline{B}_p \cdot \left(\sin \frac{x\pi}{t_s} \right)^{2n} \qquad (1)$$

with

$$n = \frac{t_s - b_s}{b_s}.$$

This expression represents for the integers n, discrete wave forms of just the type desired to take care of the ratio of slot width b_s to slot pitch t_s. The mean value follows by integration as

$$(B_p)_m = \overline{B}_p \cdot \frac{1.3.5.\ldots.(2n-1)}{2.4.6.\ldots.2n}, \qquad (2)$$

and by Fourier's analysis the amplitudes of the first and second harmonics may be determined. Their ratios to $(B_p)_m$ in (2) give the amplitude factors

$$K_{w1} = \frac{4}{\pi} \cdot \frac{2.4.6.\ldots(2n)}{1.3.5.\ldots(2n-1)(2n+1)},\ K_{w2} = \frac{2n}{2n+2} \cdot K_{w1}. \qquad (3)$$

Contrasted with those values obtained by Mr. Pollard it is to be seen that formula (3) gives lower values for kw_1 and higher ones for kw_2 than Fig. 3 of the paper; but the variation with $\frac{b_s}{t_s}$ is of the same character. This may be taken as a satisfactory check and yet may provide a means for further correction and approximation, since it is quite easy to compute even the higher harmonics.

The effect of increase of resistance of the damper loss due to alternating current has been treated with close approximation by R. Rogowski, *Archiv. f. Elektrotech.* 1913, Vol. 2, p. 81. A formula for the ratio of a-c. loss to d-c. loss is given there so that the statement regarding the absence of a rigorous treatment of this subject does not seem correct.

As to the effect of the damper slots on telephone interference may I summarize some experiences I had in 1926 when I was associated with the Siemens-Schuckert-Werke in Vienna. In an unpublished paper I proved that minimum loss and minimum interference harmonics are two nearly opposing tendencies. For slot harmonics to be a minimum the voltage wave of the generator, I could set up the condition $0.62 < \frac{t_s}{t_r} < 0.70$ and experiments as well as careful investigation of many oscillograms showed satisfactory agreement. The damper losses, however, are not at the minimum within this range of the above ratio. Although it does not make any difference whether we write $\frac{t_s}{t_r}$, or the inverse value $\frac{t_r}{t_s}$, it seems to me that generally the stator slot pitch is made smaller than the damper slot pitch and I should like to know if Mr. Pollard purposely used $t_r < t_s$.

E. I. Pollard: Mr. Weber questions the accuracy of the

No-Load Conditions of Single-Phase Induction Motors and Phase Converters, A.I.E.E. Trans., 1918, Vol. 37, p. 539.

*E. Weber, *E.T.Z.*, 1928, Vol. 49, p. 858.

curves for Kw_1 and Kw_2 in Fig. 3. As stated, these curves were calculated from flux maps which Mr. Weber states "can be very inaccurate." Concerning the accuracy of flux maps, I refer Mr. Weber to reference 1 of the bibliography. Although I am not familiar with the derivation of Mr. Weber's equations, his relatively simple solution of a complex mathematical problem indicates that various simplifying assumptions must have been made. For instance, he neglects the effect of the ratio of slot width to gap length, as I did in Fig. 3. The assumption of a considerably different ratio of bs/g would in part account for the discrepancy, although the effect is secondary importance, except in extreme cases.

The article by Von W. Rogowski referred to by Mr. Weber considers the calculation of eddy current factors for various arrangements of coils and bars in open slots, as has been done by various other investigations, notably A. B. Field, B. Hague and Gilman. But does Rogowski take into account the effect upon field distribution and eddy factor of partially closing the slot above the bar? He does not consider the case of the round bar.

In general the rotor slot pitch is smaller than the stator slot pitch, except possibly in the case of small low-voltage machines. This is because a large number of bars is desirable from the standpoint of reduced rotor reactance and better power factor for induction motor performance of synchronous motors as pointed out by Mr. Hellmund. Also, if a smaller number of larger bars were used, they would occupy a greater depth in the pole head and pole tips, requiring a deeper pole head and resulting in increased pole tip leakage.

Sine-Wave Generators

Cylindrical Rotor Winding Connections and Analysis of Rotor M.M.F. Waves

BY LLOYD P. SHILDNECK*

Associate, A.I.E.E.

Synopsis.—*This paper describes and compares various types of double layer rotor winding connections for non-salient pole machines, with a particular application to sine-wave generators. Design coefficients are obtained by a mathematical analysis of the derived equations for the rotor m.m.f. Five specific types of connections, the V, Y, X, U, and W types, are discussed, as well as the general case. The new types advocated are seen to be superior to the old, giving a greater output for a given maximum temperature.*

GENERAL

CERTAIN applications, as iron testing, meter testing,[1] cable testing, etc., require a sine wave of generator voltage under all load conditions. The voltage wave form of an ordinary salient-pole generator is usually very good at no load, but becomes distorted under load. To overcome this inherent defect, the round-rotor type of sine-wave generator was developed. The stator of this type is very similar to that of a salient-pole generator. For the field, however, a round punched rotor is used with a double layer winding distributed in evenly spaced slots and connected to give nearly a sine wave of m.m.f.

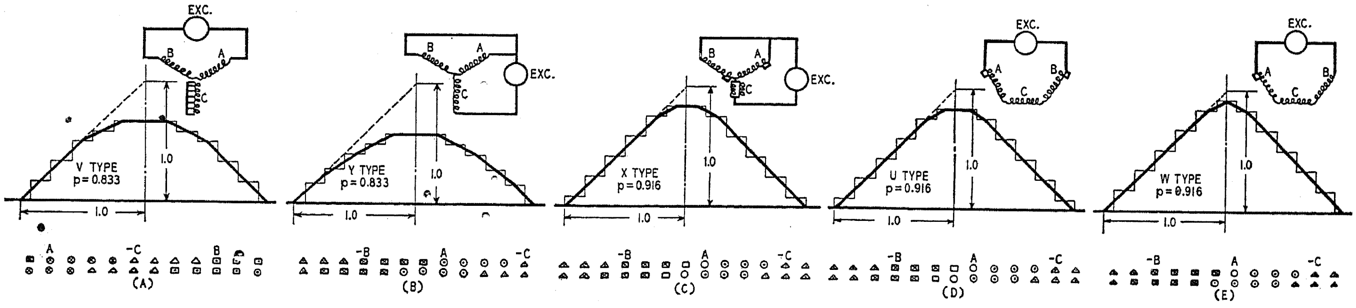

FIG. 1—ROTOR M.M.F. WAVES

Because of the equal permeances in both axes of this type, a sinusoidal air gap flux density wave is set up by any combination of sinusoidal rotor and stator m.m.f. waves. Thus a sine-wave generator has fundamentally three requirements: a round rotor, a sine wave of stator m.m.f., and a sine wave of rotor m.m.f. Of these, only the third requirement is discussed here, since the construction of the rotor is similar to that of a wound rotor induction motor, and the methods of producing a sinusoidal stator m.m.f. are well understood.[2,3,4]

The very useful, but rather tedious, *graphical* method is used to plot the flux density fields of salient-pole machines.[5] In round rotor machines, however, the permeance variable disappears, and the field m.m.f. and field flux density wave shapes are identical. Consequently, to determine the field flux density wave shape, it is but necessary to derive *analytically* the shape of the field m.m.f. wave. (The assumptions are: infinite permeability of the iron and infinite number of rotor teeth.) This method allows a direct determination for the maximum or minimum value of any coefficient, by equating the derivative to zero. The analytical method is general and applicable to all round rotor machines, but the analyses made in this paper are directly applicable only to round rotor machines with double-layer rotor coil windings. This includes the synchronous-induction motor and the sine-wave generator, but not the turbine generator.

DESCRIPTION OF TYPES OF ROTOR WINDING CONNECTIONS

A study of the two types of connections used heretofore (*V* and *Y*) revealed certain inherent undesirable features. Three new types (*X*, *U* and *W*) were developed as a direct result of eliminating these deficiencies.

V Type. This old type is obtained by winding the rotor with an ordinary three-phase double-layer winding, and connecting phases *A* and *B* in series with the exciter, as shown in Fig. 1A. Thus only two-thirds of the coils are used to produce the field m.m.f. The remaining coils are short-circuited individually to assist in damping out the backward rotating component of single-phase armature reaction. So

$$s = 0.333$$

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*A-C. Engineering Dept., General Electric Co., Schenectady, N. Y.

1. For references see Bibliography.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

is the defining equation for this type, where s = the ratio of short-circuited to total coils. This type is inferior because an insufficient ratio of coils is used in the d-c. circuit.

Y Type. In the other old type, the three phases are connected to the exciter as shown in Fig. 1B. This type has good damping features (since polyphase currents can flow to damp out the backward rotating flux), but it has serious heating limitations. The third leg C, carrying full field current, has a heating four times as great as that of either leg A or B. The heating in this "bottle-neck" leg then definitely limits the output of the rotor. The Y type has another undesirable feature in that opposing currents in the top and bottom of the same slot are used to obtain the flat top of the m.m.f. wave. (See Fig. 1B.) This, of course, increases the I^2R loss.

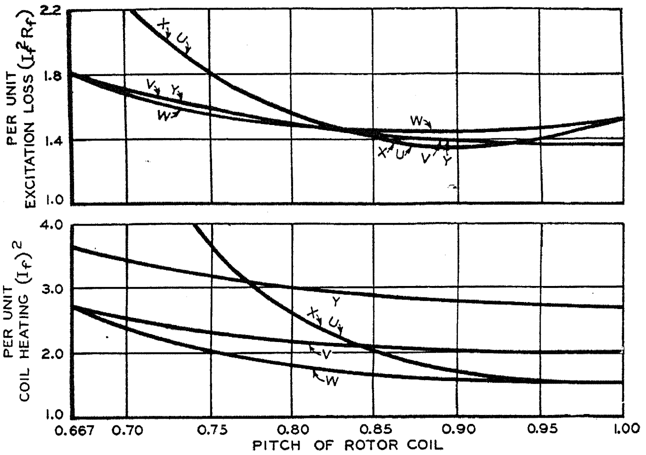

FIG. 2—FIELD EXCITATION LOSS $(I_f^2R_f)$ AND HEATING (I_f^2) FOR UNIT FUNDAMENTAL

Five specific types (V, Y, X, U, W)

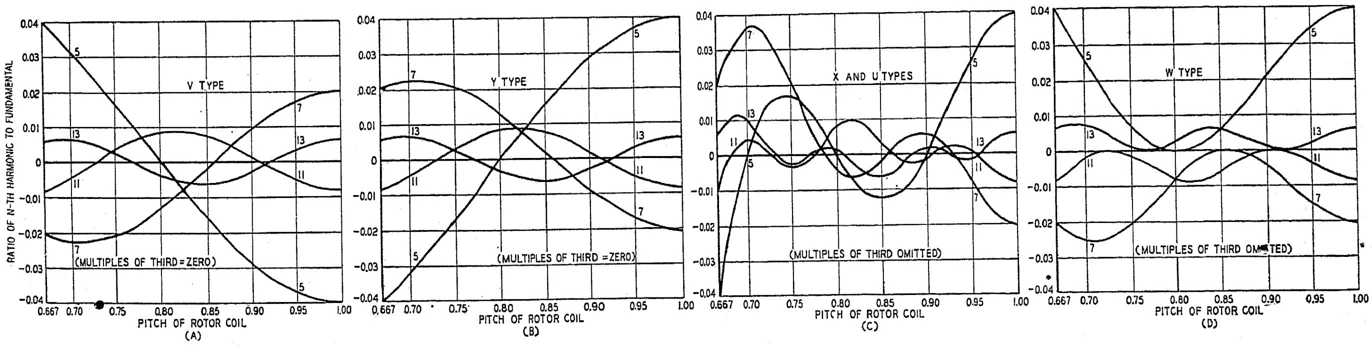

FIG. 3—HARMONICS IN ROTOR M.M.F. WAVES

Five specific types (V, Y, X, U, W)

X Type. This new type is a modification of the Y type with the following changes as shown in Fig. 1C.

Phase C is split into two parallel circuits, *thereby eliminating the bottle-neck* feature of the Y type; also the coils that carry opposing currents in the Y type are removed from the d-c. circuit, *thereby eliminating the unnecessary power loss.* From the beginning of each pole group in phase A (1-pitch) × (coils per pole) are taken and a like number from the end of each pole group in phase B, so that the two groups of short-circuited coils are adjacent. Thus

$$s = 2\,(1 - p)$$

is the defining equation for the X type, where p = rotor coil pitch.

U Type. This new type is a modification of the X type, obtained from it by joining the circuits in series, instead of in parallel, as shown in Fig. 1D. In practise this makes a very simple winding. All the coils in each pole pitch are placed in series; then those end coils which would oppose each other, are short-circuited individually. The defining equation for the U type is also

$$s = 2\,(1 - p)$$

W Type. This new type is the same as the U type, except just half as many coils are short-circuited. (See Fig. 1E.) The defining equation for this type is then

$$s = (1 - p)$$

The W type is a limiting case; any larger value of s would cause "opposing" coils, as in the Y type.

General Case. In the five specific types previously described, s is either a function of p or else is constant, enabling all the coefficients to be plotted in terms of the one variable p. In a study of the general case, however, both p and s are allowed to vary independently. Fig. 2 shows very strikingly the definite superiority of the V type (in which a series connection is used) over the Y type (in which one leg is placed in series with two paralleled legs). This superiority of the series type has also been proved by tests made by Messrs. Ganapati and Parikh.[6] Since a part series, part parallel type is inherently inferior to a series type, only the latter will be discussed in the general case. The general type of winding connections then results from placing all the coils under each pole in series, short-circuiting individually any fractional part s of them, and using any coil pitch p. It necessarily includes the V, U, and W types, but not the Y type.

It is always possible to obtain values

$$s = n\,(1 - p)$$

where n is any integer or zero. Under certain conditions of pitch, n may be fractional: $n = 1$ gives the W type; $n = 2$ gives the U or X type; all possible values of n are included in the general type.

DISCUSSION

The mathematical expressions for the m.m.f. waves of the various types of winding connections are analyzed by the method of Fourier series to obtain A_n, the value of the nth harmonic. The substitution $n = 1$ gives the fundamental (A_1) or useful part of the wave, which is proportional to the output of the rotor. The reciprocal of A_1 gives the field current (I_f) necessary to produce unit fundamental. From this value (I_f), the heating (I_f^2) and the excitation power loss ($I_f^2\ R_f$) are easily obtained. These values are listed in Table I.

Damping. The damping provided by the series types of winding connections (V, U, and W) is not very effective, because currents cannot flow in such windings so as to set up a revolving magnetic field of constant amplitude. The Y type and the X type are better fitted for damping out any backward rotating flux. However, in the case of sine-wave generators, it is deemed better practise to install an auxiliary damping winding than to make the field winding perform double duty.

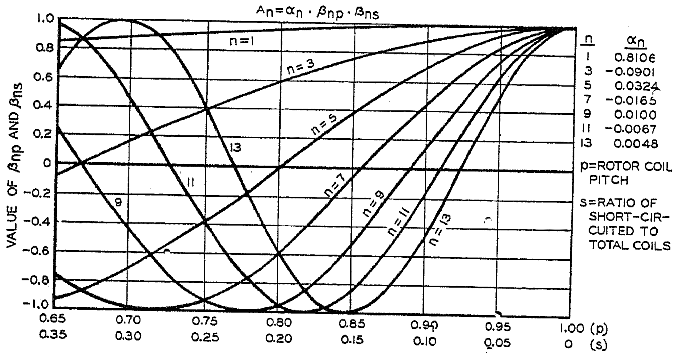

FIG. 4—MAXIMUM HEIGHT OF *N*TH HARMONIC (A_n)

(General type)

Wave Form. The harmonics* present in the m.m.f. waves of the various types are shown in Fig. 3. The following tabulation shows the extremely small ratios of harmonics to fundamental appearing in the line-to-line terminal voltage of typical machines with various types of rotor winding connections.

	Y type Stator pitch = 0.833 Stator slots = 72 Rotor pitch = 0.800	*X* or *U* type Stator pitch = 0.800 Stator slots = 60 Rotor pitch = 0.928	*W* type Stator pitch = 0.81 Stator slots = 84 Rotor pitch = 0.889
A_3/A_1	0	0	0
A_5/A_1	0	0	0.00027
A_7/A_1	0.00047	0.00003	−0.00016
A_9/A_1	0	0	0
A_{11}/A_1	0.00081	0.00026	−0.00009
A_{13}/A_1	0.00030	0.00004	−0.00014
A_{15}/A_1	0	0	0

The curves in Fig. 4 are rather unique in that they give A_n, the value of the nth harmonic, for the general type winding connections, for any value of (n) from 1 to 13, any value of (p) from 0.65 to 1.00, and any value of (s) from 0 to 0.35, *without interpolation.* This is obtained by using a double abscissa, as shown.

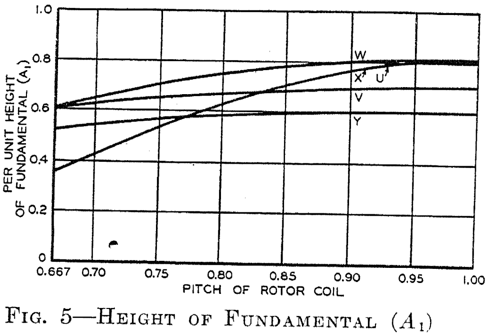

FIG. 5—HEIGHT OF FUNDAMENTAL (A_1) FOR UNIT COIL CURRENT

Five specific types (V, Y, X, U, W)

Fundamental. Fig. 5 shows the height of fundamental, or useful part of the m.m.f. wave, given by the various types of connections, for the same maximum current flowing in any part of the winding. A comparison at the pitches necessarily used to give good wave forms shows a very decided advantage in favor of the new types of connections (X, U, W).

A_1 for Y type at 0.82 pitch = 0.584
A_1 " V type " 0.82 " = 0.674
A_1 " X and U type " 0.93 " = 0.786
A_1 " W type " 0.90 " = 0.790

Thus the new types have a greater output for the same maximum temperature.

K_m and Air-Gap Ampere-Turns. The derivation of K_m illustrates how easy it is to obtain design coefficients analytically after the expressions for the m.m.f. wave are set up. K_m is simply the ratio of the maximum to the average height of the m.m.f. wave. This coefficient is very useful in determining the maximum air-gap flux density, which is proportional to the air-gap ampere-turns. The values of K_m for the general type are given in Fig. 6.

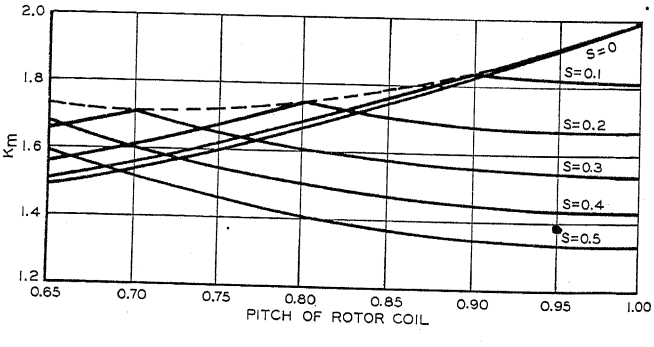

FIG. 6—$K_m = \dfrac{\text{MAXIMUM FLUX DENSITY}}{\text{AVERAGE FLUX DENSITY}}$

(General type)

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*The third and multiples of the third are omitted in the figures for the X, U, and W types, since the Y connection of the stator eliminates them from the line-to-line terminal voltage.

Excitation Power Loss ($I_f^2 R_f$). Fig. 2 shows the power required for unit fundamental with the various specific types of connections. A comparison at the pitches to give good wave form shows that the new types require slightly less power.

$I_f^2 R_f$ for Y and V types at 0.82 pitch = 1.47
$I_f^2 R_f$ " X and U types " 0.93 " = 1.39
$I_f^2 R_f$ " W type " 0.90 " = 1.44

The minimum excitation for any type occurs where the first derivative equals zero.

Minimum $I_f^2 R_f$ for Y and V types = 1.356 at p = 1.000
Minimum $I_f^2 R_f$ " X and U types = 1.385 " p = 0.902
Minimum $I_f^2 R_f$ " W type = 1.445 " p = 0.887

Fig. 7 shows the excitation power for the general type. For any value of pitch, the excitation loss decreases as s is increased, reaching a minimum at s = 0.26. Therefore, for any given pitch p, 26 per cent of the coils should be short-circuited to give minimum excitation power loss.

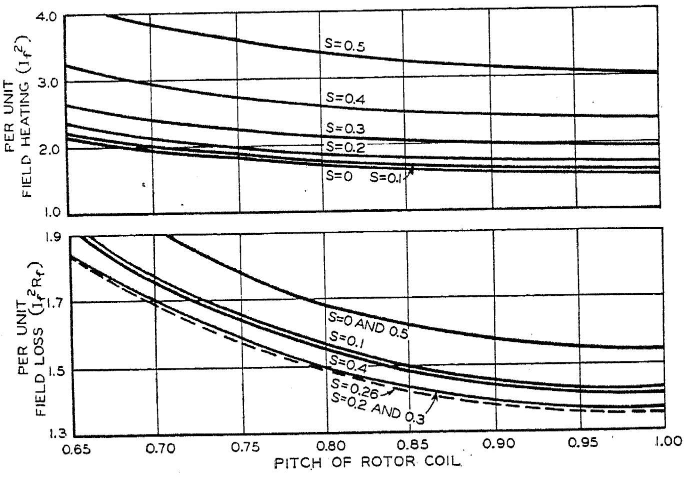

FIG. 7—FIELD HEATING (I_f^2) AND EXCITATION LOSS ($I_f^2R_f$) FOR UNIT FUNDAMENTAL

(General type)

Field Heating. The square of the maximum current in any slot gives a rough indication of the field heating. The new types of connections are decidedly superior to the old in the matter of field coil heating for the same fundamental. (See Fig. 2.)

I_f^2 for Y type at 0.82 pitch = 2.932
I_f^2 " V type " 0.82 " = 2.200
I_f^2 " X and U types " 0.93 " = 1.620
I_f^2 " W type " 0.90 " = 1.601

To obtain low coil temperature, it is seen from Fig. 7, general type, to be necessary to use a very low value of (s). The high values of pitch (and consequently, low values of s) permissible with the new types of winding account for the cool winding. Since field temperature usually limits the output of a machine, this low field heating constitutes the major superiority of the new types over the old.

CONCLUSIONS

1. Three new types of connections, X, U, and W, have been developed and shown to be superior to the old types, V and Y. By their use 15 per cent or more increase in output can be secured from the rotor without increasing the maximum temperature rise, or impairing the voltage wave shape. The Y type is decidedly inferior to the other types, because of its high maximum temperature rise.

2. The analysis made in this paper is quite general and is directly applicable to all round-rotor synchronous machines with double-layer rotor coil windings. Sine-wave generators and synchronous-induction motors are good examples of this type.

TABLE I

	A_n(For Unit I_f)	I_f(For Unit A_1)
(V)	$(8/n^2 \pi^2)(\cos n \pi/6)(\sin n \pi p/2)$	$1.425/(\sin \pi p/2)$
(Y)	$(8/n^2 \pi^2)(\cos^2 n \pi/6)(\sin n \pi p/2)$	$1.648/(\sin \pi p/2)$
(X)	$(8/n^2 \pi^2)(-\cos n \pi p)(\sin n \pi p/2)$	$2.470/(-\cos \pi p)(\sin \pi p/2)$
(U)	$(8/n^2 \pi^2)(-\cos n \pi p)(\sin n \pi p/2)$	$1.235/(-\cos \pi p)(\sin \pi p/2)$
(W)	$(8/n^2 \pi^2)(\sin n \pi/2)(\sin^2 n \pi p/2)$	$1.235/(\sin^2 \pi p/2)$
(G)	$(8/n^2 \pi^2)(\cos n \pi s/2)(\sin n \pi p/2)$	$1.235/(\cos \pi s/2)(\sin \pi p/2)$

	$I_f^2 R_f$ (For Unit A_1)
(V) and (Y)	$1.356/(\sin^2 \pi p/2)$
(X) and (U)	$1.525\ (2p - 1)/(\cos^2 \pi p)\ (\sin^2 \pi p/2)$
(W)	$1.525\ (p)/(\sin^4 \pi p/2)$
(G)	$1.525\ (1 - s)/(\cos^2 \pi s/2)\ (\sin^2 \pi p/2)$

	K_a	K_m
(V)	$1.21/(\sin \pi p/2)$	$12/[8 - 9(1 - p)^2]$
(Y)	$(3p + 1)/(9 \sin \pi p/2)$	$3(3p + 1)/[7 - 9(1 - p)^2]$
(X)	$(\pi/2)(2p - 1)/(-\cos \pi p)(\sin \pi p/2)$	$2(2p - 1)/[1 - 5(1 - p)^2]$
(U)	$(\pi/2)(2p - 1)/(-\cos \pi p)(\sin \pi p/2)$	$2(2p - 1)/[1 - 5(1 - p)^2]$
(W)	$(\pi/2)(p)/(\sin^2 \pi p/2)$	$2(p)/[1 - 2(1 - p)^2]$
(G-1)	$(\pi/2)(1 - s)/(\cos \pi s/2)(\sin \pi p/2)$	$2(1 - s)/[(1 - s^2) - (1 - p)^2]$
(G-2)	$(\pi/2)(p)/(\cos \pi s/2)(\sin \pi p/2)$	$2(p)/[(1 - s^2) - (1 - p)^2]$

	(M.M.F. wave (For unit I_f)
(V)	$[x]_0^{x_1}\ ;\ [x/2 + (p/2 - 1/6)]_{x_1}^{x_2}\ ;\ [2/3]_{x_2}^{1}$
(Y)	$[x]_0^{x_1}\ ;\ [3x/4 + (p/4 - 1/6)]_{x_1}^{x_2}\ ;\ [x/2 + 1/6]_{x_2}^{x_3}\ ;\ [p/2 + 1/6]_{x_3}^{1}$
(X)	$[x]_0^{x_1}\ ;\ [x/2 + (3p/2 - 1)]_{x_1}^{x_2}\ ;\ [2p - 1]_{x_2}^{1}$
(U)	$[x]_0^{x_1}\ ;\ [x/2 + (3p/2 - 1)]_{x_1}^{x_2}\ ;\ [2p - 1]_{x_2}^{1}$
(W)	$[x]_0^{x_1}\ ;\ [x/2 + (2p - 1)/2]_{x_1}^{1}$
(G-1)	$[x]_0^{x_1}\ ;\ [x/2 + (p - s)/2]_{x_1}^{x_2}\ ;\ [1 - s]_{x_2}^{1}$
(G-2)	$[x]_0^{x_1}\ ;\ [x/2 + (p - s)/2]_{x_1}^{x_2}\ ;\ [p]_{x_2}^{1}$

	x_1	x_2	x_3	y_1	y_2	y_3
(V)	$p - 1/3$	$5/3 - p$	—	$p - 1/3$	$2/3$	—
(Y)	$p - 2/3$	$4/3 - p$	p	$p - 2/3$	$5/6 - p/2$	$1/6 + p/2$
(X)	$(3p - 2)$	p	—	$3p - 2$	$2p - 1$	—
(U)	$(3p - 2)$	p	—	$3p - 2$	$2p - 1$	—
(W)	$2p - 1$	1	—	$2p - 1$	p	—
(G-1)	$p - s$	$2 - p - s$	—	$p - s$	$1 - s$	—
(G-2)	$p - s$	$p + s$	—	$p - s$	p	—

(G-1)—General type, region I, where $1 \geqq p \geqq (1 - s)$
(G-2)—General type, region II, where $(1 - s) \geqq p \geqq s$.

3. All practical methods of making rotor coil connections have been considered, and the final results of the study listed in Table I. This comprehensive list of analytical expressions for design coefficients has been partially expanded into curve form for greater ease in making comparisons. By virtue of the information given in this table, it is now possible to choose the proper values of p and s to fit any particular case.

SYMBOLS

A_1 = maximum height of fundamental of rotor m.m.f. wave.

A_n = maximum height of nth harmonic of rotor m.m.f. wave.

I_f = field current (= coil current in series type).

K_a = ratio of fundamental armature flux to fundamental field flux for same peak values of m.m.f.

K_m = ratio of maximum to average air gap flux density.

p = rotor coil pitch.

R_f = resistance of the field winding.

s = ratio of short-circuited rotor coils to total coils.

x_1, x_2, x_3 = x-coordinates of the first, second, and third points of discontinuity of the slope of the m.m.f. wave.

y_1, y_2, y_3 = y-coordinates of the above three points.

UNIT QUANTITIES

Unit ampere-turns = 1 ampere-turn

Unit field current = 1/(turns per pole) amperes.

Unit number of coils = coils per pole.

Unit resistance = resistance of all coils in series.

Unit distance along X-axis = (pole pitch)/2.

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Discussion

Ernst Weber: Although the author emphasizes in the first sentence of his paper the need for a sinusoidal wave form of the generated voltage under all load conditions, he does not mention the influence of saturation on the voltage curve in the new types of sine-wave generators. Is the table, giving the ratios of the various harmonic amplitudes to the amplitude of the fundamental, computed with or without regard to saturation? It is known that under the influence of saturation, it is possible to secure sine voltage at only a definite value of the exciting current, and, that distortion takes place even at no load, if the excitation current is varied. The attempt has been made to formulate the saturation curve analytically and to compute in a general way the field curves of synchronous machines by means of using a so-called "standard" magnetization curve.* It may well be possible to apply a similar procedure to sine-wave generators and to gain more information as to the required m.m.f. curves in order to secure sine voltage over at least a certain range of excitation current.

L. P. Shildneck: As stated in the paper, all saturation effects were neglected. It may well be possible analytically to include the effect of saturation on the flux density wave form, as suggested by Mr. Weber. However, sine-wave generators are usually required to operate over a wide range of conditions, and saturation effects must be reduced to a minimum. This is done by making the air-gap length relatively large, and the flux density in the teeth relatively low. Thus for sine-wave generators of current design it was considered unnecessary to include the effect of saturation.

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*"The Magnetic Field Forms of Synchronous Machines Under No-Load Condition," E. Weber, *Archiv. f. Elektrotech.*, 1927, Vol. 19, p. 193.

Engineering Features of Three-Power Locomotives

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and

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Synopsis.—Three-power locomotives capable of operating from an external distribution system or on internal power are in use by the New York Central and the Delaware, Lackawanna and Western Railroads. In one case the supply voltage is 600 volts direct current and in the other 3,000 volts direct current.

The authors discuss the characteristics of this new type of locomotive and its equipment, as well as the methods of calculating the capacity and characteristics required for the service. The coordination of the diesel engine, generator, battery, and motor equipment has been carefully worked out to insure successful operation of the complete unit.

* * * * *

GENERAL

THE three-power electric locomotive operates from any of the following energy sources:

1. External power; that is, trolley or third rail.
2. Internal power from a storage battery.
3. Internal power from a storage battery and engine-driven generator connected in parallel.

Ability to operate from these power supplies gives the three-power locomotive an outstanding degree of flexibility.

Regardless of the system voltage upon which the three-power locomotive is designed to work, the general scheme of operation is the same. When using external power, the unit performs exactly as a straight electric locomotive, although the engine-generator set may be run to charge the battery. On non-electrified tracks power may be taken from the battery alone. However, if the engine is running, traction and auxiliary power are taken entirely from the engine-generator set up to its capacity above which the battery automatically assumes the excess load. On light loads, the engine-generator may run the locomotive and charge the battery at the same time.

The New York Central Railroad tried out the first three-power locomotive in 1928, and in 1930 put 35 units into transfer and switching service on the West Side in connection with the complete elimination of steam service from Manhattan Island. The Delaware, Lackawanna & Western Railroad put two three-power locomotives into similar service in 1930 to help eliminate steam from the Bergen Tunnels with the advent of electrified suburban service from Hoboken.

Fig. 1 shows outline dimensions of the New York Central locomotive and Fig. 2 the location of apparatus in the cab. The Delaware, Lackawanna & Western locomotive is of similar mechanical and electrical design. Table I gives the pertinent data on the two locomotives and shows wherein they differ, due largely to the difference in voltages.

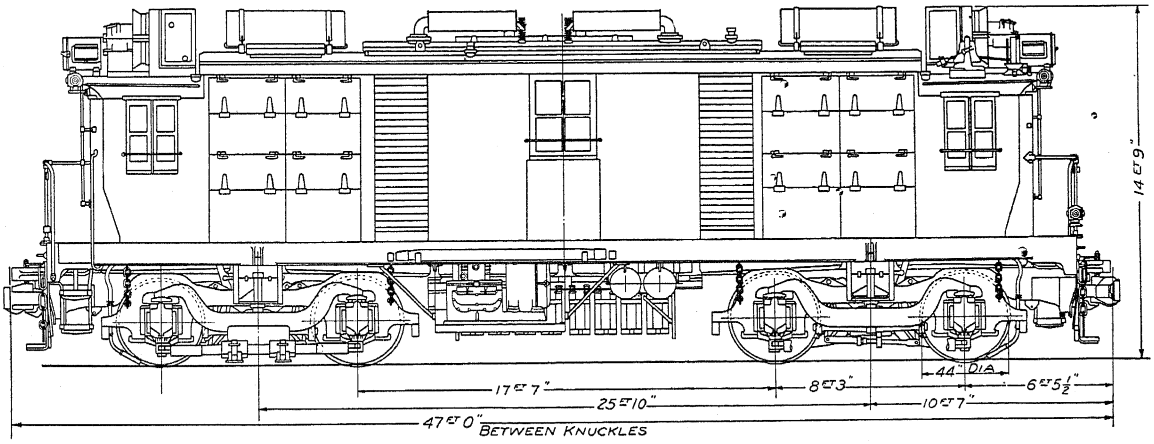

FIG. 1—OUTLINE OF NEW YORK CENTRAL LOCOMOTIVE

GENERAL REQUIREMENTS FOR INTERNAL POWER PLANT

The average power required for switching service is small while the peak demands are relatively large. The theory upon which the internal power operation of the three-power locomotive is based is that the battery will provide the excess power required for the peak demands, allowing the engine size to be cut to a minimum. Extensive tests on internal power were run in 1928 on the first New York Central three-power locomotive to determine the actual extent of these power

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[1] Transportation Engg. Dept., General Electric Co., Erie, Pa.

Presented at the Winter Convention of the A.I.E.E., New York, N.Y., January 25-29, 1932.

TABLE I—DATA

	New York Central	Delaware, Lackawanna & Western
Weight of locomotive in running order	257,000 lb.	248,000 lb.
Rated voltage on external power	600 (3rd rail)	3,000 (trolley)
Continuous horsepower on external power	1,300	1,450
Hourly horsepower on external power	1,665	1,600
Continuous tractive effort	24,600 lb.	22,200 lb.
Hourly tractive effort	34,100 lb.	25,200 lb.
Speed on external power at continuous tractive effort	19.8 mi. per hr.	24.5 mi. per hr.
Speed on external power at hourly tractive effort	18.3 mi. per hr.	23.8 mi. per hr.
Maximum tractive effort (25 per cent adhesion)	64,250 lb.	62,000 lb.
Traction motors:		
Number	4	4
Type	Axle hung	Axle hung
Voltage	600	1,500/3,000
Gearing	72/17	72/17
Ventilation	Forced	Forced
Oil Engine:		
Type	I-R solid injection	I-R solid injection
Horsepower	300	300
Traction generator	200 kw.	200 kw.
Battery:		
Type	Exide-Ironclad TL-27	Exide-Ironclad MVA-21
Number of cells	240	360
Ampere-hr., capacity at 6 hr. rate of discharge	650	340
Kw-hr. capacity at 6 hr. rate of discharge	301	242
Average voltage at 6 hr. rate of discharge	464	712
Connection of cells	Series	Series
Grounded at	Negative	Negative
Control:		
Type	Electro-pneumatic Non-automatic	Electro-pneumatic Non-automatic
Motor combinations on external power	3	2
Motor combinations on internal power	3	3

requirements in heavy switching service. The average for the maximum observed day for three-shift service was approximately 500 kw-hr. per eight-hour shift, or 62.5 kilowatts average load. The average work for 65 shifts of 3-shift service was 447 kw-hr. per shift. Frequently peak loads of 800 kilowatts were recorded, or more than twelve times the maximum observed average load per eight-hour shift.

For long battery life and reasonable maintenance the battery discharge should be limited to 125 per cent of the total capacity at the six-hour rate for each twenty-four hours the locomotive is operating in three-shift service. To limit the battery discharge to this value successfully, it is essential that the correct generator characteristic be used. This characteristic should be high enough and of the proper shape to insure that the engine carries the entire load up to the engine-generator capacity, but not high enough to allow the battery to recharge so rapidly that the engine must be shut down frequently to prevent overcharging. Without the engine, the battery must supply the entire load, which results in the discharge over a twenty-four hour period being considerably greater when the engine is shut down often than with the engine running most of the time. The initial locomotive tests showed that the engine should be run at least 70 per cent of the time to meet these discharge limitations.

A generator characteristic low enough to insure engine operation 70 per cent of the time is too low to give the battery an equalizing charge. Therefore, two generator characteristics are necessary; first, a normal characteristic for regular operation; and second, a higher or equalizing characteristic for use periodically for giving the battery a gassing charge.

The minimum size of the battery is fixed by traction motor characteristics and weight of the locomotive on one hand and speed requirements on the other. The former determine the maximum current which may be drawn from the battery as limited by adhesion, and the latter the number of cells. With traction motors in parallel, the total current values on the New York Central locomotive at various adhesions are approximately as follows:

Adhesion	Amperes
25 per cent	3,700
20 per cent	3,100
15 per cent	2,500

The battery selected for this locomotive must be large enough to deliver these currents and to carry successfully at least 4,000 amperes for a period long enough to blow the 2,500-ampere battery fuses. Because of the decreased available capacity of the battery at high discharge rates, it is not practical to operate this type of locomotive on internal power at high adhesion and high speed for any great length of time. However, the extreme load conditions given above are possible and must be successfully met for short periods in practical operation.

In case of necessity it is possible to operate the three-power locomotive at high adhesion over considerable distances by running at reduced speeds with the motors in series or series-parallel. In such cases the current to the motors is reduced to ¼ or ½ of the value which would be required with the motors in parallel for the same total tractive effort. As the engine-generator set will continue to deliver its full power, all of this reduction in current is taken from the battery discharge rate, thus greatly increasing its available capacity. In addition to this, the increase in available kw-hr. capacity of the battery increases the duration of the movement and to this extent increases the kilowatt hours delivered by the engine, thus further increasing available mileage.

For fast and responsive operation of the locomotive it is advantageous to have the battery voltage near the rated voltage of the traction motors.

Battery short time current rating—2 minutes—3,000 amperes.
Engine horsepower—300.
Assumed time engine running—85 per cent.
Maximum amount battery may be discharged in 24-hour period, 125 per cent of capacity at 6-hour rate which is equivalent to the following:
  812 ampere hours in 24 hours.
  271 ampere hours per 8-hour shift.
  114 kilowatt hours per 8-hour shift, (average discharge voltage 420).

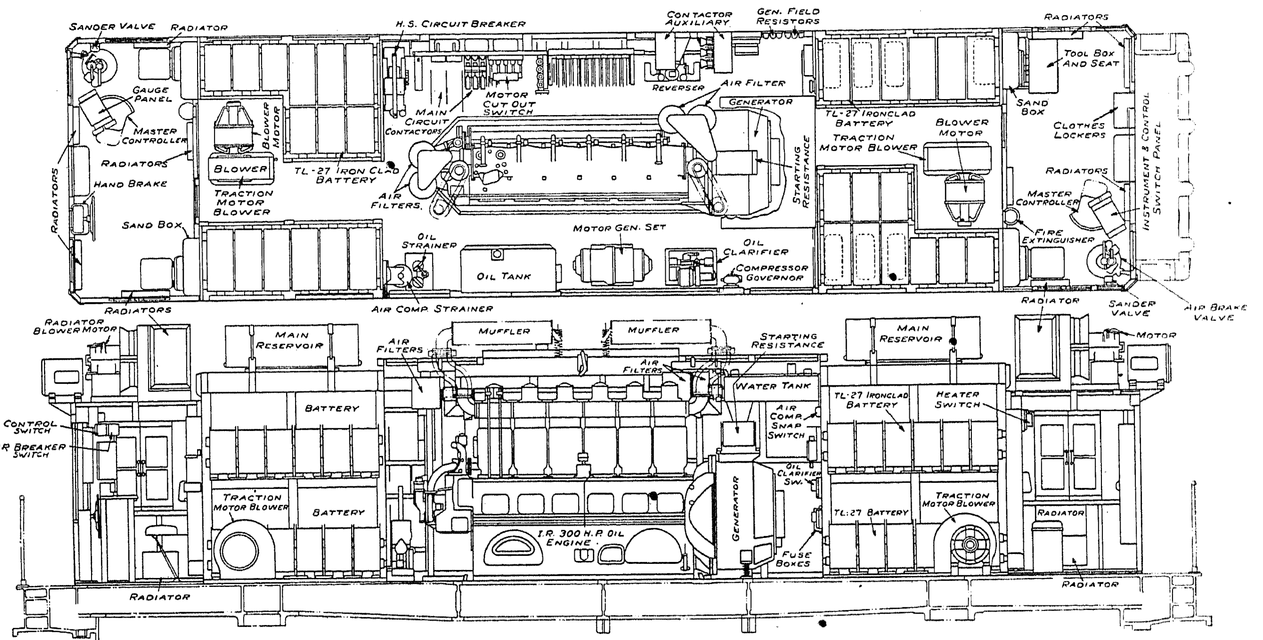

FIG. 2—LOCATION OF APPARATUS ON NEW YORK CENTRAL LOCOMOTIVE

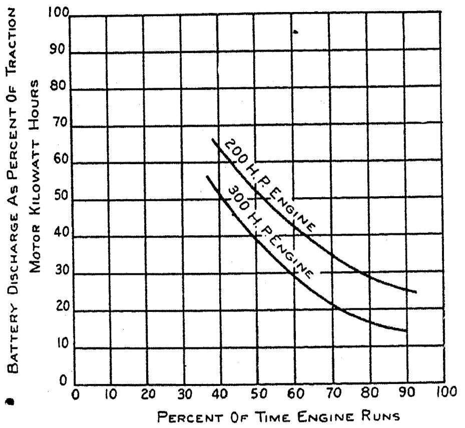

FIG. 3—RELATION OF BATTERY DISCHARGE TO TIME OF ENGINE OPERATION

Curves showing time oil engine is operated plotted against battery discharge as per cent of motor kilowatt-hours for initial New York Central locomotive in heavy switching service

SELECTION OF INTERNAL POWER PLANT FOR THE NEW YORK CENTRAL LOCOMOTIVE

As a result of the initial New York Central locomotive tests and the weight and speed considerations just outlined, a design for the New York Central locomotive was adopted to which the following data apply:
(See Table I also.)

Fig. 3 shows the effect of the time the engine operates upon the battery discharge. These curves are plotted from data taken during the tests on the initial New York Central locomotive. For the 300-hp. curve a number of points was secured between 45 and 75 per cent operation, the curve being extended somewhat beyond these values upon reasonable assumptions for 0 per cent and 100 per cent engine operation. The 200-hp. engine curve is based largely on assumptions since only one point (93 per cent) was available from the test data, this test being made by setting the fuel finger of the oil engine governor to limit the engine output to 200 hp. It is apparent, however, that this curve will intersect the ordinate with the engine running 0 per cent of the time at the same point as the curve for the 300-hp. engine, and, hence, should draw nearer to the latter curve as the per cent of the time the engine runs is decreased. Therefore, it is believed that the curve approximately represents the condition that would exist for a 200-hp. engine.

Fig. 3 shows that with a 300-hp. engine running 85 per cent of the time, the battery discharge will be equivalent to approximately 15 per cent of the power delivered to the traction motors. Therefore, the maximum average work which the locomotive can do per 8-hour shift when operating 3 shifts per day on internal power is: 114/0.15 = 760 kilowatt hours. Using Fig. 3 and interpolating for the 250-hp. engine, the following maximum kilowatt hours per shift may be calculated for different engine sizes using the present 301-kw-hr. battery:

Engine hp.	Maximum kw-hr. per shift
300	760
250	531
200	403

From this it appears that a 250-hp. engine might have been used successfully with the 301-kw-hr. battery to meet the requirement of 500 kw-hr. per shift. However, it was thought advisable to allow some margin for contingencies, a 25 per cent margin giving a maximum of 625 kilowatt hours, and with a 250-hp. engine, this would require a battery of approximately 355-kw-hr. capacity. The combination of a 300-hp. engine and a 301-kw-hr. battery was finally chosen because of the greater margin of capacity per shift as well as the ability to use an oil engine which had been thoroughly proved in railway service.

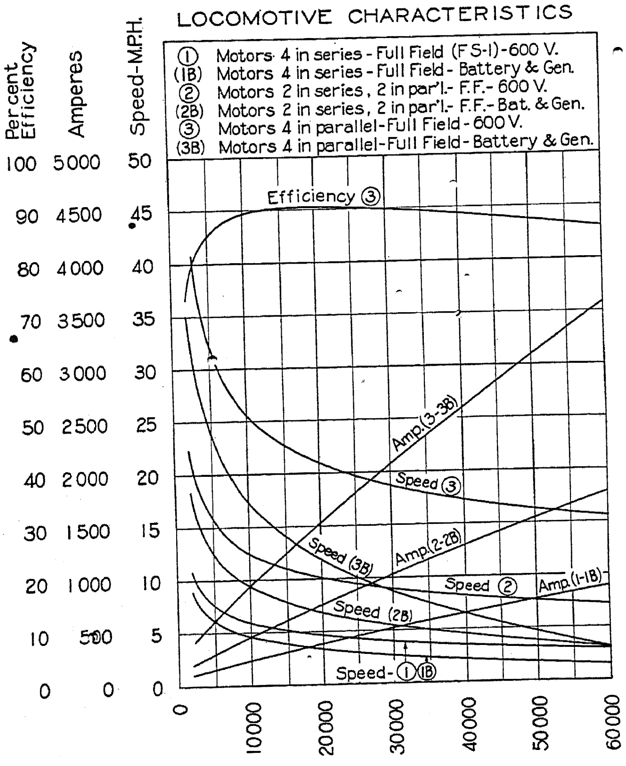

Fig. 4—Speed-Tractive Effort Characteristics of New York Central Locomotive

Fig. 4 shows locomotive characteristics on both external and internal power.

Description of Internal Power Plant of New York Central and Delaware, Lackawanna & Western Locomotives

The oil engine is the Ingersoll-Rand 300-hp., vertical, 6-cylinder, 4-stroke cycle, trunk piston, single acting type with direct fuel injection. The engine is controlled by a constant speed governor which regulates between 550 r.p.m. full load and 575 r.p.m. no load. The engine is directly connected through a flexible coupling to a 6-pole, self-excited, compound wound, 200-kw. generator of the single bearing type, with multiple self-ventilation. The generator frame is bolted to a bracket on the engine and is also supported by springs resting on the floor. The commutator end of the armature is supported by a roller bearing in the generator frame head, the other end being carried by a pilot bearing in the engine shaft. The engine-generator unit is supported on a cork cushion and held in place by bolts with springs under the nuts to prevent deflections of the cab underframe from being transmitted to the engine base flanges. The longitudinal thrust of the engine is taken by steel and wooden wedges and shear lugs cast in the cab underframe.

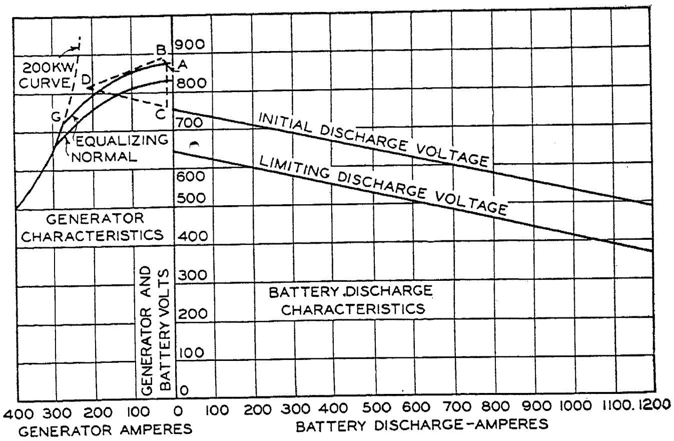

Fig. 5—Battery-Generator Characteristics of Delaware, Lackawanna & Western Locomotive

Selection of Generator Characteristic

Fig. 5 shows the generator characteristic and the battery voltampere charge and discharge characteristics of the internal power plant of the Delaware, Lackawanna & Western locomotive. These characteristics are representative of three-power locomotive practise and will be explained. When the battery is fully charged, any rate of charge however small will produce gassing, and if the charging rate be high enough, gassing will occur with the battery in any state of charge, although the rate may be reduced during the latter part of charge to a value which, unless abnormally sustained or occurring too frequently, will produce so small an amount of gassing as to be practically harmless. This value is called the "finishing rate" and is point *B* in Fig. 5. The charging rate at the end of charge should be confined between the finishing rate and one-half of the finishing rate. Consequently, line *A B* represents the voltampere charge characteristic of the battery at finish of charge condition. The ability of the battery to absorb high charging rates at the start of charge is indicated by point *D* which is the maximum charging rate recommended, and hence line *C D* is the voltampere charge characteristic at start of charge condition. Line *D B* represents the theoretical generator characteristic which would charge the battery in the minimum time and *A C* that which would produce a constant current charge at one-half the finishing rate. Therefore, the area *A B D C* includes all values of current and voltage that may be used to charge this battery under permissible conditions and the generator characteristic should intersect lines *A B* and *C D* and lie between lines *B D* and *A C*. The generator characteristic is also restricted by the 200-kw. output curve of the engine-generator set. To allow delivery of approximately full engine load over the whole range of battery discharge, the generator characteristic should pass near point *G* and lie close to the 200-kw. curve at voltages below G.

The higher generator curve in Fig. 5 meets these requirements and is suitable for giving the battery an equalizing charge, but is too high to permit the engine to run a sufficient percentage of the time to keep battery discharge within the daily limit of 125 per cent of rated capacity without overcharging the battery. Therefore, the lower curve is used for normal charging and the higher characteristic for periodic equalizing charges. The charging rate at the end of charge, when using the normal characteristic, is less than one-half the finishing rate, but with a properly shaped characteristic and the engine running almost continuously, the battery will be maintained between equalizing charges in a satisfactory state somewhat below full charge. Use of the low generator curve for normal charging has the further advantage of leaving margin for variation of characteristic between inspections without running the risk of overcharging the battery seriously, such as would happen, for example, if the engine speed increased and the equalizing characteristic were being used in daily operation.

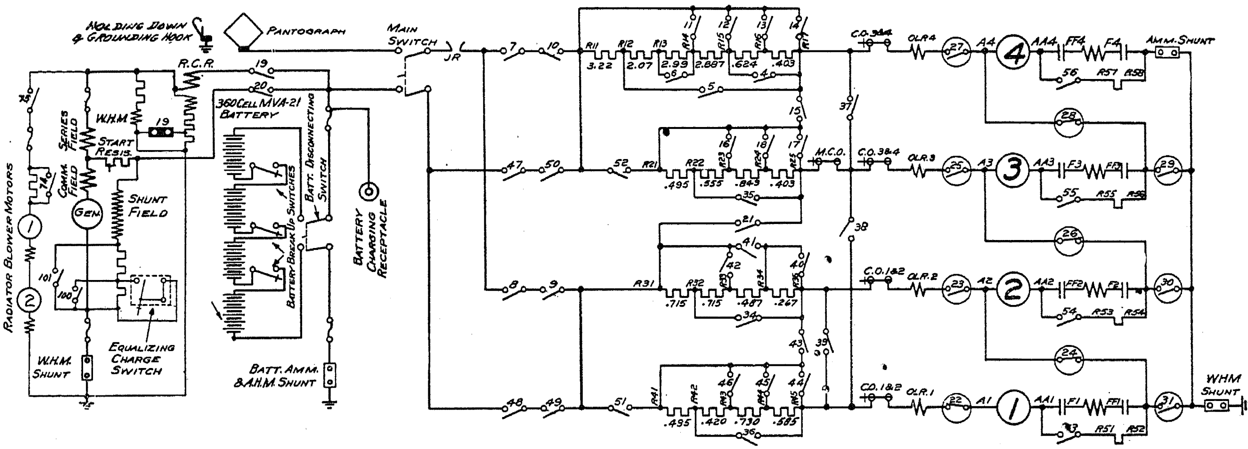

Fig. 6—Power Circuit Connections of Delaware, Lackawanna & Western Locomotive

The control is arranged so that whenever the locomotive is operating on internal power and *the traction motors are connected to the battery generator bus*, the higher characteristic is automatically brought into operation. This is permissible because usually the traction motor load is sufficient to hold the generator voltage below the point where high charging rates will occur unless the battery is in a very low state of charge. This arrangement permits the use of a more drooping curve without dropping the traction power characteristic below the point *G*. The necessity for a drooping curve may be better appreciated when it is considered that the battery is intended to deliver only 42 per cent of its capacity per 8-hour shift, whereas a generator characteristic lying along the line *B D* would give the battery a complete charge in 4 hours if continuously applied from full discharge and neglecting the small auxiliary load.

Control of the Internal Power Plant

Both the New York Central and Delaware, Lackawanna & Western locomotives have a three-speed control on internal power. The series or lowest speed connection develops maximum tractive effort with minimum current from the battery-generator unit, allowing most slow switching movements to be made almost entirely on the engine-generator set without using battery current.

Fig. 6 shows the power circuit connections of the Delaware, Lackawanna & Western locomotive. Contactors Nos. 7, 8, 9 and 10 connect the traction motors to external power, and Nos. 47, 48, 49 and 50 to internal power. On both the New York Central and Delaware, Lackawanna & Western locomotives, transfer from external to internal power is controlled by a potential relay and is automatic upon loss of external power, although the automatic feature is selective on the New York Central locomotive because of interruption of power on third rail gaps when transfer to internal power is not desired. On both locomotives, transfer from internal to external power is accomplished by turning the controller handle to the off position and then notching out, necessitated because of going from a lower to a higher voltage power source. Indicating lights show whether operation is from internal or external power and whether collector shoes or pantograph are in contact with the third rail or trolley wire. The battery may be broken up into four sections to limit the maximum voltage to ground when doing any work in the battery compartments. The engine is started by motoring the generator from the battery through contactor No. 20 and the starting resistance; maximum field for starting is secured by short-circuiting the generator shunt field resistance with contactor No. 101. The reverse current relay controls contactor No. 19 and connects the generator to the battery when the proper generator voltage has built up; it likewise disconnects the generator should the generator voltage drop below that of the battery due to loss of engine speed, generator field, etc. To give the battery its weekly gassing charge the equalizing charge switch is closed, short-circuiting a section of the generator shunt field resistance. Contactor No. 100 provides the automatic transfer to the equalizing characteristic when the traction motor load comes on.

Fuel oil is delivered to the engine through an electrically operated valve, which is closed automatically by low lubricating oil pressure or overspeed, by means of an oil pressure switch or speed limit switch respectively.

Referring to the traction motor circuits in Fig. 6 contactors Nos. 4, 5, 6, 34, 35, and 36 are used to rearrange the values of resistor steps to secure proper operation on internal power, when it is impossible to use the 3,000-volt values because of the comparatively low battery voltage. The same rheostat values are used on both internal and external power on the New York Central locomotive because the voltages are near enough together. The control of the battery-generator combination is almost identical with that on the Delaware, Lackawanna & Western locomotive.

AUXILIARIES

Auxiliaries on the New York Central locomotive operate automatically from the third rail when available, but on the Delaware, Lackawanna & Western locomotive these run at all times on internal power as this scheme is simpler and less expensive than furnishing auxiliaries for 3,000-volt operation.

Operating cabs are heated by hot water from the oil engine cooling system. Electric immersion heaters operating from third rail are used on the New York Central locomotive to heat the cooling system water for cab heating as well as prevention of freezing when working on external power. The Delaware, Lackawanna & Western locomotive has 3,000-volt heaters for trolley operation.

SERVICE CONDITIONS

Tests on the first New York Central locomotive were confined to internal power operation and the results of these tests have been covered under *General Requirements of Internal Power Plant.*

Tests were made on the Delaware, Lackawanna & Western locomotives in a combination of transfer and switching service. The transfer work consists of hauling heavy trains at relatively high speed on external power between Secaucus and Jersey City Yards, a distance of 3½ miles. The switching, done entirely on internal power, consists of hauling these trains into the yards, partially breaking them up, making up outgoing trains, charging the train line, and hauling them to the electrified tracks. The ratio of the average power required in the switching work to that in the transfer work proved to be approximately 50 kw. to 450 kw. or 1 to 9. It is believed that this is the first case where actual data have been obtained which show the relative amount of work done on the main and yard tracks. The switching service called for frequent short periods of operation at 200 to 400 kw. and peaks of 800 kw., while transfer work had peaks of 3,000 kw.

The outstanding feature of the three-power locomotive is its great flexibility. On both the New York Central and Delaware, Lackawanna & Western these locomotives are used in different classes of service and combinations of service. The New York Central West Side development between 72nd Street and St. Johns Park includes yards at 72nd and 30th Streets with many sidings leading into warehouses and industrial buildings as well as much trackage on city streets. The complete electrification of these facilities with external power would be impractical, if not almost impossible; yet the service demanded a self-propelled locomotive capable of handling the heaviest type of switching and short transfer work, as well as road freight and transfer service on third rail. The three-power locomotive was developed to meet these requirements.

This locomotive has a field of its own and is not intended generally to supplant the straight oil or gas electric unit. The work of the locomotive when operating on internal power must be sufficiently intermittent to allow the battery to be maintained in a satisfactory state of charge. Heavy switching and short heavy transfer work on internal power or a combination of both, together with external power operation, offer conditions for which this type of locomotive is best suited. The internal power feature makes this type approach a "universal" locomotive for road freight, transfer, and switching service on sections of road which, for economic or practical reasons, could not be completely electrified.

ACKNOWLEDGMENT

The authors gratefully acknowledge the assistance rendered by Mr. W. S. H. Hamilton of the New York

Central Railroad and Mr. D. P. Orcutt of the Electric Storage Battery Company in the preparation of this paper. The original analysis of the tests on the initial New York Central locomotive by Messrs. Hamilton and Orcutt has been largely drawn upon. The discussion of the selection of generator characteristics is based on an analysis of this subject by Mr. J. H. Tracy of the Electric Storage Battery Company, to whom thanks are extended for the use of this material as well as valuable comments.

Discussion

W. S. H. Hamilton: This discussion is confined to the three-power locomotives used by the New York Central around New York City.

Some question has been raised as to the size of internal power plant required on these locomotives and also whether with the present power plant they are capable of meeting emergency conditions.

The size of the internal power plant was based on the requirements in our 60th Street yard where there is heavy switching and float work. It was estimated that to do the work quickly, internal power equivalent to 900 engine hp. should be available. This makes the locomotives "snappy" even in heavy work and lets them kick and drop cars without difficulty.

It should be borne in mind that while switching service averages only about 60 kw-hr. per hour input to the traction motors, peak inputs of 600-800 kw. are frequently encountered. They are of short duration and do not represent many kw-hr. in the course of a trick. While the lack of this amount of power would only make a small difference in time in each movement, where it was required, at the same time these small differences are cumulative and have an effect on the total work done in the trick. Furthermore, in congested yards slowing down one movement even a little usually affects movements of two or three other locomotives.

There are other locations on the West Side where this amount of power is not required and where a lighter locomotive could be used but it was desired to have all locomotives alike to increase the utilization, decrease the number of spares, and relieve the yard forces of having to worry about which locomotive to get ready for each job.

As to the adequacy of the internal power plant to stand overloads or conditions not anticipated, I wish to point out that from August 1930 until June 1931, these locomotives handled 2,000-ton trains in regular road service on internal power from 72nd Street to Spuyten Duyvil a distance of about 8 miles. This should be ample demonstration of their ability, as they are not intended regularly to handle such service and with the installation of third rail down to 72nd Street, this service has been discontinued.

The assignment of these locomotives is approximately as follows:

30th Street Yard	11
60th Street Yard	11
Hudson Side-Electric Division	8
Harlem Side-Electric Division	3
In Shop	3
Total	36

The locomotives at 30th Street operate entirely on internal power, those at 60th Street mostly on internal power and the others mostly on third rail. Those assigned to the Hudson and Harlem Sides of the Electric Division are used for express trains, way freight trains, and as traveling switchers. These latter do the industrial switching, picking up and dropping cars at various stations on internal power and then run to the next station on external power. There is a number of industrial plants where it would be very difficult to install either third rail or trolley and yet a locomotive switching them must be capable of moving rapidly between stations in order to keep out of the way of other traffic.

For such service the three-power type of locomotive is ideally suited and is a very economical solution of the problem. It should be pointed out that at the continuous rating of the traction motors, these locomotives develop 1,300 hp. at the wheel rim on external power, whereas on internal power they develop only 682 hp. To duplicate the amount of power developed on external power, a 1,600-hp. engine would be required.

For the year 1931, the whole fleet of 36 locomotives was in service with crews on 61 per cent of the time, and available for service 85.5 per cent of the time. This performance with new locomotives and new crews, we consider very creditable, but look forward to bettering it as everyone becomes more used to them.

D. P. Orcutt: Information has been asked regarding the expected life of the battery on three-power locomotives and in this connection the fact was mentioned that in an investigation of a proposal for the use of a large straight storage battery locomotive for switching service, it was found that the battery life had been stated to be 4 years and that this life had resulted in an excessive battery maintenance cost for the work under consideration.

It should be remembered that with a three-power locomotive operating on internal power, a very large part of the energy used by the motors flows directly from the generator to them and does not pass through the battery.

Data collected from the operation of the first three-power locomotives on the New York Central, No. 1525, showed that in switching service about 85 per cent of the energy used by the traction motors flowed directly from the generator to the motors and 15 per cent from the battery to the motors. It is therefore evident that a very much smaller battery can be used with a three-power unit, thus greatly reducing battery maintenance cost from what it would be with a straight battery locomotive in switching service.

Incidentally it may be stated that this same data obtained from No. 1525 showed that the losses in the battery amounted to only about 5 per cent of the total energy generated while in a straight battery unit the battery losses would probably exceed 30 per cent of the energy required to keep the locomotive in service.

A straight battery locomotive may be the most economical unit to use under conditions where the size of the battery necessary to do the total amount of work does not require more battery weight than can be placed on a locomotive of the total weight required and where time is available for recharging the battery. In neither of the installations discussed by Messrs. Brehob and Craton would a straight battery locomotive give the required service which includes 24-hour operation as well as heavy transfer service.

The economies resulting from the use of any form of internally powered locomotive for use with an electrification project can be determined only by a comparison of the quality of service made available and of the cost of the entire service under consideration. A study based only on the cost of locomotive operation per hour does not give a true picture of the economies effected on the whole project.

As the paper and the discussion are confined to locomotives capable of operating on both external and internal power, we have combined data obtained from both the D. L. & W. three-power locomotives and New York Central three-power locomotives and in this way obtain the following as being approximately representative of this type of locomotive operating in connection with a railroad electrification project:

In an 8-hour shift the locomotive would operate on non-electrified tracks approximately 6 hours at an average input to the motors of 60 kw. per hour which would represent a total of 360 kw-hr.

On electrified tracks it would operate approximately two hours at an average input to the motors of 450 kw-hr. per hour which would represent a total of 900 kw-hr.

The power required for auxiliaries and all electrical losses during this 8-hour period would represent about 220 kw-hr. making the total power required for the locomotive for the 8-hour shift approximately 1,480 kw-hr.

Approximately 15 per cent of the 360 kw-hr. delivered to the motors on internal power while the locomotive is on non-electrified tracks would come from the battery or 54 kw-hr. This represents only about 4 per cent of the total power required by the locomotive during the 8-hour shift. As compared to this a straight battery locomotive would have had to deliver all the power required by the locomotive if the locomotive were to do the same work that the combination locomotive did during this 8-hour shift.

It can easily be seen, therefore, that when only 4 per cent of the work which this combination locomotive does in an 8-hour shift comes from the battery, that battery costs will be a relatively small proportion of the total operating costs of the locomotive as compared with a type of locomotive in which all the work is done by the battery.

Even when called upon to operate entirely on internal power the input from the battery to the traction motors is only 15 per cent of the total input to the motors whereas it would have to be 100 per cent (plus input to auxiliaries) in a straight battery locomotive.

It has been pointed out that this locomotive has decided advantages in connection with an electrification project:

It can be operated in warehouses and other places where it may be objectionable to run an internal combustion engine or install third rail or trolley.

It is capable of accelerating rapidly.

It is the most flexible type of locomotive tried out or suggested for this class of service.

It has high fuel economy because the engine operates with a favorable load factor.

It is our belief that the battery costs on this locomotive have been and will be entirely justified by the advantages obtained.

Hermann Lemp: To design an oil-engine required only to operate at nearly constant load and speed is a much simpler problem than to design one for adjustable speed and load, as anyone conversant with oil-engine design will readily appreciate. Similarly a storage battery, floating on a system, receiving and delivering energy continuously, on a hand to mouth proposition operates under a far better condition than when first fully charged then full discharged sometimes way down to the very lowest figure permissible. It is therefore quite conceivable, as Mr. Orcutt pointed out, that the storage battery manufacturer's guarantee of four years life can be met. This combination, when used in switching service, permits the time load factor of 15 per cent to be exchanged for 62 per cent and hence requires a smaller oil engine to perform the work of a bigger unit.

Mr. Brehob laid particular stress on the necessity of providing heavier axles and frames owing to the addition of the internal combustion engine plant, while he made no mention of the weight of the storage battery. If my records are correct the weights of the first three-power locomotives were as follows:

Total weight—257,000 lb., in which the battery weighs 34,300 lb. and oil engine generator and accessories weigh 31,500 lb.

In other words another engine equipment of 300 hp. would not cost and weigh any more than the battery, good only for four years, while the engine equipment, if properly maintained, would be available at least for ten years. To me the question is therefore one of original investment.

Railroads I think are particularly anxious to obtain an equipment which can maintain the rated output substantially continuously and which is not limited by a "time power decrease," such as a battery under a continuous full discharge rate would constitute. A similar operating condition is presented by the Hoboken Manufacturers Railroad. Most of the switching can be performed by a 300-hp. unit. But sometimes during the day, particularly when the tide is low and loaded cars have to be pulled up a stiff grade from the float, a second 300-hp. engine in reserve is started up and gives the additional power and acceleration needed. The engine units *A* and *B* are shifted every day, so as to equalize the time service of each. Such a locomotive, when climatic or service conditions call for nearly continuous full load operation, can give such a service at a lower first cost and maintenance than the battery combination.

Mr. Hamilton has expressed a fear that the oil-engine part of the equipment might prove unreliable. He stated that the actual experience with the three-power locomotive had demonstrated an availability for service of 85.5 per cent. It was brought out in a paper read by the writer before the Metropolitan Section of the A.S.M.E., April 17, 1928, that the average availability of straight oil-electric locomotives using the same oil-engine unit as the three-power locomotive had been established to be 91.5 per cent. This figure was obtained from returns to a questionnaire sent to all railroads and industrial plants using these locomotives for about three years in various locations and service.

F. H. Brehob: Due to the fact that the service requirements to be met were severe and the type of locomotive to be built was of unusual design, a number of difficult problems in mechanical design had to be solved.

The locomotive as developed represents the heaviest four-axle swivel truck locomotive in service today, and utilizes the heaviest axle loading using single-geared axle-hung motors.

Naturally, the railroad desires no more weight in a locomotive than is absolutely necessary to obtain the required adhesion for the maximum load to be handled. Generally, the basic design of the ordinary straight electric freight locomotive results in approximately the desired weight. By basic weight I mean using normal stresses and using the kind and grade of material ordinarily used for the various parts. For example, on the New York Central R-2, six-motor freight locomotive utilizing the same traction motor as on the three-power locomotive, approximately 2½ per cent of the total weight of 266,000 lb. is ballast.

The three-power locomotive as furnished the New York Central is not only a straight electric locomotive converting electrical power into mechanical power, but carries in addition a power plant with its accessories such as water tank, fuel tank, radiators and piping. The weight is thus considerably increased due to this apparatus and also to the increased weight of the mechanical portion requiring heavier supporting members to carry this load and because of the necessity of building a larger locomotive to house this apparatus. The railroad specified a minimum weight of 125 tons. By utilizing material to best advantage, reducing sections of material where feasible, and in one or two cases using a higher grade of material, it was possible to limit the weight in working order to 257,000 lb. For example, the axles were made of vanadium steel, although the selection of this material was also influenced by a limitation in the diameter of the axle to 8 in. so that the GE-286 motor used on other locomotives on the New York Central could be utilized without change on the three-power locomotive.

One of the most effective ways of keeping the weight to a minimum is to design the locomotive as compactly as possible which involves a suitable arrangement of apparatus using every bit of space to best advantage. The apparatus must be arranged to be readily accessible for inspection and repair and for easy removal from the locomotive.

The arrangement finally adopted was the one as illustrated in the paper. It will be noted that the blowers for ventilating the traction motors are placed directly over the hollow center plate into a small space recessed into the battery compartment although the upper tier of batteries is not affected. The control apparatus is mounted along one side of the locomotive opposite the engine, access being obtained from one side by removable covers on the outside of the cab. In this manner the space between the engine and the control equipment serves a double purpose; on the one hand giving accessibility to the engine for maintenance and on the other hand providing the necessary arcing clearances for the contactors. The space between the trucks is practically all taken up with the accelerating grids and the air compressor. The space not required on the roof for the hatch is occupied with engine radiators, and air reservoirs. Utilizing all the available space to advantage permits building the locomotive as short as possible with a minimum distance between center plates resulting in a minimum cross section of material in the supporting underframe, and consequently a minimum weight of material.

Owing to the fact that collection of power on the D. L. & W. locomotive is from a 3,000-volt overhead trolley, the arrangement of apparatus is somewhat different. It was desirable to enclose the 3,000-volt control apparatus somewhat more than the 600-volt apparatus on the New York Central three-power locomotive, for the sake of safety and cleanliness. This resulted in an arrangement whereby the engine was placed in one side of the cab leaving a greater space for the high-voltage equipment opposite the engine. This also permitted the use of enclosing covers on the inside as well as on the outside. Instead of locating the rheostats beneath the locomotive they were placed inside the locomotive immediately above the floor, and chimneys for heat dissipation were provided. The space underneath was used for air reservoirs instead of the roof, thereby providing space for mounting the overhead trolley. The battery trays were arranged in three tiers due to the fact that this battery was of a higher voltage and consisted of a larger number of smaller cells.

Another reason for arranging the apparatus as compactly as possible was to obtain a design which would negotiate the minimum radius of curvature of 100 ft. required by the railroad. This severe curvature is the principal reason for the swivel truck design as it is doubtful whether an articulated type could have been built which would prove successful.

Certain definite height and width limitations had to be met by the railroad, and these had to be considered especially on a curve when a displacement from normal position is encountered. This was not an easy matter to accomplish because a straight electric locomotive required to negotiate the same curve would be shorter. The design required that the distance between center plates be kept to a minimum and of even greater importance that the overhang from the center plate to the end of the coupler be kept at a minimum.

The draft rigging is, in effect, radial in that the coupler is free to swivel in a coupler extension which also swivels.

The three-power locomotives for the New York Central and D. L. & W. railroads are a modification of the original three-power locomotive jointly designed by the American Locomotive Company and the General Electric Company, and the operating experience gained by the New York Central on this locomotive and straight electric locomotive has been most helpful. It is gratifying that the combined efforts of the railroad and builder produced a new type of motive power which in a relatively short length of time has already proven successful.

F. H. Craton: Space limitations in the paper did not allow much detail regarding the interesting background behind the development of the three-power locomotive. There are 36 of these locomotives in service on the New York Central in New York, 2 on the Lackawanna and 2 on the Illinois Terminal Railroad. As the so-called West Side Improvement of the New York Central was largely responsible for the development of the three-power locomotive, it is briefly described in the following.

This project is the last step in eliminating steam locomotive operation from Manhattan Island. The New York Central's freight tracks enter Manhattan at Spuyten Duyvil which is at the northwest tip of the island at the junction of the Hudson and Harlem rivers, the line running from Spuyten Duyvil south along the east shore of the Hudson River for 12.4 miles as far as St. Johns Park. The electrification of these tracks constitutes a part of the so-called West Side Improvement. From Spuyten Duyvil to 72nd Street, which is at the entrance to the 60th Street yard, 600-volt third rail has been installed as an extension of the present third rail electric zone. The heavy freight service from Harmon, the northern terminus of the electric zone, to 72nd Street, approximately 30 miles, is handled by Class R-2 straight electric 600-volt road locomotives, 42 of which were purchased during 1930 and 1931. However, south of 72nd Street, owing to unusual conditions involving much trackage following city streets, a number of sidings entering industrial buildings and warehouses, much yard trackage and special conditions connected with float work, it was impractical to electrify with either third rail or overhead distribution systems. Therefore, the railroad management was faced with the necessity for choosing a suitable type of internally powered locomotive for the service south of 72nd Street as well as a locomotive which would operate also on third rail in order to increase its flexibility. After a detailed study of the question involving consideration of all the conditions to be met, the three-power type of locomotive was finally chosen.

Discussion of the three-power locomotive usually leads to its comparison with a two-power design, which is a unit with a straight engine-generator power plant and external power facilities. The justification for using one type or the other hinges entirely on the service requirements to be fulfilled and can be determined only after a thorough study of all the conditions involved. A power plant of the engine-generator-battery type is best fitted for operation in service in which the peak demands are high and the load factor low. The operating conditions on the New York Central West Side as well as the Lackawanna are well suited to take advantage of this type of internal power plant. The three-power type possesses the advantage of being able to operate on battery power into buildings or restricted areas where there might be an objection to an internal combustion engine because of sparks or odor, but the value of this feature would be dictated by the service requirements.

The question has been raised as to whether the internal power plant should be regarded as an auxiliary to the external power operating facilities or vice versa. This appears to be dependent upon the service conditions under which the locomotive operates on each particular application. Because of the unusual flexibility of this type, it is possible that the predominance of one type of operation over the other would shift from one to the other in operation on a single diversified application such as the New York Central where the locomotives are used with different combinations of external and internal power operating conditions but all units are interchangeable in the various classes of work.

F. H. Brehob and F. H. Craton: If the term "power plant" as referred to in the discussion has been misunderstood, it should be stated that this term includes the battery inasmuch as the battery and engine generator in multiple combine to furnish power to the traction motors. It was realized that the battery presented a design problem on account of its weight, which is one reason the battery was divided, one-half of it being placed almost directly over each center plate, where its weight would require a minimum amount of supporting structure.

Mr. Hamilton has pointed out that a locomotive without a battery would require additional engine-generator capacity together with additional accessories. This additional weight of power plant in the form of engine-generator, radiators, etc., would offset the weight of the battery.

The two-power locomotive undoubtedly has its place in the field of transportation. By two-power is meant a locomotive which will first operate from external power and secondly from an engine-generator set, but without battery for traction purposes. We believe the selection of either type can only be satisfactorily arrived at when all the service requirements to be performed are known and studied.

Motors for 3,000-Volt D-C. Multiple Unit Cars

BY JOHN C. AYDELOTT*
Member, A.I.E.E.

A NEW multiple unit car motor, incorporating many novel features of design, was developed for the electrification of the suburban service of the D. L. & W. R. R. The absence of trouble, in commencing service and in maintaining it, marked the whole electrification program. This was the first installation of multiple unit suburban service in this country employing a trolley potential of 3,000 volts direct current.

Locomotive motors to operate from a 3,000-volt trolley were first built for the C. M. & St. P. R. R. in 1914 and subsequent installations include the Spanish Northern Railway in Spain, the Paulista Railway in Brazil, the Mexican Railway in Mexico and the Cleveland Union Terminals Company. Experience gained from several years of operation with the earlier designs of 3,000-volt locomotive motors has been invaluable in perfecting the design of the Lackawanna multiple unit car motor which rounds out the line available for 3,000-volt d-c. electrification.

There are now in service on the Lackawanna Railroad 564 of these motors. The breaking in of equipments started in August, 1930. Revenue service started September 3, 1930, when the first passenger-carrying train ran from Hoboken to Montclair. With the final extension to Dover on January 25, 1931, the last of 141 units was placed in service. Traffic has so increased that units originally intended as spare cars are all needed regularly and there are times during the day when all but three units are on the road.

By June 1, 1931, 4,500,000 car-miles had been operated with only two train delays, one of 19 minutes and one of 4 minutes, chargeable to motor failure. One train was held at Hoboken Terminal 19 minutes due to a burned out armature bearing and the other was 4 minutes late arriving at Montclair.

During the period of initial operation these motors have been remarkably free from flashovers, insulation failures, and mechanical troubles.

The minimizing of motor flashing is an achievement embracing many features of motor design. Flashing may be caused by interruptions or disturbances in the voltage supplied to the motors, and many things within the motor itself may contribute to motor flashing, such as poor commutator condition, commutator roughness, oil and dirt between commutator bars, flat spots, bouncing brushes, and low motor stability.

The Lackawanna motor has 281 commutator segments. With 1,500 volts applied to the motor terminals there is an average of 21.3 volts per bar. The successful operation of this motor demonstrates the feasibility of high volts-per-bar, higher than previously used in any 3,000-volt railway motor, and depends upon commutating characteristics that will keep the commutator always in first class condition, and upon the ability of the motor to withstand frequent wide and sudden variations of trolley voltage on full or weak field with a minimum of spit at the brushes.

Good commutation hinges first upon keeping the reactance of the armature conductors at a minimum so that the energy involved in the reversal of the current in a conductor is low. Then the compensation, depending upon the quantity of commutating pole flux, must have the required value at all loads. The design of this motor employs a one-turn armature winding. Careful attention was given to the number of slots and the proportions of the slots, air-gap and pole faces. The armature bars are clipped at the back permitting a "breaking up of the slots" which has long been a feature of good railway motor design. Non-magnetic steel binding wire is used to reduce slot reactance. At light loads the effect of non-magnetic binding wire as compared with steel binding wire is particularly noteworthy. With non-magnetic binding wire the slot reactance flux and slot reactance are not only low in value but are essentially proportional to load current. With steel wire the slot reactance flux is proportionately very high at light loads until the binding wire saturates and is increased by the amount of flux carried by the wire at all loads. When the slot reactance voltage is not proportional to load it cannot be correctly compensated at all loads.

It is well known that good commutation can usually be obtained at some one motor load according to the adjustment of the commutating field strength and it has been common practise to adjust compensation for minimum sparking at some load around the one-hour rating. If the commutating poles were somewhat saturated at this point the motor would be over compensated for all lighter loads. The design of this motor includes ample commutating pole cross section. Commutation is good from the lightest load to four times the continuous rating. Light loads are mentioned particularly because no sacrifice of commutation at light loads has been made to obtain correct compensation at heavy loads.

This motor has ability to withstand flashover from interruption to an unusual degree. This may be attributed largely to the use of highly responsive commutating poles. Extra turns are wound on the commutating coils and the extra magneto-motive force is consumed under steady load condition in non-magnetic shims at the back of the commutating pole pieces. Thus the commutating pole flux follows very quickly any change in the armature current and tends to maintain correct compensation even under extreme transient conditions.

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Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

32-36

A strong exciting field winding adds to the stability of the motor. The use of an inductive shunt rather than field tapping to obtain weak field gives any motor a greater ability to withstand flashover from interruption during reduced field operation. In the case of the Lackawanna motor the inductance of the field shunt has been so proportioned that the ability of the motor to withstand flashover from interruption on reduced field is superior even to its performance on full field.

Motor flashing that might be caused by dirt is avoided by the ventilating scheme developed for this motor. Dirt may lodge between commutator segments causing short circuits which, especially with high volts per bar, will lead to flashovers. Dirt that causes brushes to stick in the holders inevitably leads to flashing. It is also highly desirable that all other parts of the motor as well as the commutator and brushholders be kept as free from dirt and moisture as possible.

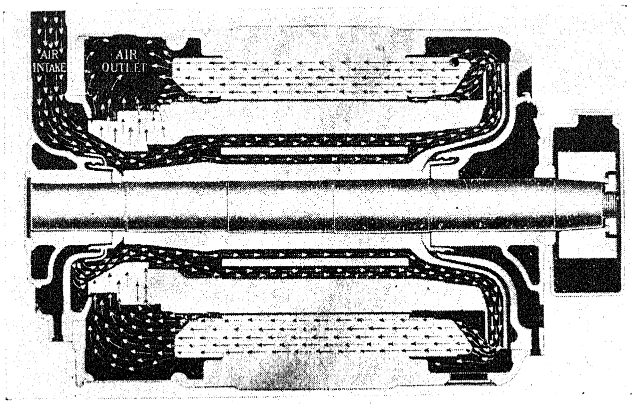

FIG. 1—VENTILATING SCHEME FOR D. L. & W. R. R. MOTOR SHOWING AIR CIRCULATION INSIDE THE FRAME

The most successful 3,000-volt locomotive motors have been blown, clean air being taken from the cab; and a blast of air has been directed straight against the motor commutator serving to prevent any accumulation of dust from the brushes in the mica grooves. The increased pressure in a blown motor tends to prevent the entrance of brake shoe dust or other dirt, or snow from the lower levels of the air surrounding the running gear of a locomotive in motion.

No external blowers are used on the D. L. & W. multiple unit cars but with the system of ventilation used the motor frame operates at all times either at or slightly above atmospheric pressure. The motors are self-ventilated as shown in Fig. 1. The cooperation of the car builder permitted the ventilating air to be taken from settling chambers over the vestibules to which it is admitted through louvers at each corner of the car roof. A separate duct to each motor goes down through the side and along under the floor of the car to a point over the commutator end bearing as shown in Fig. 2. A flexible air connection made of cloth sewed over a coiled wire leads the air from the duct on the car to the motor inlet in the commutator end framehead. The flow of air is maintained by a fan on the motor shaft. The air enters the motor under the commutator and passes first through the ducts in the armature. It is then returned by the fan through the field structure of the motor and exhausted at the commutator end. The air outlet opening is covered with an expanded metal screen and a hood so that foreign material cannot fall or be thrown into the motor. This method maintains pressure inside all the exposed surfaces of a self-ventilated motor, which means that track dirt, fine snow, or brakeshoe dust cannot enter the motor. It should also be noted that before the ventilating air reaches the commutator chamber of the motor it is filtered, so to speak, through both the armature and field coils. Furthermore, any carbon dust that originates in the commutator chamber is carried directly to the exhaust openings and does not enter any other parts of the motor. Routine inspections have shown these motors to be unusually clean. Surplus oil from the bearings is led through ducts to the outside of the motor and none finds its way inside.

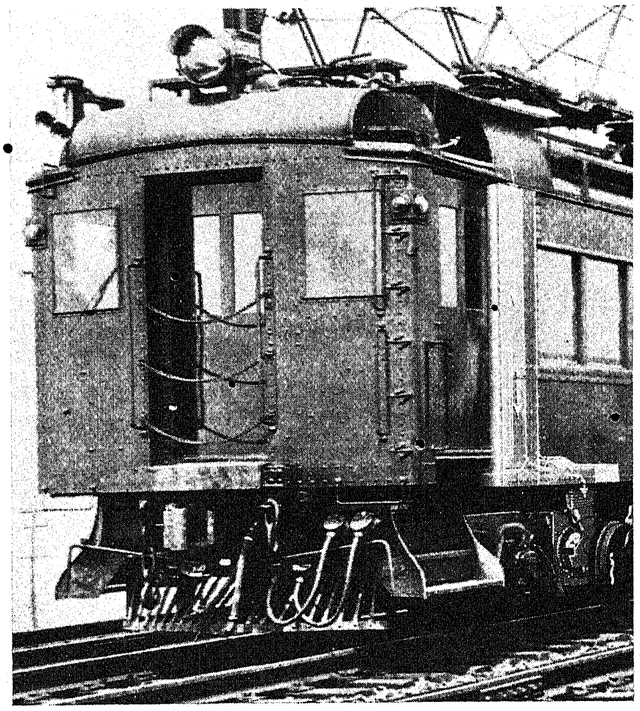

FIG. 2—VENTILATING SCHEME FOR MOTOR CAR SHOWING AIR INTAKE AT THE ROOF AND PATH TO MOTORS

Smooth commutators depend not only on good commutating characteristics but also on good mechanical design and construction. The use of four brushholders and the low current necessary because of the high voltage permit the commutator to be compact and strong. The commutator diameter is 16¼ in. and there is a brush surface of 2¾ in. Six bolts hold the commutator cap and shell together. The cars operate at 70 mi. per hr. at least once during every round trip. With 35-in.

wheels and 59/22 gearing the motor reaches 1,800 r.p.m. at 70 mi. per hr. This gives a peripheral speed at the commutator of 7,620 ft. per minute.

When the motors were first put into service there was some brush breakage at high speeds but in a short time the commutators seasoned and brush breakage is now practically eliminated. The brushholders play their part in maintaining good commutating conditions. Pressure fingers are of the steel clock spring type, of low inertia, maintaining their tension permanently, and are centrally located by recessed adjusting sleeves clamped by the first turn of the springs. A non-resilient brush is used to maintain contact with the commutator at high speed. No abrasive action is necessary to keep the commutator smooth. Thus the brush may be comparatively soft, of low friction coefficient and high contact drop.

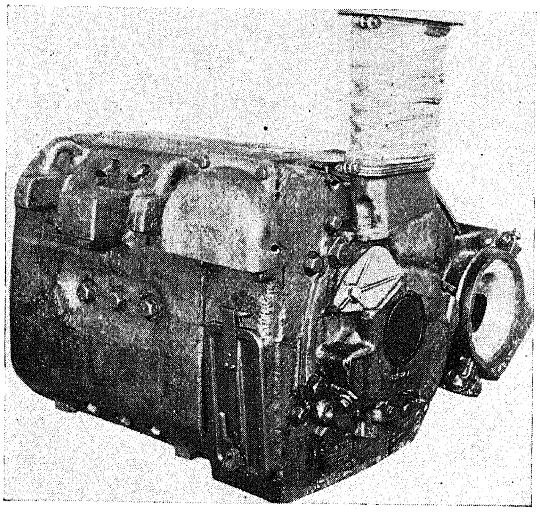

FIG. 3—GE-700-A 1,500/3,000-VOLT RAILWAY MOTOR SHOWING AIR INTAKE AND OUTLET

The insulation of the motor throughout is in accordance with recognized standards for 3,000-volt railway motors. While the absence of insulation failures in the motors thus far covers a comparatively short period, experience has shown that the same kind of insulation in other 3,000-volt motors lasts indefinitely, failures seldom occurring from any other cause than mechanical damage.

The ratings of the motors, when ventilated as in service, are as follows:

	Volts	Field strength	Shaft hp.	Amps.	R.p.m.	Rise deg. cent.
Continuous	1,500	50 per cent	181	97.0	1055	105
One hour	1,500	50 per cent	255	137.0	913	120

Although during the time that it is connected, the inductive shunt carries 50 per cent of the current passing through the motor, it has a continuous rating of only 34 amperes. The field shunt is designed with a temperature rise characteristic which at the r.m.s. current obtained in continuous suburban service will give a temperature rise approximately equal to the corresponding temperature rise of the traction motor exciting fields. Cold equipments coupled in train with hot equipments will share the load equally.

On certain length runs the r.m.s. current of the shunt slightly exceeds the rating, reaching a peak value at runs of approximately 3,000 ft. in length, where the calculated r.m.s. is approximately 37 amperes. However, under these conditions the r.m.s. current of the motor exceeds the continuous rating for 105 deg. cent. rise by resistance. Not only was the present electrification of the D. L. & W. R. R. considered in studying shunt capacity but also the possibility of extensions into more mountainous territory involving long uphill runs. These studies indicate that these runs do not require a shunt rating more than one-third the continuous motor rating for 50 per cent field strength operation. The reduction in shunt rating also permits a saving in weight.

Views of the complete motor are shown in Figs. 3 and 4. Four motors are used on a motor car weighing 81.5 tons loaded, which together with a trail car weighing 56.5 tons forms a two-car unit of 138 tons. The motor weighs complete with gear, gear case and axle linings 6,850 lb., giving a weight per horsepower of 26.9 lb. The four motors give a capacity for the two car units of 7.4 hp. per ton. Trains for use in short run commuter service may be made of 2, 4, 6, 8, 10 or 12 cars, half of which in each case are motor cars. A typical six-car train has a free running speed of 67 mi. per hr. on level track (Davis Friction) and a maximum permissible speed of 70 mi. per hr. Some of the trains shown on the time tables make the following stops and schedule speeds.

Stations	Miles	Stops	Avg. run	Schedule speed
Hoboken to South Orange	13.85	10	1.38 mi.	28.7 mi. per hr.
South Orange to Hoboken	13.85	10	1.38 mi.	29.7 mi. per hr.
Hoboken to Summit	20.10	13	1.55 mi.	30.9 mi. per hr.
Hoboken to Morristown	29.77	18	1.65 mi.	31.3 mi. per hr.
Morristown to Hoboken	29.77	17	1.75 mi.	31.9 mi. per hr.
Dover to Hoboken	40.49	22	1.84 mi.	32.8 mi. per hr.
Hoboken to Montclair	13.12	6	2.19 mi.	34.2 mi. per hr.
Dover to Hoboken	40.49	14	2.89 mi.	40.0 mi. per hr.
Hoboken to Dover	40.49	7	5.78 mi.	41.2 mi. per hr.
Montclair to Hoboken	13.12	1	13.12 mi.	41.4 mi. per hr.

The motors will permit the use of as high an accelerating rate as desired, limited only by the weight on drivers and the slipping point of the wheels. Normally an automatic acceleration of 1.5 mi. per hr. per sec. is maintained corresponding to an adhesion of 12.8 per cent. Acceleration tests have shown an equipment to be capable of reaching 30 mi. per hr. in 20 seconds and 47 mi. per hr. in 60 seconds.

In frequent stop service a four-motor equipment is well loaded by a motor car and trailer unit, and the motor heating approaches the permissible limit for class B insulation. Experience has shown that for railway motors, class B insulation will have a longer life if the motors are run hot enough to keep them free from moisture. In long run service the motors will have sufficient excess capacity to permit the handling of additional trailers. Such train make-ups as two standard units and one trailer, or three standard units and three trailers are contemplated. With such combinations the accelerating rate will be reduced but in infrequent stop service these trains will be capable of fast schedules.

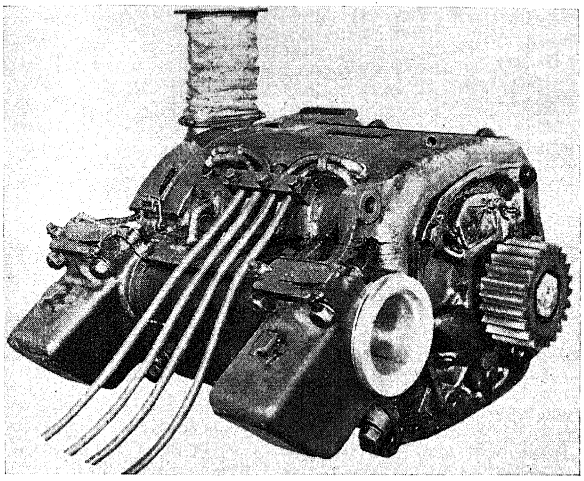

Fig. 4—GE-700-A 1,500/3,000-Volt Railway Motor Showing Pinion and Axle Preparation

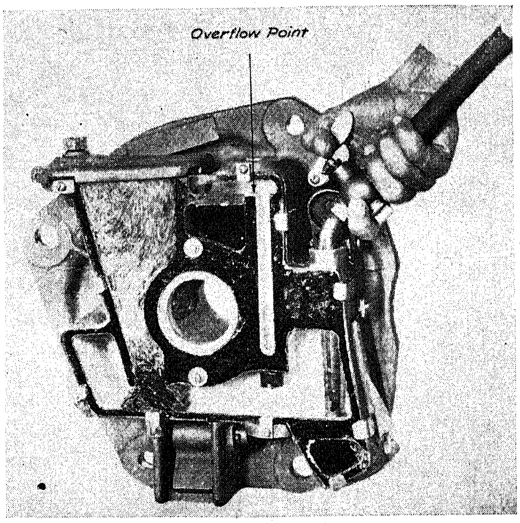

Fig. 5—Model Showing Principle of Constant-Oil-Level Bearing for Railway Motors. Oil in Reservoir at Overflow Point During Filling Operation

Conservative interpretation of experience characterizes every feature of the mechanical design, contributing to low maintenance. Stresses in all working parts are low. Both armature and axle bearings are of the sleeve type and embody the constant oil level feature permitting oiling periods to be extended to several times the usual period. There is a layer of babbitt on the wearing surfaces of the bore and flanges of all the bearings. The almost complete absence of hot bearings during the initial stages of the electrification was due not only to design, but to the fact that all of the linings were line reamed after assembly in their respective final positions and due to the careful packing of the bearings, oiling and breaking in.

In the bearings of the motor the oil level is maintained practically constant in the waste chamber as long as there is any oil in the reservoir. There are separate openings for waste and oil, hence there is no chance for dirt to fall into the waste chamber when replenishing the oil supply. The principal of operation of the constant oil level bearing is illustrated in Fig. 5 showing a view of a sectional model.

This type of bearing has been designed to operate between inspections without addition of oil. Should it become necessary to add oil on the road, the oil can be poured into the bearing through the filler opening and the bearing will operate as an ordinary bearing.

The operation of the gearing on the motors is quiet, promising long life. Gearing is of special heat treated steel, General Electric Company grade F, 2¼ pitch, with long and short addendum tooth form. The gear teeth were ground after treatment to insure correct profile. The possibility of giving off ringing notes from the gear has been overcome by a non-resonant feature shown in Fig. 6. This is obtained by snapping rings of small cross section into annular grooves machined in the under side of the rim. These rings are of a size having a different period of vibration from that of the gear section thereby counteracting or destroying the vibrations in the gear. The manner in which the rings are assembled provides an annular pocket for the retention of gear grease. This also has a decided damping effect. After assembly the rings are welded at the joint to prevent any possibility of their coming loose.

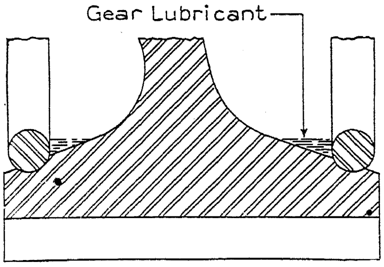

Fig. 6—Section of Non-Resonant Gear

Gear cases are of malleable iron with three-point support.

Six months to a year of operation gives only an indication of what the maintenance expense will be. A longer time will be necessary before sufficient data are accumulated and ready to publish. The absence of trouble thus far, however, promises unusually low maintenance for this first multiple unit suburban electrification employing a trolley potential of 3,000 volts direct current.

Temperature Rise of Ventilated Railway Motor Armatures[†]

BY D. A. LIGHTBAND*

Associate, A.I.E.E.

Synopsis.—*The calculation of temperature rise and of time-temperature curves of a railway motor for which only preliminary design data are available, usually presents considerable difficulty. Modern railway practise demands increasingly large power units in a limited space, so that accurate methods are necessary for the predetermination of temperature rise as a basis for guaranteed ratings and of time-temperature curves for the application of a motor to a complicated service cycle.*

For calculating the temperature rise of the armature in a new design of motor for any time interval and load condition, the author presents a method which is shown to give consistent results when applied to motors of widely varying size. A brief outline is given of the method by which the equations are derived from the usual theory of heat flow between related bodies. It is shown that by making certain assumptions the armature can be reduced to a simple equivalent model consisting of two conducting bodies separated by an insulating wall, and that the equations resulting from an analysis of this thermal system form a practical method of calculating time-temperature curves of an armature from preliminary design data. A detailed mathematical treatment is presented in an appendix.

By substitution of existing test data into part of the formula it is shown how a curve can be obtained which renders the equations applicable to a very wide range of motor sizes. To illustrate the practical application of the formula, a number of calculated time-temperature curves for existing motors is shown and compared with data from actual temperature tests of these same motors.

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THE accurate predetermination of temperature rise for loads of short duration in a new design of traction motor is of the utmost importance since the application of all such motors involves service of an intermittent nature. The usual method of using a r.m.s. current over a given load cycle as a basis of motor application assumes that the temperature rise varies as the square of the motor current over a given time interval. Experience has shown that this is not the case, although the use of the r.m.s. current will always result in a margin of safety in the final result. Where the load cycle is not of a very widely varying nature the r.m.s. current will give a fairly close approximation of the actual conditions. In such service as mine haulage or yard switching where a widely varying load cycle has been definitely specified by the purchaser, greater economy of space and cost can be accomplished if an accurate method is available for calculating the time-temperature curves for any load condition. Such a method can then be applied successively to each load condition of the cycle and a true picture of the motor heating will be obtained.

In many cases a motor designer is called upon to guarantee ratings which allow very little margin for guesswork in estimating temperature rise and consequently feels the need of a reliable means for its calculation. Railway motors are usually worked much closer to the limits allowed by existing insulating materials than machines used in other applications. Also, the exacting space limitations imposed upon motors placed under locomotives or rail cars, together with the present demand for still greater capacity within these limitations calls for closer designing than is generally required for other types of machine. Consequently, there is an urgent need for more accurate predetermination of temperature rise if a constant advance in the art is to be maintained.

The development of highly efficient ventilation of railway motors in recent years has rendered the estimating of short-time temperature rise more difficult than was the case with the older type of motor of larger thermal capacity. In the newer designs, the value of the nominal one-hour rating as a true indication of thermal capacity is not very great since the much more effective ventilation has considerably decreased the margin between the nominal one-hour rating and the continuous rating. The rate of temperature increase for short intervals cannot be safely regarded as the function of an unvarying time constant in a motor of this type since the rate of heat dissipation, which is largely dependent on the speed, has a considerable influence on the so-called thermal time-constant. In the following treatment a method of predetermining the thermal characteristics of ventilated motors is derived which takes into account the influence of the rate of heat dissipation on the thermal capacity as well as the dissimilar nature of the materials contained in the structure of the armature.

In a well-designed motor having an efficient ventilating system, the temperature of the armature windings will increase more rapidly than that of the field windings so that the armature is the more sensitive to changes in load. On this account it becomes essential to know the thermal characteristics of the more complicated rotor windings if a close estimate of the behavior of the motor under a varying load cycle is desired. This treatment, therefore, is devoted entirely to the more difficult problem of arriving at suitable equations from which the temperature rise of the armature can be determined for any desired load and speed.

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[†] Part of the material in this paper was submitted as a thesis to the University of Pittsburgh in 1930.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

In order to arrive at anything approaching a simple result, it is necessary to reduce the armature to the simplest possible equivalent model. Nearly all railway armatures are designed so that a portion of the cooling air is drawn through ventilating ducts running axially through the armature punchings underneath the slots. Since the axial length of the armature core is usually long in comparison to the radial depth of the punchings, the heat flow across the plane of the laminations will be very small compared to the flow parallel to the plane of the laminations. Also, the temperature of the end windings in modern railway motors is usually fairly close to that of the embedded windings over most of the working load conditions and, on account of the comparative length of the embedded part, the temperature variation along the whole length of armature winding is comparatively small. From these considerations, the following assumptions are made:

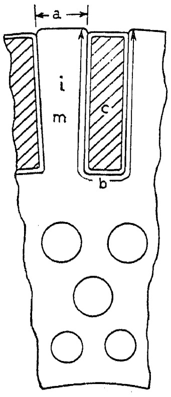

Fig. 1

A. The temperature of the copper in the slot is assumed to be uniform in all three dimensions.

B. The temperature throughout the iron of the armature punchings is assumed to be uniform in all three dimensions.

C. The rate of heat transfer to the ventilating air in the axial ducts of the core is taken as being one-half that at the outer surface. This is a purely empirical assumption and is found to give the most consistent results in the calculation of the surface heat transfer constant from temperature test data. It is also based on the supposition that the air in the ducts moves relatively to the surface of the ducts with a velocity which is about one-quarter to one-third the velocity of the air relative to the armature surface. The rate of heat transfer in axial ducts for various conditions of air velocity and nature of surface has been obtained experimentally by Luke.

D. The flow of heat through the insulation at the top of the slot is neglected.

The first two of these assumptions reduce the problem to a two dimensional one so that it is only necessary to consider the flow of heat in the armature in the plane of the laminations, that is, perpendicular to the axis of the armature. There are two main component parts of the armature, the iron of the core, represented by i in Fig. 1, and the copper c embedded in the slot. The iron and the copper, whose thermal characteristics differ from each other quite considerably are separated by a comparatively thick insulating wall m. It is the interchange of heat between the iron and the copper through this insulation that is usually neglected in most methods of predetermining temperature rise in railway motor armatures. The arrangement of the component parts of the armature as shown by Fig. 1 can be represented by the simple thermal model shown in Fig. 2. The body c represents the copper separated by the insulation m from the body i which represents the iron of the core. The equations given below are derived from the mathematical consideration of this system presented in the Appendix.

The following symbols are employed:

a = width of tooth at the armature surface in inches.
b = periphery of the slot in inches.
K_1 = rate of heat conduction through the slot insulation in watts per sq. in. per deg. cent. $= \frac{\kappa}{\tau}$
K_2 = coefficient of heat transfer from the armature surface to the ventilating air in watts per square in. per deg. cent.
κ = conductivity of the insulation in watts per in. per deg. cent.
p = sum of the peripheries of all the axial ventilating ducts through the armature core divided by the number of slots.
S_c = specific heat of copper in watt-hours per lb. per deg. cent. = 0.0485.
S_i = specific heat of the armature iron in watt-hours per lb. per deg. cent. = 0.0605.
t = time in hours.
T_c = temperature reached by the copper in time t.
T_i = temperature reached by the iron in time t.

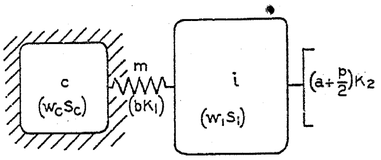

Fig. 2

T_a = mean temperature of the ventilating air adjacent to the armature surface. (It is assumed that this temperature will be the same in the vent ducts as at the armature surface.)
T_k = temperature rise of the copper above the average temperature of the ventilating air after infinite time.
τ = thickness of insulation between copper in the slot and iron at sides and bottom of slot.
θ = temperature rise of the copper above the ambient air temperature at time t.
θ_a = average temperature rise of the ventilating air above the ambient temperature at time t.
θ_c = temperature rise of the armature copper above the average temperature of the ventilating air at time t.

θ_k = temperature rise of the armature copper above the ambient temperature when t is infinite.

V = rate of air flow through the motor in cu. ft. per min.

v_s = peripheral speed of armature surface in ft. per min.

W_c = loss in the embedded conductors in one inch length of one slot in watts

W_i = core loss in watts per inch length of core per slot.

w_c = weight of copper in one inch length of one slot + 50 per cent of weight of insulating material in one inch length of slot. (In a motor insulated for 750 volts with single-turn coils this will be equivalent to 1.3 times weight of copper in one inch length of one slot.)

w_i = weight of armature punchings in lb. per in. length of core per slot.

Mathematical consideration of the interchange of heat between the two component bodies c and i leads to two simultaneous linear equations of the first order:

$$A \frac{d\,T_i}{d\,t} + (B + C)\,T_i - B\,T_c - C\,T_a - W_i = 0 \quad (1)$$

$$D \frac{d\,T_c}{d\,t} + B\,T_c - B\,T_i - W_c = 0 \quad (2)$$

where A, B, C, D are constants (see Appendix).

Since railway motors are now usually rated on the basis of the temperature rise of the windings as measured by increase in resistance we are only interested in the copper temperature T_c. Eliminating T_i from the above equations we obtain a second order equation in T_c:

$$L \frac{d^2\,T_c}{d\,t^2} + M \frac{d\,T_c}{d\,t} + N\,T_c = K' \quad (3)$$

where L, M, N, K' are constants.

The solution of this equation is of the form

$$T_c = C_1\,e^{-\lambda_1 t} + C_2\,e^{-\lambda_2 t} + \frac{K'}{N} \quad (4)$$

where C_1 and C_2 are constants of integration and $-\lambda_1$ and $-\lambda_2$ are roots of the equation $L\,D^2 + M\,D + N = 0$. Introducing the original constants into equation (4) and making certain simplifying approximations, the temperature rise of the armature copper at any time t is given by the following set of equations:

$$\theta_c = T_k\,(1 - e^{-\lambda_1 t}) + C_2\,(e^{-\lambda_2 t} - e^{-\lambda_1 t}) \quad (5)$$

where:—

$$C_2 = \frac{w_c\,S_c\,W_i - w_i\,S_i\,W_c}{(2\,w_c\,S_c + w_i\,S_i)\,b\,K_1} \quad (6)$$

$$\lambda_1 = \frac{\left(a + \frac{p}{2}\right) K_2}{\left[\left(a + \frac{p}{2}\right)\frac{K_2}{b\,K_1} + 1\right] w_c\,S_c + w_i\,S_i} \quad (7)$$

$$\lambda_2 = \frac{b\,K_1}{w_c\,S_c} + \frac{\left(a + \frac{p}{2}\right) K_2 + b\,K_1}{w_i\,S_i} \quad (8)$$

$$T_k = \frac{\left[\left(a + \frac{p}{2}\right)\frac{K_2}{b\,K_1} + 1\right] W_c + W_i}{\left(a + \frac{p}{2}\right) K_2} \quad (9)$$

θ_c is the temperature rise of the armature copper above the mean temperature of the ventilating air at time t. Since the ventilating air temperature varies quite considerably, it cannot be conveniently considered as a constant over the period of time under consideration, so that some means is necessary to take its variation into account. The following empirical relation is found to be a close approximation in modern self-ventilated motors for peripheral speeds not less than 1,000 ft. per min.:

$$\theta_a = T_k \left(\frac{700}{v_s + 1000}\right)\left(1 - e^{-\frac{v_s{}^3 t}{2\times10^{10}}}\right) \quad (10)$$

Introducing this into equation (5)

$$\theta = T_k\,(1 - e^{-\lambda_1 t}) + C_2\,(e^{-\lambda_2 t} - e^{-\lambda_1 t}) + T_k \left(\frac{700}{v_s + 1000}\right)\left(1 - e^{-\frac{v_s{}^3 t}{2\times10^{10}}}\right) \quad (5a)$$

The correction for rise in ventilating air represented by equation (10) is only approximate and can, of course, be made more exactly by the introduction of a more complicated expression. The rise in ventilating air temperature, however, is relatively small in comparison with the copper rise so that a correction of the above type is close enough for most applications. In all the foregoing, the mean temperature of the ventilating air is taken as the arithmetic mean between the temperature of the ingoing and outgoing air so that θ_a actually will be one-half the temperature rise of the ventilating air through the motor.

In equation (9), T_k is the temperature rise of the copper above the mean temperature of the ventilating air when t is infinite, that is, after a continuous run under the load condition represented by the losses W_c and W_i. Equation (9), therefore, gives us a means of arriving at values of K_2, by substitution of data from actual temperature tests into the equation, for various conditions of speed and ventilation. By taking a number of temperature runs on each of a wide range of motors of varying size it was possible, from equation (9), to find a relation between K_2 and the peripheral speed of the armature or other condition affecting the rate of heat dissipation. Fig. 3 shows the values of K_2 so obtained for a range of self-ventilated motors varying in size from 50 hp. to 310 hp., plotted against the armature peripheral speed. In these calculations and in all subsequent calculations, the quantity W_c was taken as the $I^2\,R$ loss calculated from conductor resistance plus the eddy current loss in the conductors as calculated by Field's method. As the tests from which these points are taken were ordinary test floor runs in which no special precautions for absolute accuracy were taken as would be the case in laboratory tests, the points are very favorably consistent for a curve of this type. The temperature rise by resistance was measured by taking a four point cooling curve immediately after shutting down the motor and projecting the curve back to the shutdown point. By measuring V, the rate of air flow through the motor in forced ventilated machines and, in the case of self-ventilated motors, estimating it from the losses and air rise for each test, and assuming that the axial velocity of the air over the armature surface is approximately $\frac{V^*}{0.3\,D^2}$, the velocity of the air relative to the armature surface will be, approximately, $\sqrt{v_s^2 + \left(\frac{V}{0.3\,D^2}\right)^2}$. Fig. 4 shows K_2 plotted against this relative velocity for a range of motors including both self and forced ventilated machines varying in size from 50 hp. to 800 hp. and in armature diameter from 11 to 30 in. With the aid of the curves of Figs. 3 and 4, the temperature rise above the mean ventilating air temperature after a continuous run can be predetermined from equation (9). However, equation (9) can be modified to correct for the rise in ventilating air temperature. For the line of self-ventilated motors mentioned above, this is done by adding the right hand member of equation (10) when t is infinite:

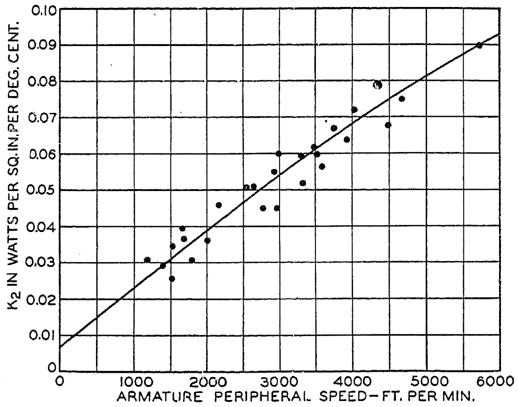

FIG. 3—K_2 FOR SELF-VENTILATED RAILWAY MOTORS

$$\theta_k = \left(\frac{\left[\left(a + \frac{p}{2} \right) \frac{K_2}{b\,K_1} + 1 \right] W_c + W_i}{\left(a + \frac{p}{2} \right) K_2} \right) \times \left(1 + \frac{700}{v_s + 1000} \right) \qquad \textbf{(9a)}$$

In determining the curves shown in Figs. 3 and 4 the value of κ, the conductivity of the insulation between the copper and the walls of the slot, was taken as 0.0025 watts per in. per deg. cent. This corresponds to an average value for the combination of mica, paper and varnish which is used for armature coil insulation. The temperature rise of the copper as will be seen by an inspection of equation (9) is considerably influenced by the value of K_1, the rate of heat transfer through this insulation; so that, considering the possibility of variations in the conductivity due to air pockets and incomplete drying out of compound, it is probable that the variations from the curve of the points of Figs. 3 and 4 can, to some extent, be ascribed to variations in the condition of the insulation.

The right-hand side of equation (5), from which time-temperature curves of a motor armature can be calculated, consists of two parts, $T_k\,(1 - e^{-\lambda_1 t})$ and $C_2\,(e^{-\lambda_2 t} - e^{-\lambda_1 t})$. The equation was arranged in this form because it illustrates clearly how the time-temperature characteristics vary from the usually accepted simple logarithmic function which the first of these two expressions represents. The second expression, $C_2\,(e^{-\lambda_2 t} - e^{-\lambda_1 t})$, is very small compared with the first when t is greater than $\frac{1}{\lambda_1}$ and disappears when t becomes infinite so that for finding the temperature rise over intervals of time exceeding the usually accepted thermal time constant, the second expression can be neglected. The equation then assumes the familiar form:

$$\theta_c = T_k\,(1 - e^{-\lambda_1 t}) \qquad \textbf{(11)}$$

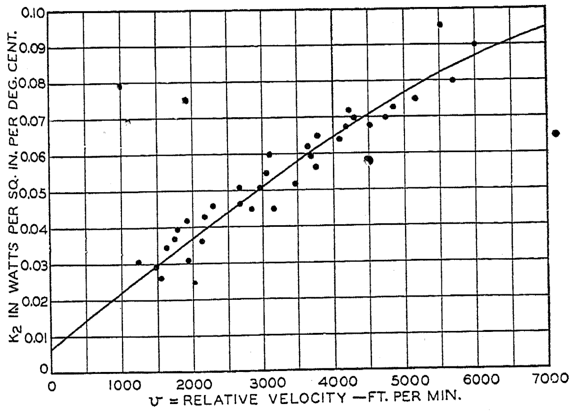

FIG. 4—K_2 FOR BOTH SELF- AND FORCED-VENTILATED RAILWAY MOTORS

For short time intervals, however, this second expression becomes very important and cannot be neglected. Figs. 5 to 10 show several time-temperature curves which were calculated by means of equation (5) for three motors, one a forced-ventilated motor of 450 hp. and the others, self-ventilated motors rated at 100 hp. and 50 hp. respectively. These curves were calculated for the same load and speed at which some actual temperature test giving consistent results was taken. The points plotted on the diagrams show the actual temperature rise above the mean ventilating air temperature obtained in the corresponding temperature run.

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*Where D is armature diameter in inches.

It is recognized that a simple logarithmic curve of the form $\theta = T_k\,(1 - e^{-\lambda_1 t})$, involving a single thermal time constant, could be found which would pass through any such set of two, or even more, test points. But it has been found very difficult to discover a sufficiently simple law which would connect the constants obtained experimentally of such an expression with the main design dimensions of a wide range of armature sizes. It has been shown that the equations (**5**) to (**9**) can be made, with the aid of a curve or set of curves of the type shown in Figs. 3 and 4, to represent within a reasonable degree of accuracy, the thermal characteristics of a very wide range of machines. The only feature which such a group of motors need have in common is that the path of the cooling air through the motor should be essentially similar in all machines of the group. The chief purpose of this method is the predetermination of the thermal characteristics of a projected motor design when only preliminary data are available. The method may appear unwieldy at first sight but with a little practise these equations can be handled quite easily and the extra time and labor involved in applying them will be justified in most cases by the smaller margin of safety with which it will be found that a new design of motor can be applied.

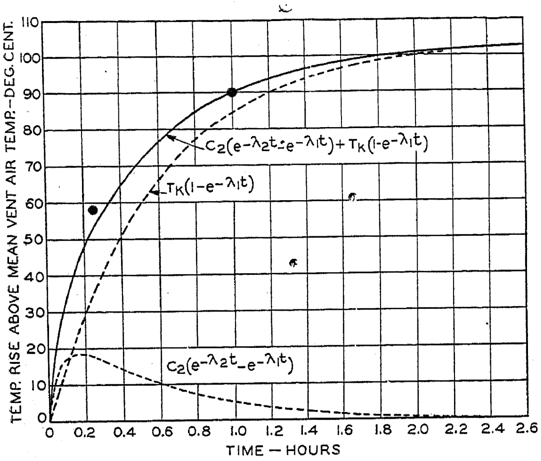

Fig. 5—450-Hp. Motor (One Hour Temperature Run)

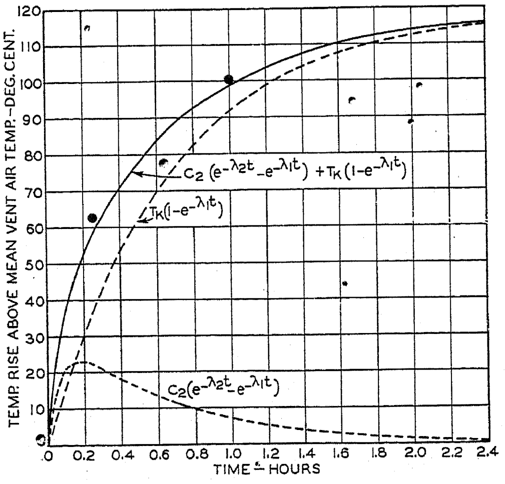

Fig. 6—450-Hp. Motor (One Hour Temperature Run)

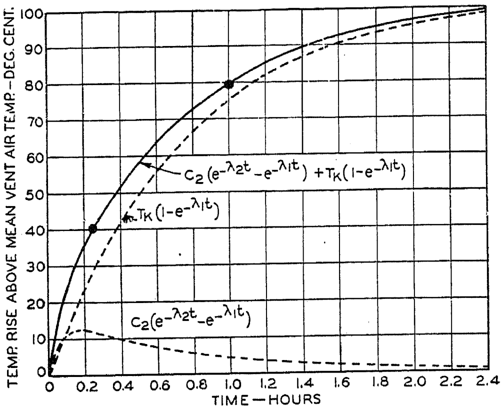

Fig. 7—450-Hp. Motor (One Hour Temperature Run)

Appendix

Since the flow of heat through the insulation at the top of the slot is neglected, all the heat escaping from the copper will be transferred to the iron. Considering only one inch length of core, the rate of heat flow through the insulating wall will be $b\,K_1$ watts per deg. cent. Referring now to Fig. 2, the heat increment in time $d\,t$ in the body i will be:

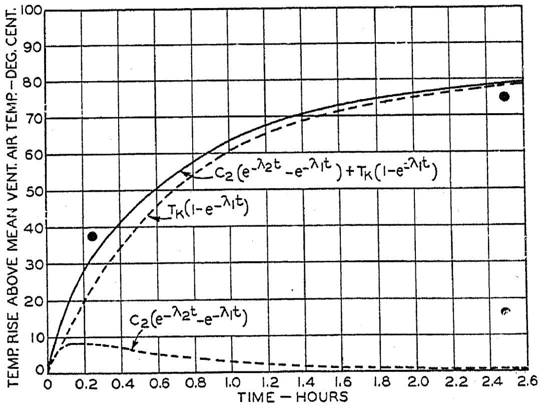

Fig. 8—450-Hp. Motor (Continuous Temperature Run)

$$W_i\,d\,t + (T_c - T_i)\,b\,K_1\,d\,t \qquad (\mathbf{12})$$

The heat stored in time $d\,t$ will be

$$w_i\,S_i\,d\,T_i \qquad (\mathbf{13})$$

The heat imparted to the ventilating air will be

$$\left[a K_2 (T_i - T_a) + p \frac{K_2}{2} (T_i - T_a) \right] d t \qquad (14)$$

Equating these values and rearranging:

$$w_i S_i \frac{d T_i}{d t} + \left[b K_1 + \left(a + \frac{p}{2} \right) K_2 \right] T_i$$

$$- b K_1 T_c - \left(a + \frac{p}{2} \right) K_2 T_a - W_i = 0 \qquad (15)$$

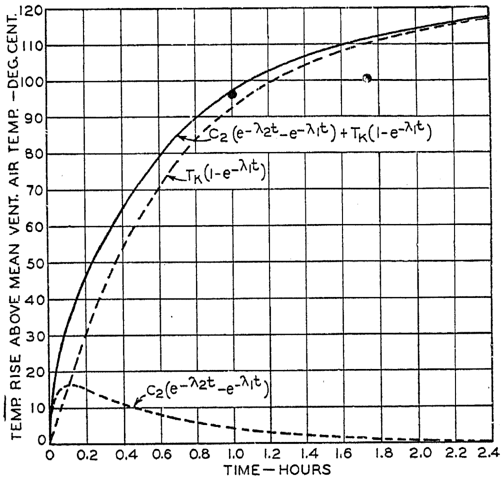

FIG. 9—100-HP. MOTOR (ONE HOUR TEMPERATURE RUN)

In the body c, the heat increment will be:

$$W_c \, d t$$

The heat transferred to the body i will be:

$$b K_1 (T_c - T_i) \, d t \qquad (16)$$

The heat stored will be:

$$w_c S_c \, d T_c \qquad (17)$$

Equating these quantities and rearranging

$$w_c S_c \frac{d T_c}{d t} + b K_1 T_c - b K_1 T_i - W_c = 0 \qquad (18)$$

Let

$$w_i S_i = A$$
$$b K_1 = B$$
$$\left(a + \frac{p}{2} \right) K_2 = C$$
$$w_c S_c = D$$

Then we have:

$$A \frac{d T_i}{d t} + (B + C) T_i - B T_c - C T_a - W_i = 0 \qquad (15a)$$

$$D \frac{d T_c}{d t} + B T_c - B T_i - W_c = 0 \qquad (18a)$$

From (18a) we have:

$$T_i = \frac{D}{B} \frac{d T_c}{d t} + T_c - \frac{W_c}{B} \qquad (19)$$

Differentiating (18a) with respect to t:

$$D \frac{d^2 T_c}{d t^2} + B \frac{d T_c}{d t} - B \frac{d T_i}{d t} = 0$$

or

$$\frac{d T_i}{d t} = \frac{D}{B} \frac{d^2 T_c}{d t^2} + \frac{d T_c}{d t} \qquad (20)$$

Substituting these values for T_i and $\frac{d T_i}{d t}$ in equation (15a) and rearranging

$$A D \frac{d^2 T_c}{d t^2} + [A B + (B + C) D] \frac{d T_c}{d t}$$

$$+ B C T_c - B C T_a - (B + C) W_c - B W_i = 0 \qquad (21)$$

Let

$$A D = L$$
$$A B + (B + C) D = M$$
$$B C = N$$
$$B C T_a + (B + C) W_c + B W_i = K'$$

Equation (21) becomes:

$$L \frac{d^2 T_c}{d t^2} + M \frac{d T_c}{d t} + N T_c = K' \qquad (21a)$$

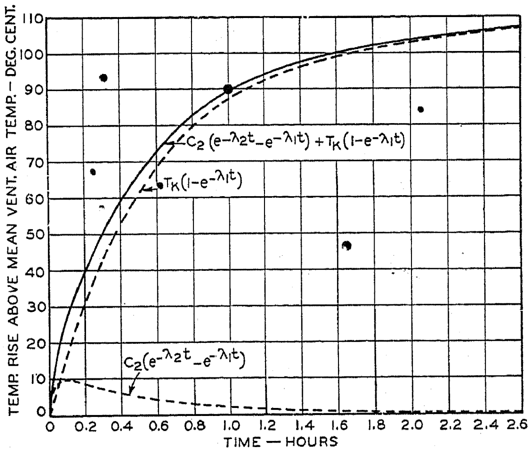

FIG. 10—50-HP. MOTOR (ONE HOUR TEMPERATURE RUN)

The solution of this equation is:

$$T_c = C_1 e^{-\lambda_1 t} + C_2 e^{-\lambda_2 t} + \frac{K'}{N} \qquad (22)$$

Where:

$$\lambda_1 = \frac{M}{2L} - \sqrt{\frac{M^2}{4L^2} - \frac{N}{L}} \qquad (23)$$

$$\lambda_2 = \frac{M}{2L} + \sqrt{\frac{M^2}{4L^2} - \frac{N}{L}} \qquad (24)$$

$$\frac{K'}{N} = T_a + \frac{(B + C) W_c + B W_i}{B C}$$

Substituting this in equation (**22**) and calling:

$$\frac{(B + C)\,W_c + B\,W_i}{B\,C},\ T_k:$$

$$T_c - T_a = C_1\,e^{-\lambda_1 t} + C_2\,e^{-\lambda_2 t} + T_k \qquad (\mathbf{25})$$

When

$$t = 0,\ T_c - T_a = 0$$

$$C_1 + C_2 + T_k = 0$$

or

$$C_1 = -\,C_2 - T_k$$

so that

$$T_c - T_a = C_2\,(e^{-\lambda_2 t} - e^{-\lambda_1 t}) + T_k\,(1 - e^{-\lambda_1 t}) \qquad (\mathbf{26})$$

From equation (**19**), knowing that when $t = 0$, $T_c = T_i = T_a$:

$$T_a = \frac{D}{B}\,\frac{d\,T_c}{d\,t} + T_a - \frac{W_c}{B}$$

or

$$\frac{D}{B}\,\frac{d\,T_c}{d\,t} = \frac{W_c}{B} \qquad (\mathbf{27})$$

Differentiating equation (**26**) with respect to t:

$$\frac{d\,T_c}{d\,t} = C_2\,(\lambda_1\,e^{-\lambda_1 t} - \lambda_2\,e^{-\lambda_2 t}) + T_k\,\lambda_1\,e^{-\lambda_1 t}$$

Putting $t = 0$ and $\dfrac{d\,T_c}{d\,t} = \dfrac{W_c}{D}$ (from equation (**27**);

$$\frac{W_c}{D} = C_2\,(\lambda_1 - \lambda_2) + T_k\,\lambda_1$$

or

$$C_2 = \frac{\dfrac{W_c}{D} - T_k\,\lambda_1}{\lambda_1 - \lambda_2} \qquad (\mathbf{28})$$

$$\lambda_1 = \frac{M}{2\,L} - \sqrt{\frac{M^2}{4\,L^2} - \frac{N}{L}}$$

$$= \frac{1 - \sqrt{1 - \dfrac{4\,L\,N}{M^2}}}{\dfrac{2\,L}{M}}$$

$$\frac{1 - \left(1 - \dfrac{2\,L\,N}{M^2}\right)}{\dfrac{2\,L}{M}}$$ (approx.) since $\dfrac{4\,L\,N}{M^2}$ is of the order of 0.2

$$= \frac{N}{M}$$

$$= \frac{B\,C}{A\,B + D\,(B + C)}$$

$$= \frac{b\,K_1\left(a + \dfrac{p}{2}\right)K_2}{w_i\,S_i\,b\,K_1 + \left[\,b\,K_1 + \left(a + \dfrac{p}{2}\right)K_2\right]\,w_c\,S_c}$$

$$= \frac{\left(a + \dfrac{p}{2}\right)K_2}{\left[1 + \left(a + \dfrac{p}{2}\right)\dfrac{K_2}{b\,K_1}\right]w_c\,S_c + w_i\,S_i} \qquad (7)$$

$$T_k = \frac{(B + C)\,W_c + B\,W_i}{B\,C}$$

$$= \frac{\left[1 + \left(a + \dfrac{p}{2}\right)\dfrac{K_2}{b\,K_1}\right]W_c + W_i}{\left(a + \dfrac{p}{2}\right)K_2} \qquad (9)$$

$$\lambda_2 = \frac{M}{2\,L} + \sqrt{\frac{M^2}{4\,L^2} - \frac{N}{L}}$$

Using the previous approximation for the radical:

$$\lambda_2 = \frac{1 + \left(1 - \dfrac{2\,L\,N}{M^2}\right)}{\dfrac{2\,L}{M}}$$

$$= \frac{M}{L} - \frac{N}{M}$$

$$= \frac{A\,B + D\,(B + C)}{A\,D} - \frac{B\,C}{A\,B + D\,(B + C)}$$

Neglecting the last term which is very small in comparison with the first:

$$\lambda_2 = \frac{B}{D} + \frac{B + C}{A}$$

$$= \frac{b\,K_1}{w_c\,S_c} + \frac{b\,K_1 + \left(a + \dfrac{p}{2}\right)K_2}{w_i\,S_i}$$

$$C_2 = \frac{\dfrac{W_c}{D} - T_k\,\lambda_1}{\lambda_1 - \lambda_2}$$

$$\lambda_1 - \lambda_2 = -\,2\sqrt{\frac{M^2}{4\,L^2} - \frac{N}{L}}$$

$$= -\frac{M}{L}\sqrt{1 - \frac{4\,L\,N}{M^2}} = -\frac{M}{L}\left(1 - \frac{2\,L\,N}{M^2}\right) \text{ approx.}$$

$$= \frac{2\,B\,C}{A\,B + D\,(B + C)} - \frac{A\,B + D\,(B + C)}{A\,D}$$

$$T_k\,\lambda_1 = \frac{N}{M}\,\frac{(B + C)\,W_c + B\,W_i}{N}$$

$$= \frac{(B + C)\,W_c + B\,W_i}{M}$$

$$\therefore C_2 = \frac{\dfrac{W_c}{D} - \dfrac{(B + C)\, W_c + B\, W_i}{M}}{\dfrac{2\, B\, C}{A\, B + D\,(B + C)} - \dfrac{A\, B + D\,(B + C)}{A\, D}}$$

$$= - \frac{\dfrac{A\, B}{D}\, W_c - B\, W_i}{\dfrac{A\, B^2}{D} + \dfrac{D}{A}\,(B + C)^2 + 2\, B^2}$$

Putting $(B + C)^2 = 2\, B^2$, dividing through by B^2 and neglecting D^2

$$C_2 = \frac{D\, W_i - A\, W_c}{B\,(A + 2\, D)}$$

$$= \frac{w_c\, S_c\, W_i - w_i\, S_i\, W_c}{(2\, w_c\, S_c + w_i\, S_i)\; b\; K_1} \qquad (6)$$

Putting $T_c - T_a = \theta_c$ in equation (**26**), the temperature rise of the armature copper above the mean temperature of the ventilating air at any time t is given by the set of equations (**5**) to (**9**) inclusive.

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Discussion

John Basta: We know how difficult it is to obtain a simple method of calculation of heating curves on assumptions following closely the experience. With those machines where the exchange of heat from winding to air cannot be neglected, the heating curve differs widely from a simple exponential curve. However, Mr. Lightband found it justified to neglect the exchange mentioned with armatures of railway motors and succeeded in showing that the heating curve for them is composed from two exponential curves. The actual heat runs used for verifying the calculated curves, however, do not give more than two, and in some cases even one point only. That is why we would suggest to Mr. Lightband that he undertake another series of tests in which complete curves would be obtained. This, of course, should not be done by measuring the resistances of armature windings, as in the tests presented, but rather by placing temperature detectors into the winding, which would be connected through slip rings to meters. With such an arrangement, the distribution of temperatures along the winding could be ascertained at the same time.

Carl J. Fechheimer: It has been recognized for several years that the usual transient equation for temperature in an armature in which there is one time constant is inadequate, particularly during the first part of the period after the load on the machine is changed. The thermal time constant is given by the ratio of the energy stored to the power dissipated, and that ratio is usually considerably different for the copper and the iron. The errors that may arise for short time ratings, if only one time constant is considered, may be of considerable magnitude. Mr. Lightband has brought out these points clearly.

The general forms of the transient equations when two bodies are considered are analogous to those of the two electric circuits which are magnetically coupled, and when self and mutual inductances are considered. In either case, it is essential to solve two differential equations simultaneously. The solutions for the thermal circuit may be obtained by making use of the principles of superposition and reciprocity. The solutions obtained thereby are perhaps somewhat more general than the forms obtained by Mr. Lightband.

Mr. Lightband's solutions are for the heating state, but they might easily be extended to the cooling state. When the cooling curve is plotted, it is found that the initial part is similar to the "sub-transient" for the decay of current in a suddenly short-circuited alternator, and like it, it dies out relatively soon, and only the "transient" state then obtains, and then only the second time constant is needed. Mr. Lightband has shown for the heating curve that the influence of the first time constant diminishes fairly rapidly, and subsequently only the influence of the second is of importance.

The third assumption made in the paper, that the rate of heat transfer in the axial ducts is one-half that at the outer surface, limits the final equations to a particular type of ventilation. This should be borne in mind by anyone who plans to use the equations. Undoubtedly, the equations could easily be modified so as to make them of more general form.

It should be recalled that the problems involved in heat flow become too involved to secure an accurate solution. For example, in many machines the influence of the longitudinal heat flow in the copper has a very large effect upon the copper temperature. While the differential equations for this case can be written, the solutions for the transient state become too involved to admit of placing them in practical form. It is essential to make assumptions such as Mr. Lightband has done, although his assumptions may not be the ones to adopt for some other class of apparatus. Thus, his results are all based upon the average copper temperature as measured by resistance. For machines having long cores, the copper at the middle is usually at considerably higher temperature than the average, and consequently his assumption B would introduce too large an error.

P. H. Powell: A point which is not quite clear in Mr. Lightband's paper is the definition of K_2 and the surface of the armature regarded as radiating. His argument seems to be based on the assumption that all the heat is dissipated by the surfaces of the tops of the teeth and by the surface of the ventilating ducts; on the other hand, K_2 is defined with respect to the armature surface again somewhat indefinitely. If one regards the outside barrel surface of the armature (circumference multiplied by overall length) as being the only radiating surface, the value of K_2 for a speed of 3,000 feet per minute is about 0.1 for open type machines. If one adds up the external barrel surface, the internal barrel surface, and the ducts, the figure for K_2 appears to be about 0.04. Mr. Lightband's result seems to indicate a figure of about 0.05.

There are, of course, differences in types and in ventilating arrangements that make a comparison almost impossible, but it would seem that if only the tops of the teeth and the iron surface of the ventilating ducts are allowed as radiating surface, the figure for K_2 should be considerably higher than 0.05. However, it is very interesting to find a method involved which will enable the short-time rating to be better defined.

D. A. Lightband: It is pointed out that in machines where direct flow of heat from the winding to the air cannot be neglected, the heating curve differs widely from a simple exponential curve. I have shown that in railway motors, even though this heat flow is neglected, there will still be considerable deviation from the simple exponential curve. Attention has been drawn by Mr. Basta to the use of only two test points as an illustration of the closeness of the method. It should be pointed out that the two points are really three, since the origin must be included. Any kind of exponential curve cannot be drawn through three such points, which would represent a reasonable characteristic heating curve of a motor. Even a simple exponential curve drawn through these three points has been found to differ very widely from an exponential heating curve estimated from the one hour and continuous temperature tests.

The suggestion by Mr. Basta that test curves be taken by temperature detectors in the winding rather than by measuring the temperature rise by the resistance method would be rather defeating the object of this method since it is on temperature rises measured by resistance that the ratings of all modern railway motors are based. Measurement of distribution of temperature along the winding longitudinally is beyond the scope of this paper.

Mr. Fechheimer mentions the fact that the solutions are for the heating state only. In the consideration of cooling, it should be pointed out that if the different time constants in the copper and iron are to be taken into account, a knowledge is necessary of the exact nature of the heating immediately prior to the commencement of the cooling curve. This is necessary because the nature of the cooling curve will depend on the initial relation between the temperature rise of the iron and that of the copper. The following equations will give the cooling curve when both the initial temperature rise of the copper and of the iron are known.

Equation for cooling with no power on:

$$\theta_c = \theta_o e^{-\lambda_1 t} + C_4 \left(e^{-\lambda_2 t} - e^{-\lambda_1 t}\right) \qquad \text{(c)}$$

where θ_o = initial copper temperature

λ_1 and λ_2 defined by equations (7) and (8)

$$C_4 = \frac{(w_i S_i + w_c S_c)(\theta_o - \theta_i) - w_c S_c\left(a + \frac{p}{2}\right)\frac{K_2}{bK_1}\,\theta_i}{w_c S_c + 2 w_i S_i} \qquad \text{(e)}$$

θ_i = initial iron temperature

For cooling with power on:

$$\begin{aligned}\theta_c &= T_k\left(1 - e^{-\lambda_1 t}\right) + C_2\left(e^{-\lambda_2 t} - e^{-\lambda_1 t}\right) + \theta_4\, e^{-\lambda_1 t} + C_4\left(e^{-\lambda_2 t} - e^{-\lambda_1 t}\right)\\ &= T_k + (\theta_o - T_k)\, e^{-\lambda_1 t} + (C_2 + C_4)\left(e^{-\lambda_2 t} - e^{-\lambda_1 t}\right) \qquad \text{(f)}\end{aligned}$$

Mr. Powell has brought up the point that my definition of K_2 is somewhat indefinite since I do not specifically state whether it is based on the heat radiating from the whole external and internal radiating surface of the armature. He is of the opinion that if this constant refers to the radiating surface obtained by adding up the external surface of the armature core and winding, the internal surface of the armature coils, and the surface of the ducts, it should have a value higher than that given in the paper.

I have made the definition of K_2 as the coefficient of heat transfer from the armature surface to the ventilating air. By "armature surface" is meant the external surface of the armature core and the internal surface of the ventilating ducts. Since I have assumed that no heat flow has taken place from the armature core to the windings in a longitudinal direction, it is quite unnecessary to include the outer surface of the windings as part of the radiating surface. The amount of heat which flows through the insulation at the mouth of the slots is a very small proportion of the total heat flow from the armature and has been neglected, and therefore the only radiating surfaces which need be considered are the tops of the teeth and the ventilating ducts.

The figures for K_2 which are calculated from temperature tests by means of the equations given in the paper and shown in Figs. 3 and 4, correspond very closely to values of rate of heat transfer for various kinds of rotors given in Luke's paper referred to in the bibliography. Luke's results were obtained by means of an exhaustive series of experiments and offer a very sound basis of comparison for heat transfer constants in rotating armatures.

High Capacity Rectifier Efficiency Improves by Sectionalizing

By A. L. ATHERTON*

Member, A.I.E.E.

Synopsis.—The reasons for adopting the sectional construction are given and discussed at some length. The new features of design and construction which make the sectional arrangement feasible are described. One arrangement of sections into a unit is described and alternative arrangements are discussed briefly. Performance characteristics are described in broad terms and the extent of the improvement over previous type is pointed out.

A forecast of future development is indicated.

THE idea of the sectional type mercury arc rectifier is based on the obvious, but apparently heretofore unrecognized, fact that the best possibilities inherent in the principle are realized only in the smaller sizes. Efficiency, reliability, economy, and flexibility of both application and use are all far better in small than in large units. The advantages appear to be fundamental and, therefore, permanent.

The rectifier is not alone in this. Many familiar things are inherently at their best in small sizes. The established and common "unit type" things are so well known that we have come to overlook the limitations and inefficiencies introduced by the subdivision and to accept the scheme as natural and right. In suspension insulator units, a large part of the insulation material is shunted by metal, assembly operations are duplicated many times in a single unit of use, and the distribution of potential along the string is wrong unless corrected; but the mechanical strength, flexibility, adaptability to all voltages, and the economy of concentrating manufacture and stocks in a single size of unit are so much more important that the disadvantages almost vanish. Because the benefits outweighed the disadvantages, the introduction of the idea of sectionalization in line insulators created a new utility, and because the increase in utility was great, the whole of the associated art of long distance transmission received a major stimulus.

So with the metal tank mercury arc rectifier. Subdivision into unit sections introduces both advantages and disadvantages, but the advantages so far outweigh the disadvantages that, once the conception is seriously considered and the sectional design actually worked out, doubts disappear and it seems the right and natural way to proceed.

Development through the past twenty-nine years has resulted in rectifiers of small capacity, 500 kw. for example, with a high degree of reliability, a reasonable cost, and an efficiency which makes available for the higher voltage conversion applications the unique quietness and freedom from mechanical wear which has provided the major incentive throughout the development. When size is increased to get larger output capacity, however, the quality decreases markedly. Reliability decreases not only in proportion to size as might be expected, but more rapidly, because the complexity of the problem of control of the hurricane flow of vapor from the cathode is increased many fold when size is even doubled. The "outage time" per year with 3,000-kw. rectifiers is several times as great as with 500-kw. units. Cost per unit of output does not decrease with increase of size, as it does with some apparatus, because manufacturing problems increase faster than in proportion to size, and because the total requirement for this kind of equipment, with capabilities and limitations as they are even in the smaller sizes, will not support organized "line manufacture" for the larger capacities. Efficiency falls off with increase in size because of the longer arc and greater exposure to deionizing surfaces, and because the fundamentally less reliability makes necessary an increase in loss producing devices to get acceptable quality. With present designs, the internal losses for a 500-kw. unit are approximately 25 per cent less than for a 3,000-kw. unit, both based on 600-volt operation.

The major disadvantages which go with size appear to be fundamental and permanent, whatever advances may be made in the art. Larger units will always be less efficient, and the reliability and cost disadvantages will probably persist.

If we think of multiple installations of conventional small capacity units of the sort which have been available in the past, we realize that the project is impracticable. The space required is too great and the multiplicity of control and protective devices appears questionable. But when we think of subdivision of a large capacity, taking advantage of space economies resulting from recent advances in the art, building the structure in sections but installing, controlling, protecting and using it as a whole, the project becomes feasible and the desirable qualities of the small rectifier are made available for the larger capacities while the disadvantages of sectionalizing are so reduced as to become negligible.

Our work with rectifiers of conventional size and characteristics has resulted in the conviction that the limitation in capacity for a given design, as for example

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the 500-kw. unit shown in Figs. 1 and 2, lies in local limitations of the various parts, for example, the limits of conductivity and heating at the anode, and not in the perhaps more fundamental and difficult tendency to arc back at the higher currents. Such a unit carries a nominal current of 833 amperes continuously, 1,250 amperes for two hours, and 2,500 amperes for one minute. In all our testing, which includes use at currents up to 17,000 amperes on short circuit, and which includes considerable experimental experience of loads of 2,000 to 2,500 amperes for periods of 5 to 30 minutes, dependent on temperature rise, we have not experienced arcing back as a limitation. Extended service experience with a limited number of installations gives the same result. Evidently, change in the details, leaving the general arrangement unaltered, would make it possible materially to reduce the size of the containing tank without affecting the ability of the unit to carry the loads required by its rating. Experiments along these lines have verified this conclusion.

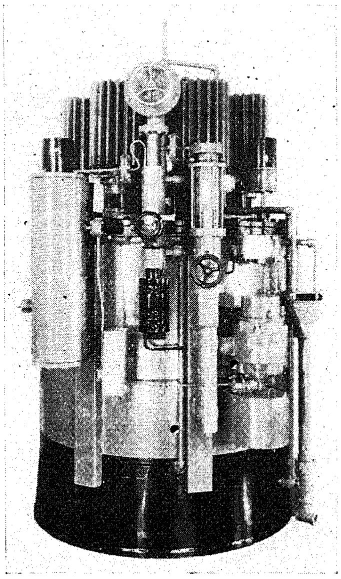

Fig. 1—Conventional Type 500-Kw., 600-Volt Rectifier

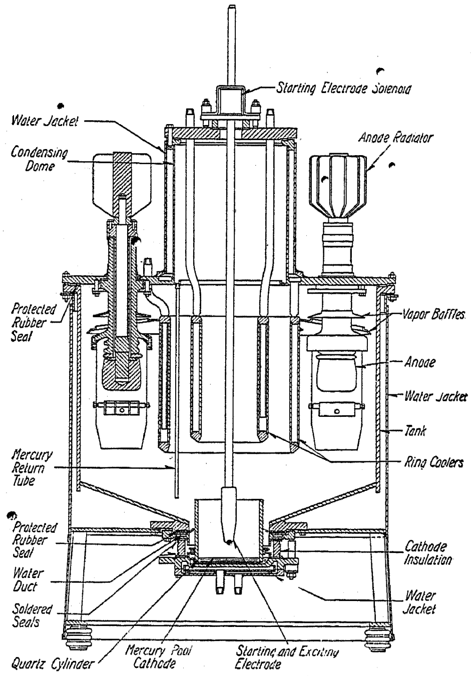

Fig. 2—Section Showing Arrangement of Parts in Conventional 500-Kw., 600-Volt Rectifier

The diameter is 54 inches and the overall height is 104 inches

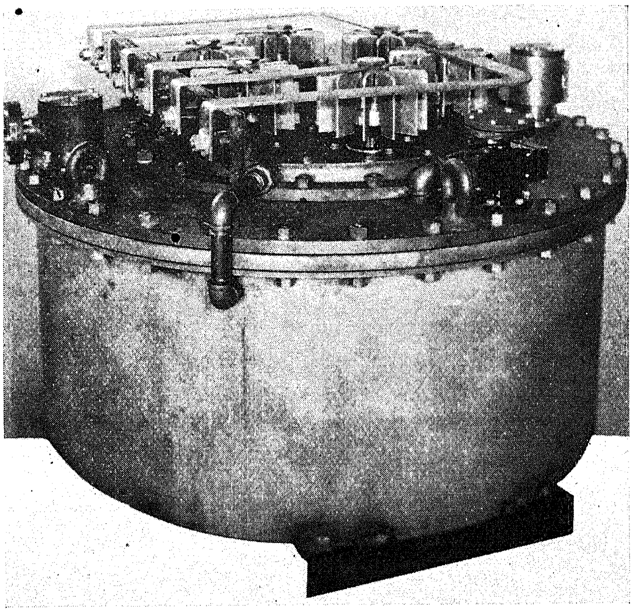

Fig. 3—Rectifier Section, 750-Kw., 600-Volt Rating

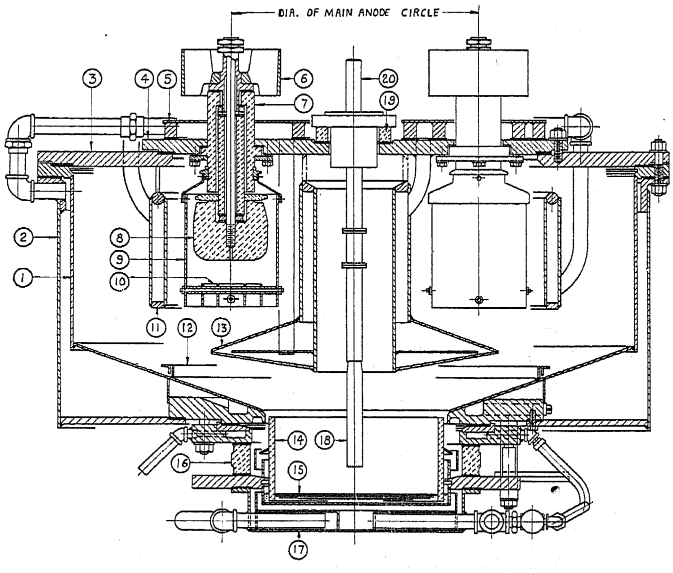

Fig. 4—Section Showing Arrangement of Parts of Unit Shown in Fig. 3

The diameter is 54 inches and the overall height is 41½ inches. The kw. rating per unit volume is thus over 3 times as great as for the conventional unit

As a further step, a rearrangement of parts made possible a still further reduction in size for a given rating, and at the same time brought about some reduction in the internal losses. The rearrangement was based on the knowledge gained in our experimental experience as to the effect of the way the vapor flows from the cathode to the condensing wall and the position of the anode structures in relation to this stream of ionized vapor on the limiting current of which the rectifier unit is capable, without excessive arcing back. Although the whole mechanism of vapor flow and its effect on arcing back are obscure as yet, several factors which influence performance have been established. The design shown in Figs. 3 and 4 is an attempt to take these as yet rather intangible and, it must be admitted, somewhat uncertain factors into account in the most satisfactory way possible at the present time.

The anode support and shielding arrangement also offered an opportunity for progress and a rather extended series of experiments resulted finally in the anode structure shown. Processing to eliminate foreign material and keep the gas evolution under operation at as low a value as possible, have progressed along with the design development.

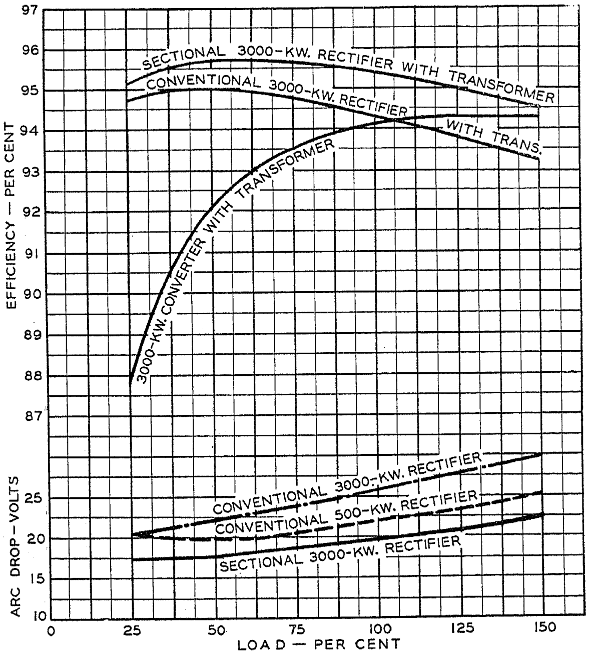

Fig. 5—Representative Arc Drop and Efficiency Curves for Conventional and Sectional Rectifiers

As a result of these improvements, the rectifier section shown in Figs. 3 and 4 is capable, with a reasonable margin, of a standard nominal rating of 750 kw. at 600 volts, and even of higher short-time overloads as sometimes required.

This section, at 750-kw., 600-volt rating, is required to carry a normal current of 1,250 amperes, with 50 per cent overload, or 1,875 amperes for two hours. Owing to the fact that final temperatures are reached in times of the order of two hours, it is felt that the two-hour capacity can best be indicated by continuous operation at the 50 per cent overload value and our experimental tests have been made in this way, thus including in a single test some margin of safety along with a degree of acceleration of the test. Double load, or 2,500 amperes is required by standard nominal rating for a period of one minute following full load. Experimental tests have been made at this current up to 5 minutes duration.

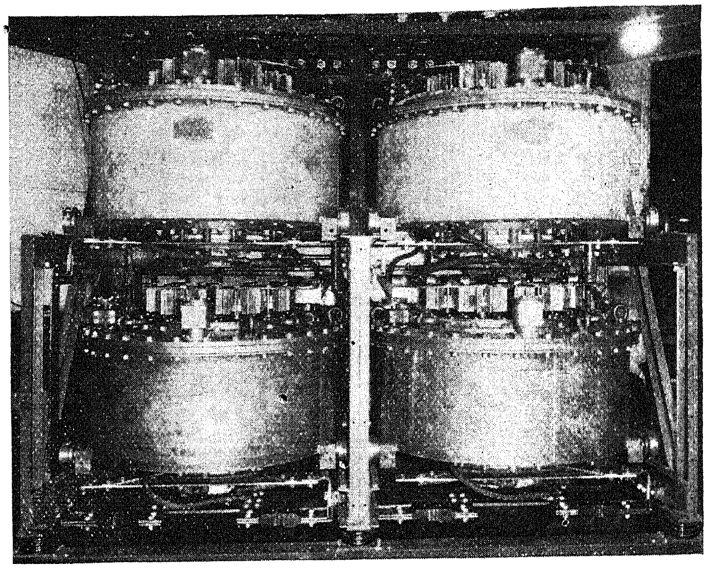

Fig. 6—Sectional Rectifier, 3,000-Kw., 600-Volt Rating

This unit consists of four sections each of 750-kw. rating, the supporting structure and the necessary auxiliary equipment for maintaining the vacuum and dividing the load. Some control and protective devices are also conveniently mounted on the structure

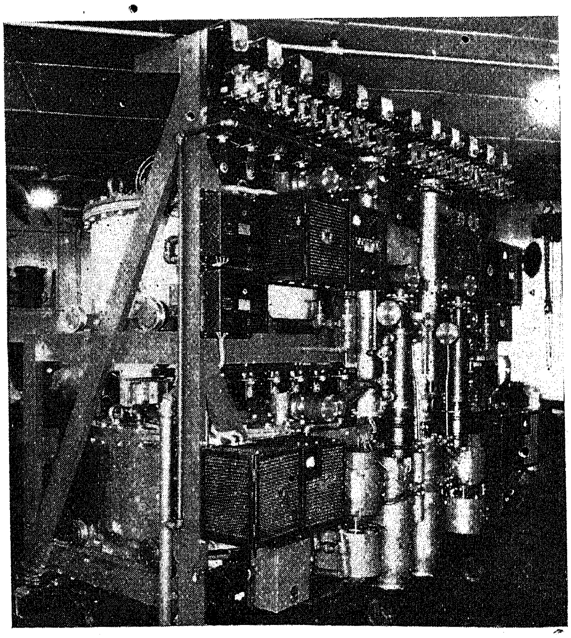

Fig. 7—View of Rear of 3,000-Kw., 600-Volt Sectional Rectifier Showing the Arrangement of the System for Maintaining the Vacuum

In all these operations, arcing back is very infrequent if it occurs at all and it is a requirement that the rectifier can be put into operation again immediately following such an occurrence.

The internal losses in the rectifier are determined by the arc drop, or voltage from anode to cathode during the conducting period. This value varies with load current and can be indicated by a curve of voltage drop against load current. The value that determines efficiency is the average for the various anodes. Fig. 5 shows average arc drop curve for the unit indicated by Figs. 3 and 4. The dotted curve shows the value for the older, conventional, unit shown by Figs. 1 and 2. As the size of a rectifier is increased, the arc path becomes longer and losses per kw. output become greater. Experimental rectifiers of 3,000 kw. with a single tank have had arc drop values as shown by the dot and dash curve of Fig. 5. It is seen that at full loads the difference between these experimental single tank 3,000-kw. rectifiers and the section shown in Figs. 3 and 4 amounts at full load to approximately 6 volts or 1 per cent, on the basis of 600 volts output. With ordinary load factors and rates for power, this difference in 3,000-kw. rating saves from $600 to $1,500 per year power cost, which represents an added value of from $4,000 to $10,000.

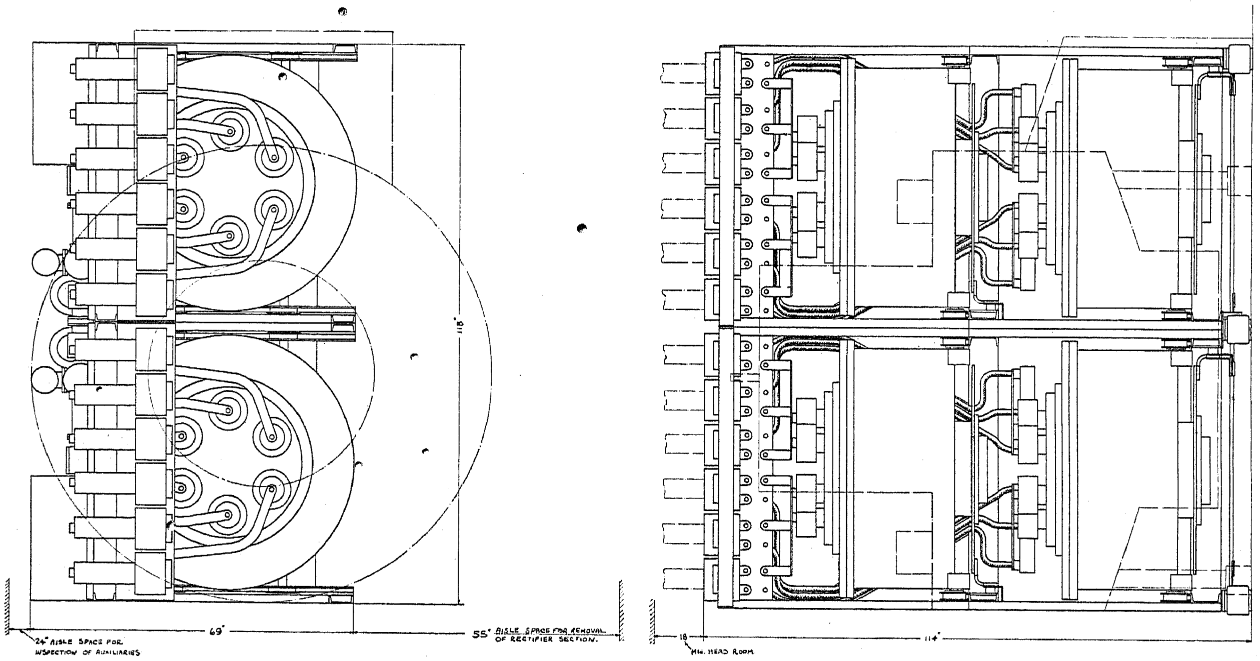
118"
69"
24" AISLE SPACE FOR INSPECTION OF AUXILIARIES
55" AISLE SPACE FOR REMOVAL OF RECTIFIER SECTION.
18
MIN. HEAD ROOM
114"

FIG. 8—OUTLINE OF 3,000-KW., 600-VOLT SECTIONAL RECTIFIER

Approximate dimensions of an experimental single tank unit are indicated by the dot and dash outline

The proportions of the new units are such as to make it feasible to mount them in service one above another. Thus, a 3,000-kw. unit can be made of four 750-kw. sections mounted two side by side and two deep. Such an arrangement is shown in Fig. 6. The floor space required for such an assembly is approximately 118 inches wide, by 68 inches deep over all parts, including auxiliary equipment. The over-all height is 113 inches, and the ceiling height of the room necessary to accommodate this equipment need not exceed this value except for electrical clearance of a few inches and whatever may be necessary to provide for adequate working space certainly not more than two feet. It is evident that there is a considerable space economy.

Such an arrangement is likely to prove the most convenient for usual applications. There is no reason, however, why other arrangements cannot be used where available space places a limitation on one or more dimensions. For example, the auxiliary equipment and wiring can be arranged between the units at the center, at some penalty from the standpoint of accessibility, making the floor dimensions approximately 150 by 55 inches. Or the four units may be mounted in a single column with floor dimensions of about 65 by 70 and a height of 210 inches. Or the parts may be separated in two or groups, possibly at some expense in auxiliary equipment.

The problem of paralleling is the same as in the case of a single tank rectifier using the same number of anodes and, except in detail, is the same as for any rectifier using twelve or more anodes. Load balancing is inherently approximately correct owing to the fact that the voltage across the arc rises with increasing current after a minimum at about 40 per cent load is passed. To get a closer balance, anode balancing coils are used and these are so arranged as to introduce no appreciable impedance into the circuit under normal operation. In the event that it is desired to take a section out of service, for maintenance or any other reason, the corresponding balancing coil sections are short circuited by connection links provided for the purpose.

One arrangement of vacuum pumping is indicated in Fig. 7. Use is made of two independent pumping systems, connected through manifolding to the various sections and with cross connections so arranged as to permit the use of either pumping system on any or all of the tanks. The various sections of the rectifier are connected to the vacuum manifold through two valves of unique design, provided with a sealing plate as an extra precaution against outside air. Even if such a valve should develop a leak, this safeguard would prevent any leakage into the tanks except at a very slow rate and for the minute or less while the valve position is being changed. The use of two valves permits the separation of a single section from the balance of the structure without admitting air to either part. In the event that maintenance work is required, it is thus possible to put the repaired section in condition by pretreatment, then install it and by proper manipulation of the valves, exhaust it completely without admitting air to either that unit or any part of the operating system. With the pumping arrangement shown, this requires the operation of one of the other units with valves closed for some time, but experience has indicated that if properly made and treated out the rectifier unit will carry normal loads without pumping for several hours or even several days without damage.

Sectionalizing inherently provides the utility that any damage which occurs is likely to be limited to a relatively small part of the total structure and that the load can be carried during the period of maintenance with the balance of the structure. With the arrangement in Fig. 7, a faulty section is segregated from the others by removing the connection links and closing the valves, requiring only a few minutes, before load can again be carried. Although a maintenance operation is required to remove a unit from the structure for repairs requiring two hours or so, this can be done during a light or no load period. It would be possible to arrange with jack connections and separate systems of auxiliary equipment or flexible pumping and water connections, so that the operator could pull the unit forward a few inches and go on with operation with the balance of the equipment after an interruption of one or two minutes. The choice between these two plans depends on the frequency with which interruptions are likely to occur. The inherent quality in the design, together with the experience thus far indicate that the interruptions requiring maintenance will be so infrequent as to make the simpler plan preferable.

The project for sectionalizing, together with the development in working out sections of suitable dimensions and proportion, thus makes available an increased degree of reliability, increased flexibility in application and use, improvement in manufacturing requirements, and an efficiency advantage of 1 per cent or more. Beyond this is the belief that as time goes on and as we acquire a clearer understanding of the fundamental knowledge gained from our research, but not yet fully used in design, further improvement will be made in respect to efficiency and size. It is to be expected that, as this progress occurs, the development will be along the lines of sectional structures rather than along the lines of single unit arrangements.

It is obvious that a development such as this is one to which essential contributions are made by many individuals. The successful completion of such a project requires a variety of knowledge, experience and talents, no part of which can safely be omitted. The author wishes to acknowledge essential contributions to this development by the research, substation, switchboard, transformer, and manufacturing engineers of the Westinghouse Electric & Manufacturing Company as well as by the design engineers whose work has made possible the design improvements. Special acknowledgment is also made of the enthusiastic and courageous support and encouragement of our executives, especially Messrs. Rugg and Kintner.

Discussion

A. W. Hull: This discussion relates to the fundamental limitation of rectifiers as given in the paper: namely, the prophecy that "the major disadvantages which go with size appear to be fundamental and permanent. Larger units will always be less efficient and probably less reliable."

The theory of rectifiers and our experience up to the present does not seem to me to justify this conclusion. The statement that "efficiency falls off with increase in size because of the longer arc" does not appear to me to apply to large rectifiers. Efficiency does not depend fundamentally on arc length but upon the ratio of length to width, or diameter, of arc path. When the dimensions of a rectifier are doubled, the width of the arc path is doubled as well as the length, and the total arc drop remains unchanged. This is in agreement with the theory of the nature of arc drop and with all experimental evidence, so far as I am aware. If large rectifiers are at the present moment less efficient than small ones, it is, I believe, a mere chance which is due to the relative state of development, and not a fundamental limitation of large rectifiers.

The same is true in regard to reliability. The only theoretical prediction regarding reliability, as a function of size, is that the arc does not tend to concentrate on a single point of a large anode but, after reaching a definite density, which is the same as the density in small rectifiers, it spreads uniformly until it covers the whole anode at full load. Hence, size does not introduce danger from concentration. There is no evidence, with which I am familiar, either theoretical or experimental that would indicate any fundamental reduction in reliability with increase in size, or the need of increased baffling in order to obtain reliability.

C. A. Butcher: It is perhaps quite generally known that I have not been an enthusiast for mercury arc power rectifiers. During eighteen years of experience having much to do with power supply for electric railways, I have witnessed many improvements in the design of substations. In that time, high reluctance commutating poles and high-speed circuit breakers solved the flashover problem in the operation of 60-cycle synchronous converters. The perfection of automatic control including supervisory, effected economies and improvement in operation.

Shortly after the first display of enthusiasm for the rectifier in America, following a lapse of a few years during which development on the part of American manufacturers ceased, I made a careful review of the available literature on the subject. This included the early development experience of my own company as well as that of one European manufacturer—also reports covering the operation and performance of a number of European rectifier installations. In a paper* I compared the rectifier with the synchronous converter. It did not appear that the conventional design of rectifier which has changed but little since then could on economic grounds replace the converter, in the 600-volt class. The demand for conversion apparatus for higher voltages is at present not of itself such as to be conducive to commercial development. The inefficiency of the rectifier at lower voltages has limited its application with few exceptions to the voltages of 600 and above. Even in the 600-volt class, it has not had the advantage of equality from the standpoint of service availability or first cost. Only at light loads is there any margin in efficiency favoring the rectifier. This advantage is very much discounted in metropolitan areas where conversion units are added to and taken out of service as demands require, which effects an average load per unit of from 75 to 100 per cent of rating.

It has not been demonstrated that the maintenance of metal tank mercury arc rectifiers will be any less than for other types of conversion apparatus.

The auxiliary equipment required with rectifiers is much greater than with synchronous converters.

On the whole, large rectifiers of the conventional type are perhaps in a class with 600-volt, 60-cycle synchronous converters before the introduction of high reluctance commutating poles and high-speed circuit breakers.

It is certain that if the rectifier is to stand as a distinct and economic advance in the art, something must be done to assure fewer service interruptions. The record established by modern synchronous converters in this respect will be difficult to equal. Also definite improvement in full load efficiency is fundamental to successful exploitation; otherwise the synchronous converter will be continued as a competitor with consequent economic loss.

Mr. Atherton has described a new development—a sectional rectifier. Tank volume per unit of capacity has been so reduced that an assembly of four 750-kw. units requires considerably less space than a single 3,000-kw. unit of conventional design. Brief as has been experience in actual service following extensive research laboratory work and testing, indications are strong that a high order of availability is to be expected. Efficiency is definitely one per cent better than that of a single tank unit of equal rating. One or more sections may be removed from service without impairing service from the remaining sections. Only a few minutes are required to disconnect a section. Division of load between sections is practically perfect. A sectionalized assembly minimizes the necessity for spare conversion equipment which is no small advantage in its favor considering capital investment in real estate, buildings, and switching equipment.

The multiplication of auxiliaries might seem a formidable objection, but surely not more so than the number of devices that go into an automatic substation. Having been intimately connected with this latter development, the success of which is unquestionable, I feel certain that there is very little cause for concern.

Truly, for the first time, I am enthusiastic about a rectifier.

W. E. Gutzwiller: The idea of the sectional type rectifier is not new. Previous to the year 1926, when 1,000 amperes was about the maximum rating per rectifier tank, sectional arrangements were used by all manufacturers wherever larger capacities were involved. Practically all rectifier installations in this country between the years 1923 and 1926 had two rectifier tanks connected to a common transformer. In Europe at about the same period as many as six tanks were operated from one transformer. Then rectifiers were usually assembled side by side on a common frame and were exhausted by a common vacuum pump set. As soon, however, as rectifiers of larger capacity had been developed sufficiently, it was found that their manufacturing cost, floor space requirements, and attendance were considerably less than for a sectional arrangement of smaller units and their reliability greater. The result was that the sectional idea was almost entirely abandoned.

The smaller sizes of rectifiers have been undergoing a process of development for a great many years, whereby certain of their features have been sufficiently perfected resulting in a slight gain in efficiency when compared with the larger sizes more recently produced. Without doubt, however, further development of the larger types will follow along similar lines so that the present slight difference in efficiency will probably disappear. Where this slight gain in efficiency is a factor of considerable commercial importance as for instance in electrolytic plants, then the sectional idea can and has been adopted.

The new feature of the rectifier described in the foregoing paper is the reduced overall height which permits at least for lower voltages, the compact assembly of a number of tanks side by side or one above the other in a cubicle form. This reduction in height is made possible apparently by placing the condensing dome into the arc chamber whereas in the conventional type rectifier, it is mounted on top of the arc chamber. The diameter and height of the arc chamber are the same as for our conventional 750-kw. 600-volt rectifier. But in my opinion this advantage of compactness has been purchased at too high a price for it has necessitated the placing of additional water cooling cylinders at points within the chamber where they are much exposed to the heat of the arc—an innovation that is perhaps dangerous.

The claims of this new sectional type rectifier are based on comparison of the conventional 500 and 3,000-kw. rectifier with a 3,000-kw. sectional type unit all at 600 volts direct current. No comparison is made between this rectifier and the conventional type in case of say 1,500 and 2,000-kw. 600-volt units or large rectifier units operated at 1,500 or 3,000 volts direct current. In other words, the author picked out the most favorable condition for his comparison and omitted all reference to medium size 600-volt units and units at higher voltages.

The arc drop curve for the sectional rectifier shown in Fig. 5, is substantially the same as for the conventional 750-kw. 600-volt rectifier built by other manufacturers. It is stated by the author that with increased tank capacity, the quality and reliability decreases and further that the outage time per year with 3,000-kw. rectifiers is several times as great as with 500-kw. units. My experiences do not check with this; in fact, I find the reverse to be true. This is independently confirmed by the 1931 Report of the A.E.R.A. which shows a 2 per cent higher availability factor for the 2,500 and 3,000-kw. units of which a total of ten units is listed as against twelve 500-kw. units. The sectional type 3,000-kw. unit has more electrodes, more seals, more weldings and several times as many auxiliaries and temperature control devices than the conventional type rectifier. One would expect, therefore, that its reliability would be inferior to the single tank arrangement.

As far as space requirements are concerned, there is a slight gain in floor space for a 3,000-kw. 600-volt sectional unit. Since, however, space for transformer equipment, switchgear, and control remains the same as for a single tank unit and the rectifier seldom occupies more than 35 per cent of the total substation floor area, the total gain in space is not of much account. The headroom requirements of the above mentioned unit are somewhat less than for a single tank type. Here again the space advantage is less pronounced in the case of 1,500 and 2,000-kw. 600-volt units and does not exist for 1,500 and 3,000-volt rectifiers which the author omitted to mention.

Contrary to statements made by the author, it has been found that the cost per unit of output decreases with larger capacity rectifiers. I find that for a 3,000-kw. 600-volt single tank unit, it is from 30 to 40 per cent lower than for a 500-kw. unit. Furthermore, in case of small units the auxiliary apparatus represents a large percentage of the total unit cost. This, however, is not the case with large rectifiers as they require about the same amount of auxiliaries as small tanks.

**Application of Mercury Arc Power Rectifiers*, by C. A. Butcher, A.I.E.E. Journal, May 1927, p. 446.

The demand for rectifiers so far is not in sufficient volume that much could be gained from mass production. Also in any event it would seem that even this small volume will require more than one tank size to take care of all requirements; *i. e.*, low-voltage and high-voltage units. Mass production, therefore, is hardly feasible; nevertheless, individual parts, such as, anodes, etc., can and are produced in quantities even with conventional type rectifiers.

It appears to me that the claims made for this rectifier have been somewhat over emphasized. Experience has shown that small sectional types have usually been later supplanted by large single tank types for reason of simplicity and reliability except in very special cases where other considerations were dominant. This is particularly true on fluctuating loads, such as heavy traction service where a real gain in reliability is of more importance than a slight gain in efficiency.

R. E. Hellmund: Some of the discussion refers to the comparative reliability and availability of sectionalized designs versus large unit designs. A somewhat similar example taken from the transportation field is given as follows.

In the early days of a-c. main-line electrification, a great deal of discussion took place on the relative merits of locomotives having one or two large motors and those having many small motors, such as the New York & New Haven locomotives having the multiple twin-motor arrangement. Practical experience showed very quickly that the reliability and availability with the large motors were very low because such unavoidable accidents as the breaking of a single carbon might damage the large commutator and keep the locomotive out of service a great part of the time. In contrast to this, a similar occurrence in the locomotives having many small motors, that is, "sectionalized power," permitted the quick replacement of the damaged motor and resulted in much greater availability of the locomotives. This was not a matter of theory but rather one of absolute fact and experience, and naturally it has practically eliminated any desire on the part of designers or operators for locomotives having large single motors.

The same tendency would not be quite so marked in apparatus having but very few parts to get out of order, such, for example, as synchronous machinery, and in such cases the tendency is toward a few large units rather than toward sectionalization. The mercury arc rectifier, however, in its present stage of development is not in this latter class. It involves the fulfilment of various severe and difficult requirements, such as tightness of a great many welded and other joints, as well as absolute cleanliness both during the construction of the rectifier and during repairs in the field. The smallest defect in either direction results in troubles and in failure. I have no doubt that under such conditions as these the sectionalized design will in the near future demonstrate its superiority over the large unit design from the standpoint of both maintenance and availability.

A. L. Atherton: Mr. Hull indicates disagreement with the reasons on which the sectional arrangement of large capacity rectifiers has been based. In view of his standing as a scientist, we may accept his fundamental statements, especially since they deal with a point which is relatively well established and generally known. It seems, however, doubtful to me if we may so readily accept the application of these fundamentals. That part of the arc path within the shield around the anode is not increased in cross section with increase in size of the rectifier. If a single set of six anodes was used for the same total output in the large and small capacity rectifiers, we might assume this relation to be as he has stated but this is not the case in today's practise. The 3,000-kw., 600-volt rectifiers are made usually with either 12 or 18 anodes which gives a rating of 1,000 to 1,500 kw. per set as against 500 or 750 for the sectional arrangement and for the smaller conventional units. Similarly, throughout the arc the effective cross section is not increased with length as the tank size is increased. The barriers, constructional or otherwise, to retaining the same design parameters in the larger units as were used in the more fully developed smaller units, are likely to prove sufficiently permanent to prevent this kind of working out of large single-tank units.

As Mr. Hull has indicated, reliability is probably affected by the same factors as efficiency, since it depends on conditions in the gas path. I do not think that we have anything so simple to deal with as the obvious but rather negative thought that the arc does not tend to concentrate on a single point. The amount and condition of vapor available for conductivity, its state of rest or motion as a body, the interaction between arcs which are carrying current and paths which are expected to withstand back voltage, and many other complicating and complicated factors enter the question of reliability.

As is usually the case in a development or investigation in which all of us know a relatively small share of what is to be known, we shall each of us have to be guided by our available knowledge and beliefs at the time. The course of events will determine which is the correct procedure. This much seems to me to be quite definite; in its present state of development, the sectional rectifier does have a material and valuable advantage in efficiency over the single-tank units of large capacity. I think that the same relation holds for reliability factors. I think that the convenience of concentrating the manufacturing problem in a reduced number of sizes will offset the fundamental disadvantage of the requirement for a greater number of pieces and a greater amount of material. I may be wrong in believing that these advantages will be retained as the developments progress. Time alone can tell.

Mr. Gutzwiller's reference to the experience in making up larger capacities of parallel installation of smaller capacity units, adds to our overall picture and it demonstrates anew the fact that valuable procedures are not always recognized at first.

He refers repeatedly to the gain in efficiency as slight. This gain at 3,000 kw., 600 volts amounts to approximately 1 per cent. For average load conditions in metropolitan traction service and with usual power rates, this gain in efficiency is worth from $750 to $1,500 per year, or capitalized at 15 per cent, from $5,000 to $10,000 in first cost. This is from 10 to 25 per cent of the normal cost of a rectifier of this capacity. The magnitude of the gain is evidently a question of point of view.

Here again, we shall have to depend on time to give the final answer. In the meantime, we who think of sectional designs as desirable can take considerable comfort in the example Mr. Hellmund has given of a well known and definitely established practise of sectionalizing for the sake of just these same gains that we feel are secured by sectionalizing the large capacity rectifiers.

Preparing Substations for Electric Operation
Philadelphia to Trenton Electrification, P.R.R.

BY L. B. CURTIS[1]

Member, A.I.E.E.

INTRODUCTION

INVOLVED in the extensive electrification program recently announced by the management of the Pennsylvania Railroad between New York and Washington are approximately forty substations. Eleven of these, including those covered by this paper, are already in operation. A definite program for testing these substations before placing them in operation had evolved, and it was thought advisable to combine these tests in a single set of instructions. This has been elaborated somewhat in this paper.

Since Edgely substation is typical of the majority of the forty substations, it is used to exemplify the testing. The other substations included in the Trenton electrification are Frankford Junction, Cornwells, and Morrisville. The standard apparatus consists mainly of 4,500-kva. step-down transformers, high-speed trolley breakers, necessary 132-kv. and 11-kv. switches and breakers, and control equipment.

The testing may be separated into the following three groups:

A. Preliminary tests without d-c. control and a-c. power.
B. Tests with d-c. control but no a-c. power.
C. Tests with d-c. control and a-c. power.

Group A—Preliminary Tests without D-C. Control and A-C. Power

CURRENT TRANSFORMER POLARITY TEST

Due to the fact that differential protection is employed in some instances it is imperative that the polarities of current transformers installed in the bushings of all power transformers and circuit breakers be correct. All protective relays receive their actuating current from these transformers. The end of the current transformer corresponding to the top of the bushing is lettered *a*, and is understood to have the same instantaneous polarity as the top of the bushing terminal.

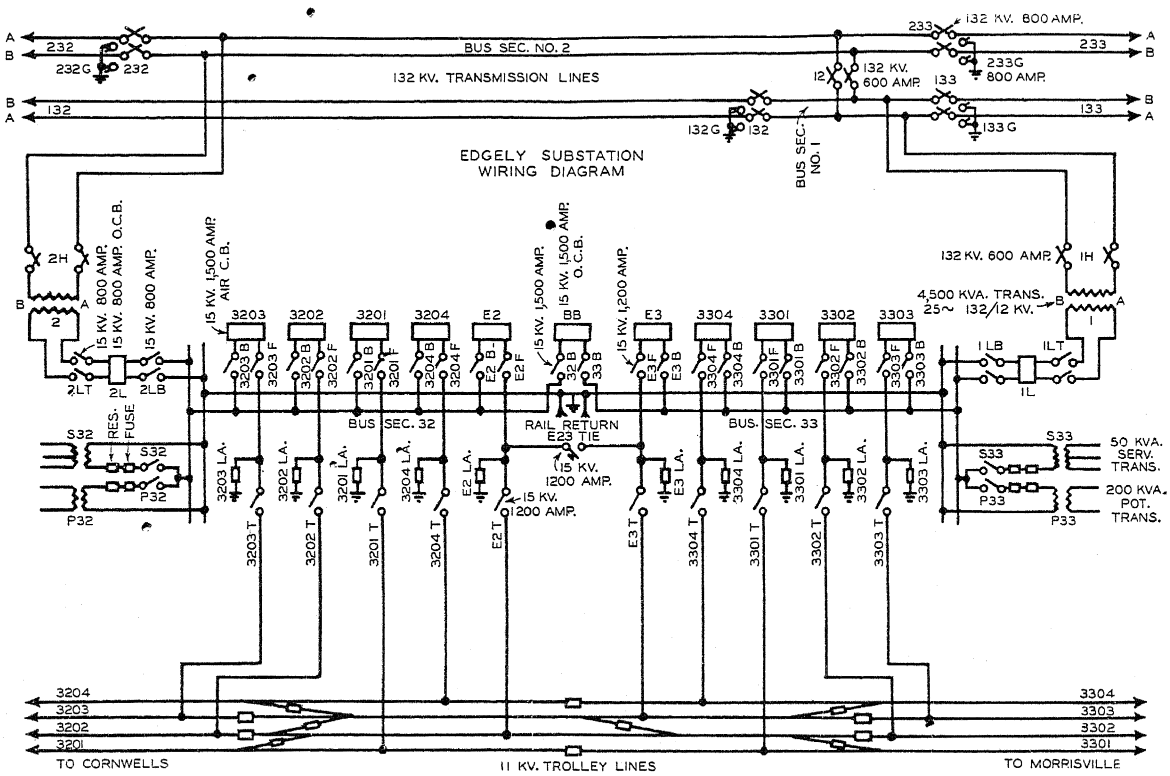

FIG. 1—EDGELY SUBSTATION WIRING DIAGRAM

Fig. 1 shows the connections for checking polarity. A millivoltmeter is connected across the secondary of the current transformer about to be tested with the plus terminal attached to the *a* side. Two or three dry cells connected in series have their negative terminal connected to the bottom of the bushing terminal. When

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Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

the positive terminal is touched to the top of the bushing, or on the "make" there should be a positive deflection on the millivoltmeter. A negative deflection takes place on the "break." When the current transformer is connected through to the control and relay board, the above test is repeated with the millivoltmeter connected across the test switch behind the board. Every current transformer is tested in a similar manner.

CIRCUIT BREAKER AND TRANSFORMER OIL TEST

Samples of oil from all oil-filled equipment must be tested for electrical breakdown strength before the equipment is energized. At least six samples are taken in each case. With the electrodes in the test set spaced 0.1 inch apart the average breakdown value must not be less than 25,000 volts.

TRANSFORMER MEGGER TEST

All transformers are meggered to test their insulation. Readings are taken from high winding to low winding, from high winding to ground, and from low winding to ground. All readings must be sufficiently high to insure the absence of moisture, short circuits, and grounds.

POWER TRANSFORMER TAP CHANGING DEVICE TEST

The power transformers are equipped with a tap changing device on the top cover in order to vary the voltage ratio between the high-voltage and low-voltage windings. A definite ratio is decided upon and all step-down transformers on the system are set on this tap.

MANUAL OPERATION OF ALL SWITCHING EQUIPMENT

All manually and electrically operated switches must be carefully adjusted for alignment and smooth operation by hand. No electrical operation of motor operated switches should be attempted before perfect operation is performed manually.

ELECTRICAL INTERLOCK TEST

In order to secure safety for the operators, and protection for the equipment, some of the apparatus are interlocked. The physical features of the interlocking parts should be carefully inspected so that all moving parts move freely.

PHYSICAL CHECK OF ALL RELAYS

All protective relays are given a predetermined setting after an examination as to their condition. The relays are operated by hand to determine the contact adjustment and if any friction exists between movable parts.

Group B—Tests with D-C. Control but No A-C. Power

D-C. ENERGIZING OF RELAY AND CONTROL BOARD

The combined relay and control board is now energized with the 125-volt control battery, one section at a time. Two lamps connected between the negative and positive buses and ground indicate whether any grounds exist.

ELECTRICAL CONTROL AND INDICATION TEST

In the switch house on the control board there is a double pole switch that throws the control either on "station control," that is, control from the switch house, or "tower control" which is control from the nearest railroad tower. Every electrically operated piece of equipment in the substation is operated by means of the drum control handles on the control board both in the substation and in the tower. A green light indicates "open" and a red light "closed." Every operation and indication must be perfect.

ELECTRICAL INTERLOCK TEST

The apparatus that is interlocked together, some to open a load circuit in a breaker instead of in a disconnecting switch, and some to prevent a ground switch from being closed on a live circuit is now operated from the control board. The interlocking features must work every time.

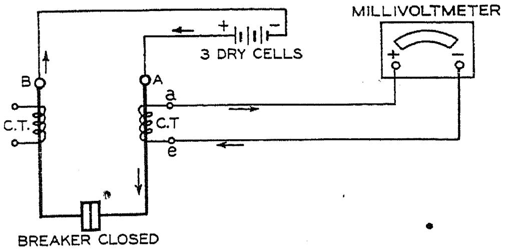

FIG. 2—CONNECTIONS FOR CHECKING CURRENT TRANSFORMER POLARITY

MANUAL RELAY OPERATION TEST

The last test of this group is to check the relay operation of all equipment by causing the relays to function by hand. Inasmuch as the operation of these relays will be described under the a-c. tests it will be omitted here.

Group C—Tests with D-C. Control and A-C. Power

PRELIMINARY

It is necessary to take the following precautions and preparations before these tests begin:

a. The entire catenary and transmission lines to be tested must be inspected for visible grounds and broken insulators.

b. A direct telephone connection must be had with the power director.

c. All temporary grounds at the substation must be removed except those on trolley feeders for tests one to eight inclusive.

d. Sufficient number of ground sticks for applying grounds must be on hand.

e. All necessary instruments for testing must be on hand.

f. A program of procedure must be set up and all involved in the test must have a copy so that it is thoroughly understood by all concerned what is to take place.

g. All interested in the test must be previously notified and be present when and where required.

Energizing Test of Transmission Line

The first of the a-c. tests is to build up the potential on the transmission lines. All 132-kv. line switches (132, 133, 232, and 233 in Fig. 1) are closed. The 132-kv. transformer horn-gap switches (1*H* and 2*H*) are opened. The voltage is then raised at Lamokin where power is obtained to ¼, ½, ¾, and full voltage, checking for grounds at each step. These would show on ammeters at Lamokin. When full voltage is reached it is allowed to "soak" for five minutes. Then the power is removed and applied two or three times suddenly at full voltage.

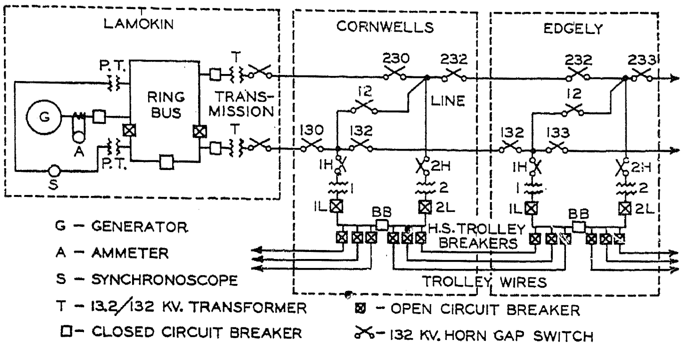

Fig. 3—Schematic Diagram for Phasing Tests

Energizing Test of Power Transformers

Both transformer horn-gap switches (1*H* and 2*H* in Fig. 1) are now closed, in addition to those in test 1. The voltage is built up as before, including the step-down transformers in the energizing.

Phase Test between 132-Kv. Lines and 11-Kv. Buses

a. The first step is to phase out the 132-kv. transmission lines. The 132-kv. line and tie switches (132, 133, 232, 233, and 12 in Fig. 3) are closed and all other switches and breakers opened. The voltage is built up and fed into one bus section at Lamokin from the generator, over one transmission line through the 132-kv. tie switch, and back to Lamokin over the other transmission line. If the pointer of the synchronoscope stops in the proper position the lines are in phase.

b. The next step is to phase through the 11-kv. bus instead of the 132-kv. tie switch. The tie switch 12 is opened and the transformer horn-gap switches 1*H* and 2*H*, the transformer low side breakers 1*L* and 2*L*, and the bus tie breaker *BB* are closed. The voltage is built up at Lamokin as in the preceding test.

Energizing Test of Entire Substation

All switches and breakers in the substation are now closed except the T-disconnect switches where the feeders leave the substation. The substation is energized with full voltage to allow the equipment and buses to "soak" for several hours in order to show up any defective insulation.

Calibration of Potential Device

While the substation is energized the potential devices connected to the 132-kv. bushings of the step-down transformers are calibrated. These devices furnish potential to the transmission line ground protective relays and must be calibrated for the particular relay used. A phase meter is used to read the phase relation between the system or reference voltage and voltage of the potential device. The watt component of the phase meter's burden should closely approximate in value that of the relay burden. The relay is blocked so it cannot operate and is disconnected during the test. The test connections are shown in Fig. 4. A condenser *C* is used across the phase meter in order to make the burden of the latter approximately that of the relay. A high resistance voltmeter is used to read the voltage across the potential device. The reference voltage is obtained from a potential transformer which is connected to the 11 kv. bus and should be constant during the test. A 100-watt lamp is used to obtain approximately one ampere 100 per cent power factor load. The values of reactors, transformer ratio, and condensers in potential device are varied so as to get a combination that will give approximately half reference voltage across the potential device, and the lowest possible phase difference. When this condition has been obtained the phase meter is disconnected and the relay reconnected.

The potential device on the other side of the transformer is then checked as above, and the same results obtained. The blocking may then be removed from the relay.

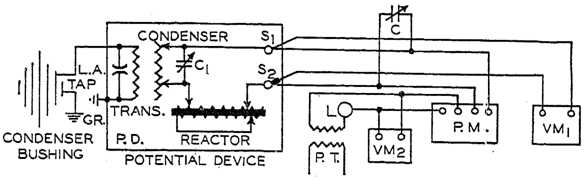

Fig. 4—Potential Device Test Connections

Reduced Voltage Ground Test on High-Speed Breakers

At Lamokin, jumpers are placed across the low- and high-voltage bushings of a step-up transformer so that generator voltage is impressed directly on the transmission line. The generator voltage is built up to approximately 11,000 volts which is reduced to 1,000 volts at the substation. With the 11-kv. bus dead, a high-speed breaker is grounded on the feeder side to the substation steel with a ground stick. This is indicated by × *U* in Fig. 5. Then all switches and breakers in the substation are closed except the disconnect switches on the out-going feeders, and the high-speed trolley breakers. The grounded breaker is then closed from the control board in the switch house.

The oil breakers are controlled from a set of three relays; impedance, load compensating, and instantaneous and time delay which are fed from current transformers located on the bus side of the trolley breaker. The load compensating relay can be considered as a part of the impedance relay, by changing the resistance value in series with the restraining coil of the latter. The impedance setting of the impedance relay is equal to 9/10 of the impedance of the trolley section to which the corresponding breaker is connected in order to avoid getting any overlapping of protection caused by inaccurate calculation. There is a directional element in series with the impedance element so that the relay will operate only when the fault is on the trolley or feeder side of the breaker.

The instantaneous and time delay relay has two overload elements, one high-speed with a current setting, and the other time-delay with a lower current setting. When a ground occurs close to one substation such as the one imposed above, the impedance relay will usually open the breaker feeding the fault. At the next substation, the breaker at the other end of the line will be opened by the high-speed element of the instantaneous and time delay relay. The impedance relay at this point being outside the setting zone will not operate. When the ground occurs midway between two substations the breakers feeding the faulty line will be opened by the impedance relay. The time delay element of the instantaneous and time delay relay acts as a back-up protection and also functions to clear stub end faults at minimum generating capacity.

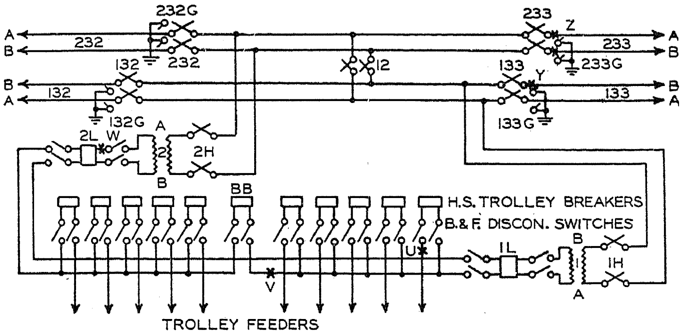

Fig. 5—Location of Grounds—Edgely Substation

The high speed air trolley breaker operates from either of two independent devices serving different purposes, a high speed device which is a part of the breaker, and a directional overcurrent, undervoltage relay.

The high speed device operates on a sudden current increase which is characteristic of a ground. It does not operate in the case of a gradually increasing current. The directional overcurrent, undervoltage relay takes care of this condition. This relay is fed from a current transformer on the trolley feeder side of the breaker. As the current demand increases, the voltage drops, and if this condition reaches a predetermined value the relay will operate to open the breaker.

If the oil or air breaker as the case may be operates satisfactorily, the bus is killed, and the ground stick applied to the next breaker. Every high speed trolley breaker is tested in this manner.

Reduced Voltage Ground Test on 11-Kv. Trolley Buses

A ground is now imposed on one of the 11-kv. trolley buses as shown in Fig. 5 at *V*. All breakers and switches in the substation are closed except the switches on the outgoing feeders, and the two transformer breakers 1*L* and 2*L*. A bus ground is caused by closing these two breakers simultaneously.

The bus differential protective relay is connected to current transformers located on the side away from the section to be protected of the bus tie breaker, the corresponding transformer breaker, and the trolley feeder breakers. The sum of the currents leaving the bus through the trolley breakers connected to one bus section under normal conditions is equal to the current entering the bus through the transformer breaker plus or minus the current flowing through the bus-tie breaker. The secondaries of all the above current transformers are connected together in such a way that under normal conditions there is no current flowing through the relay. On a bus short circuit or ground, however, this equilibrium is disturbed and the relay operates. This energizes a set of auxiliary relays which opens all breakers connected to the faulty bus section, and also a lock-out relay which locks open all closing circuits of these breakers so that they cannot be closed from the control board until the lock-out relay is reset by hand. The other bus section is tested as above.

Reduced Voltage Test for Transformer Internal Faults

A ratio differential relay is used to protect the 4,500-kva. step-down transformers against internal faults. A ground stick is attached near the trolley side of the low voltage bushing as shown at *W* in Fig. 5. If the test is made on No. 2 transformer the transmission line horn-gap switches 232 and 233 (Fig. 5) of the transmission line feeding this transformer should be opened. The other transmission line is energized with the reduced voltage, and 1*H*, 2*H*, 1*L* and *BB* are closed. The ground is caused by closing breaker 2*L*.

The ratio differential relay is energized by two sets of current transformers, one set consisting of two cross-connected current transformers on the high voltage side of the step-down transformer, and the other of a similar combination on the bus side of the transformer breaker. The relay has an operating coil and a restraining coil, there being no current flowing in the former under normal conditions. When a fault occurs in the transformer the balance between the two sets of current transformers is disturbed and the unbalanced current flows through the operating coil causing the relay to operate. This causes the transformer breaker $2L$ to open and after a sequence of operations which allows the fault current to decrease to safe value, the transformer horn-gap switch $2H$ opens thus isolating the faulty transformer. The test is then repeated on No. 1 transformer.

PHASING AND INSULATION TEST OF CATENARY SYSTEM

The reduced voltage tests having been completed the catenary system is now tested for insulation and phasing using the same system set-up as for test 3, except that the ring-bus at Lamokin is not sectionalized. The temporary grounds on the out-going trolley feeders are removed and all breakers and switches are closed. The voltage is raised to ¼, ½, ¾, and full voltage, thus energizing the entire catenary system as far as the test is to go. If the generator ammeter at Lamokin registers zero the trolley system is properly phased and the insulation is good. If there is a current flowing the trolley feeder breakers should be opened one at a time until the faulty section is located.

FULL VOLTAGE GROUND TEST ON TRANSMISSION LINES

After full voltage ground tests are made on high-speed breakers similar to test 6, the transmission lines are tested at full voltage. Reduced voltage tests are also made but as the operation is the same for both tests one description will suffice.

The transmission line ground protection as stated in test 5 is obtained with a differential relay which receives its potential from condenser potential devices. The ground is obtained by closing one leg of the 132-kv. ground switch, the other leg being previously disconnected from the motor-operated mechanism when the system is dead. The location of this ground is shown at Y in Fig. 5.

When a ground occurs on one side of the transmission line the voltage on that side of the potential device approaches zero depending upon the nature of the ground, and the voltage on the other side approaches 110 volts. The balance in the relay being disturbed, it operates, causing the corresponding step-down transformer breaker to open. This same operation takes place at each substation feeding into the faulty line. In Philadelphia at D-1 substation, the line is sectionalized by a 132-kv. breaker which also opens, thus disconnecting the faulty transmission line from all sources of power. The same test is performed on the other transmission line.

FULL-VOLTAGE SHORT-CIRCUIT TEST ON TRANSMISSION LINES

The 132-kv. transmission lines are protected from short circuits by means of directional control relays. The short circuit is caused by closing both sides of the 132-kv. ground switch. The trolley feeder network supplies power to the short circuit through all step-down transformers connected to the faulty line. The power flow is thus in the reverse direction from normal and is used as an indication of fault conditions.

The directional control relay has an overcurrent element which receives its current from a current transformer located on the low-voltage side of the 4,500-kva. step-down transformer, and a power directional element receiving potential from a 11,000/110-volt potential transformer connected to the corresponding trolley bus section. The directional element contacts must be closed before the overcurrent element can start to operate. The operation of this relay causes the corresponding transformer circuit breaker to open. Likewise all transformer breakers connected to the faulty line open as well as the sectionalizing breaker at D-1 substation, thus disconnecting the faulty line from all sources of power. This test is repeated on the other transmission line.

CONCLUSION

It is hoped that this paper has shown the completeness and thoroughness of the substation testing required by the Pennsylvania Railroad to insure a minimum of train delay with a maximum of protection under electric operation.

ACKNOWLEDGMENTS

Acknowledgment is hereby made to Mr. J. Westby for his care and interest in reading over this paper in its preliminary form and in offering valuable contributions.

Instructions prepared by Gibbs & Hill, Consulting Engineers, for use in the operation of the above electrification were of assistance in preparing this paper, particularly in describing the relay operation.

Acknowledgment is also made to two manufacturers who furnished most of the equipment in the above substations for reading over this paper and in offering many helpful comments.

Time and Its Measurement

BY ERNEST W. BROWN*
Non-member

THE nature of time and its relation to space in the theory of relativity, is not discussed here but it is assumed that, for our purposes, the Newtonian definitions give a sufficiently accurate picture.

DEFINITION OF A TIME-KEEPER

A time-keeper, whether terrestrial or celestial, may be considered as consisting essentially of a fixed dial and a moving pointer. A complete observation can be regarded as a comparison of the angles described by the pointers of two time-keepers between two signals, at each of which the positions of the pointers are simultaneously observed. This comparison of angles is then translated into a comparison of intervals of time by means of certain assumptions made as to the nature of time. In our daily life one of the time-keepers is used as a standard for judgment of the other which may, for example, be the interval between the departure and arrival of a train or that between the successive demands of the body for nourishment. Every observation demands the presence of two or more time-keepers.

In order that an observation may be made, the circumference of the dial is divided into two or more equal parts or it may have a single mark only. With some time-keepers the observations are made only when the pointer is opposite a mark. When a simultaneous position of the second pointer cannot be well estimated, a simultaneous observation with a third time-keeper, having a highly divided dial or a rapidly moving pointer, is necessary.

The pointer is actuated by a mechanism which is governed by the laws of motion. In some mechanisms the law of gravitation also plays a leading part: in others, physical laws of a different nature are mainly responsible for its motion. Most of the terrestrial mechanisms are constructed by human agency and are therefore under a certain degree of control. Celestial mechanisms can not be controlled.

ERRORS

Since our ability to deduce results from these mechanisms is limited and is subject to error, the discussion resolves itself from the outset into a consideration of the errors which may exist when an observation is made.

It will be convenient to divide these errors into three classes:

1. Lack of precision in the statements of the laws under which the mechanism is supposed to move.
2. Errors in the applications of these laws to the mechanism.
3. Errors in the reading of the pointer, usually called errors of observation.

From the practical point of view, Newton's three laws of motion on which our chief time-keeper, the earth, rotating on its axis, is mainly dependent, may be regarded as possessing an accuracy far beyond our powers of detection. The same may be said of the Newtonian law of gravitation in the practical applications to time measurement. Less accurately known are the laws governing the kinetic reactions of matter usually stated under the terms, elasticity, rigidity, hysteresis, etc. Perhaps in the same class can be placed the phenomena classified under the terms electricity and light. With the earlier time-keepers these have been used only for the transmission of signals where the accuracy for time-keeping purposes is very high; in the latest time-keeper, the oscillating crystal, electricity has become part of the mechanism

Errors in the application of the laws are usually numerical or mathematical; these can be eliminated. A more frequent source of error is the omission of the effect of known or unknown reactions of matter, such as friction, elasticity, etc. Errors of observation of terrestrial time-keepers are largely under control and can be eliminated to almost any extent by the development of suitable devices. Those of celestial observations are numerous: some follow known laws and can be allowed for to a greater or less extent; others are unknown and their effects can be discovered by indirect methods only; still others which appear to have no systematic character are supposed to follow certain laws of error and are reduced by multiplying the number of observations.

STANDARD TIME-KEEPERS. DEFINITIONS OF ERROR AND CORRECTION

The use of the word error implies the existence of a perfect time-keeper. No such mechanism exists. We can only make comparisons between different time-keepers. Any definition to be useful in practise must take this fact into account.

The error of a time-keeper at any instant is the difference between the reading of its pointer and that of another time-keeper which has a similar dial and whose pointer is constructed to have the same angular velocity, the latter being used as a standard. The angular velocity of the pointer of the standard may be different from that of the first time-keeper provided its dial is proportionately divided. The "accumulated error" in any interval is the difference of its readings at the beginning and the end of the interval, less the corresponding difference of the standard. The error is taken to be positive when the pointer is in advance of that of the standard. Thus, the error of a time-keeper includes that of the standard. They may be partially separated only by the use of the theory of errors applied to numerous time-keepers observed on many occasions. With an actual mechanism as a standard, the position of its pointer must be corrected for any errors which are known.

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Fundamentally, all our observations can be considered as those of angles, but it is convenient to express them in time. The angular error of a time-keeper is expressed in time by its ratio to the angle described by the pointer of the standard in a given interval, multiplied by this time interval. Since the errors with which we deal are usually small, whether of the time-keeper or of the standard, the error of the standard has little effect in this translation of an error of a time-keeper from angle to time and can ordinarily be neglected. In future it will be supposed that this translation has been made.

The *correction* of a time-keeper is the amount which must be applied to it in order that it may agree with the standard. It has, therefore, the opposite sign to the *error*.

DEFINITION OF RATE AND CHANGE OF RATE

The *rate* of a time-keeper is less easy to define accurately. The average rate over a given interval is perhaps best defined as the ratio of the difference of the readings at the beginning and end of the interval to the corresponding difference of the standard in the same interval. The real rate at a given instant is the limit of this ratio as the interval approaches zero, but as this is not measurable, average rates must be used. Unfortunately, the word "rate" is used sometimes in this sense and sometimes as meaning the difference of this ratio from unity, expressed in terms of the standard. Thus if we talk of a time-keeper having a daily rate of $+1^s$ we mean that at the end of a day the reading is behind that of the standard by an angle which is 1/86400 that described in a day by the standard. The rate of a time-keeper thus defined is positive when it is losing on the standard.

A daily change of rate is the difference of the rates thus measured on successive days.

TIME-KEEPERS WHICH DIVIDE INTERVALS OF TIME

In the description of the dial and pointer illustration it was mentioned that with some time-keepers the position of the pointer could be observed only when it was opposite a mark. Different kinds of time-keepers are needed, some of which can be observed accurately at more frequent intervals than others. The dial may be divided into more parts or the pointer may revolve more rapidly. We thus have a second function of the time-keeper, that of dividing an interval into equal parts.

Our fundamental standard is, in theory, observed once a day and the chief function of the terrestrial clocks has been to divide this into 86,400 equal parts called seconds, the equality of the parts being a fundamental aim of the mechanism. For smaller divisions, various devices known as chronographs, are employed, the accuracy with which these can be constructed to divide the second into any number of equal parts being far in excess of practical needs.

THE FUNDAMENTAL STANDARD

As stated, the fundamental standard for time measurement is the rotation of the earth about its axis. According to the laws of motion, certain ideal bodies will rotate about their axis with constant angular velocities. Most of the apparent deviations of the actual earth from such an ideal are calculable or measurable or both, and can be applied to the observations. One, which will be mentioned later, has been made definite only within the last few years and is of considerable importance.

With this standard, the pointer is the telescope of the transit instrument and, in theory, the dial is marked with a single star only, so that the observations are made once only in each revolution, that is, when the pointer is on the star. In practise, however, many stars are used, the time from one to the next being carried by the astronomical clock whose rate is afterward determined by the observations.

ERRORS OF THE FUNDAMENTAL STANDARD

The errors of the dial of this time-keeper are those of the angular positions of the stars relatively to one another and of their relative motions; these have been reduced to very small quantities by numerous observations and thus the relative position of and motions of these "clock stars" are known with high accuracy. Next comes the personal error of the observer in recording the time on the clock when the pointer is on the star. This error again, partly by mechanical devices, partly by experience, and partly by measurement, can be nearly eliminated. Next the errors due to movements of the instrument and of its parts relative to the ground on which it is placed, are under a certain degree of control and measurement. Finally, there are anomalous refraction errors due to the movements of the atmosphere and others due to the local movements of the ground relative to the whole earth which are probably very small but which are not under control.

Under the best conditions none of these circumstances should produce serious accumulated errors over periods of a few days or longer. Meridian observatories of the first class expect to be able to give the daily time within one or two-hundredths of a second. But considerable doubts concerning the actual accuracy must be entertained as long as these observatories show differences amongst themselves running up to one or two-tenths of a second and persisting for weeks or months at a time. There is too little known about the nature or causes of these differences for any one observatory to be able justly to claim substantial superiority over all the others in the accuracy of its time determination. The day-to-day accuracy permits of tests; the week-to-week or month-to-month accuracy is doubtful.

CHANGES IN THE RATE OF THE FUNDAMENTAL STANDARD

The important exception in our knowledge of the deviations of the rate of rotation of the earth from constancy has been demonstrated by observations of the moon, sun, planets and satellites. In the comparison of gravitational theory with observation of these bodies, they agree substantially with one another in their measure of the time and disagree with the earth. The nature of these variations is such that errors in theory or observation as their source are highly improbable. Quantitative calculations also indicate that they arise from forces which act below the earth's crust. Calculation and dynamical theory then indicate that one result of their action must be changes in the external radius of the crust.

These changes of rate of our principal time-keeper are irregular. Ordinarily they are very small and can be detected as having occurred only in the discussion of observations of the moon. But there have been two occasions when no doubt concerning their orders of magnitude can be raised. During the interval 1893 to 1900 the rate can not have diminished by less than one part in 30,000,000 and was probably greater. Again within two or three years of 1917 it increased again by about one part in 40,000,000. The chief doubt is the interval during which each change was completed; it may have been anything less than seven years in the first case and less than five in the second.

The complete technique by means of which these changes are demonstrated is so highly specialized and so laborious to carry out that few living investigators have been able to examine it in detail. The results depend for their reliability on several investigations which have been published within the last eighty years and which have been carried out by different methods. The observational effects are so large, speaking relatively, that no doubt can be entertained regarding them. The rejection of them implies that an astronomically determined unit rate may sometimes differ from its average value by as much as one part in 30,000,000: their acceptance permits the error to be later reduced to about one-tenth of this amount.

COMPARISONS BETWEEN TIME-KEEPERS

It will be seen that there are considerable differences in practise between the accuracy which is attainable in the comparison of two or more time-keepers in the past, at the present moment and in the future. High accuracy in many astronomical determinations depends on the multiplication of the observations. Slow systematic changes of rate can only be determined by observations spread over a considerable length of time. Other changes of rate cannot at present be predicted. No method of judging the accuracy of a time-keeper is of general application unless it specifies the nature of the errors under consideration, the length of the interval over which it is to be judged, and the standard with which it is to be compared. While mention of an ideal time-keeper may give mental satisfaction, it has no place in actual measurements. Indeed, the tendency of modern physics appears to lean towards the abolition of all consideration of hypotheses which assume the existence of phenomena which cannot be observed.

MEAN SOLAR TIME

The direct use of the fundamental standard is confined to the astronomical observatory: it is more convenient for the layman to use an intermediate standard which is related to the position of the sun. For purposes of definition we can regard this time-keeper as a pointer referred to a mark on a dial which is made to move with constant angular velocity, this velocity relative to the fixed stellar dial having been previously determined by observation and gravitational theory.

In the early stages of human history, the mark on the moving dial was the actual sun. It was later found that such a mark judged by the stars was in advance or behind its average position by as much as one per cent in the course of a year. Later still gravitational theory enabled the astronomer to calculate the deviation very accurately and finally, by observations extending over a long interval of time, to obtain the average velocity also with high accuracy. The moving dial makes a complete circuit in the same time that the pointer takes to make some 365 circuits.

In practise, the astronomer compares the time shown by transit observations of the stars with his "sidereal clock" constructed to have the same rate. For the layman he translates this into civil time by calculation. The instant at which the day shall begin is a matter of agreement and definition.

The mean solar day is the interval between successive coincidences of the pointer with the moving dial. The mean solar second is 1/86400 of this.

GRAVITATIONAL EFFECTS ON PENDULUM TIME-KEEPERS

This paper concludes with a few remarks concerning the astronomical and geophysical forces acting on certain types of terrestrial time-keepers. All of the latter which are in practical use for accurate time-keeping are oscillators. These are of two types, those which oscillate principally under the action of the laws of gravitation and those whose oscillations are controlled chiefly by that property of matter which we call elasticity. As far as we know, the latter are practically independent of gravitational action or can be made so. They need not, therefore, be further considered in this connection.

The pendulum clock depends mainly on gravitational action for its motion and any change in this action may alter its time-keeping properties. As far as the gravitational actions of outside bodies are concerned, no practical effect can be detected. Just lately, for the first time, the action of the moon, which is the greatest external effect, was measured by the Loomis chronograph and shown to give accumulated errors which were always less than two ten-thousands of a second as indicated by theory. This also included those indirect effects caused by the moon such as tidal action both in the oceans and in the change of shape of the earth, etc.

A much more serious effect can be produced by a change in the distance of the pendulum from the center of the earth. If the clock be raised one foot, its rate will be changed by one part in twenty million and the accumulated loss as a time-keeper will be one and a half seconds in a year. The changes in the rate of rotation of the earth have been mentioned previously. Under two extreme hypotheses, the outer radius was changed near 1897 by 5 in. or 12 ft. The former change would have meant an accumulated error of the clock of 0^s6 in a year; the latter 18^s. In either of these two cases the star transits would have shown 1^s as against the moon. Thus the comparison of the clock with star transits would have shown an apparent gain of 0^s4 in the one case and an apparent loss of 17^s in the other. If any such change as the latter takes place in the future within an interval of a year, it can certainly be detected by the averaging of results of the Shortt clocks which are now in operation under the best conditions. Even if the change took three or four years to complete, it probably could still be detected. It must be remembered, however, that it is an extreme hypothesis constructed to account for a certain phenomenon, without any collateral evidence as yet to support it. The hypothesis may be good for a smaller value or it may have to be rejected altogether. At present, it appears to be necessary qualitatively, whatever the quantitative value may be.

Discussion

For discussion of this paper see page 550.

Modern Developments in Precision Clocks

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AT THE beginning of the present century it was believed by many that the limit of accuracy attainable by mechanical clocks had been reached. This presumption was based on the performance of specific mechanisms which had indeed been brought to a high state of perfection, but this presumption did not allow for the possibility of other *types* of mechanism for measuring time nor for certain refinements that have since then more than doubled the accuracy and reliability of the best clocks then available.

In 1911 Lord Grimthorpe wrote[1] "It seems to follow that the extent to which astronomical clocks can be made accurate, *viz.*, 1/30 of a second average variation from their mean daily rate . . . is a degree of accuracy sufficient for present purposes, and it seems rather doubtful whether mechanical science will, in the case of clocks, be likely to reach a much higher figure."

As the words clock, chronometer, etc., have come to have definite technical meanings in the art, we shall use the more general term "Timekeeper." In the broadest sense a timekeeper may be defined as any physical means that can be used for the subdivision of time into intervals known in terms of a standard time interval. Thus any periodic phenomenon can be adopted in a timekeeper, provided its period can be recorded in terms of another. In this sense the earth and the moon are timekeepers, as well as vibrating pendulums, tuning forks, and other sustained oscillators. As all man-made timekeepers are essentially "oscillators" we will consider first the equation of motion of a simple oscillator which can be expressed as

$$M\ddot{x} + R\dot{x} + Sx = A \sin(\omega t + \phi)$$

where M corresponds to the mass, R to the resistance or damping and S to restoring force proportional to displacement x. The right hand term represents the sustaining force required on account of the dissipation of energy in the oscillator.

For an oscillator to be a perfect timekeeper, all of the quantities, M, R, S, A and ϕ must remain invariable. While M and S are the chief rate governing quantities, the others are, from a design point of view, the most difficult to control.

The ultra precise practical timekeepers of today can all be contained within two classes, depending on whether S (the restoring force) is

(a) Gravity (as in pendulum clocks)
(b) Elasticity (as in quartz crystal oscillators)

The second class is a development of the last decade and was made possible by the development of the vacuum tube and associated electrical circuits. Intercomparisons between timekeepers of the two classes provide a most valuable means for the study of changes in gravity and related phenomena. These two classes are discussed separately.

GRAVITY TIMEKEEPERS (PENDULUM CLOCKS)

Unfortunately the motion of a pendulum is not quite simple harmonic motion since the restoring force varies as the sine of the angular displacement (instead of directly as the displacement) and therefore the period is not truly independent of the amplitude. This can be seen from the equation of motion of the pendulum which in its simplest form, neglecting damping and sustaining forces, is

$$Ml\theta + Mg \sin \theta = 0$$

the free period T for any given angular displacement can be expressed by the series:

$$T = 2\pi\sqrt{\frac{l}{g}}\,(1 + 0.0625\,\alpha^2 + 0.00358\,\alpha^4 + \ldots)$$

Where α is the maximum deflection from mid swing expressed in radians. The corresponding change in rate expressed in seconds per day is shown in the curve Fig. 1.

Various attempts have been made to construct pendulums having a period independent of arc. Huygens[2] demonstrated that a pendulum constrained to move in the arc of a cycloid under the influence of gravity would have a rate independent of arc, but mechanisms to accomplish this result have always introduced larger irregularities than those which they attempted to cure. Suggestions have also been made that an elastic restoring force could be used to supplement gravity in such a way that the sum of the two would make the restoring force directly proportional to the displacement. Loseby first made this suggestion in 1851.[3] But such an additional control robs the gravity pendulum of one of its greatest advantages, as the rate is markedly affected by changes in the elastic restoring force.

Practical design has been confined to keeping the amplitude small and to providing a governing action so that any slight increase in amplitude causes an opposing tendency to decrease the amplitude, and *vice versa*.

The old anchor recoil escapement[4] accomplished this by increasing the friction on the pendulum with increase of amplitude. The inertia escapement[5] in the Shortt clock attains a similar result without the serious disadvantage of introducing friction. This inertia escapement is based on the principle that a weight rolling down an inclined plane applies a greater horizontal force

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[1] For references see Bibliography.

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to a slowly moving plane than to a faster moving one. A suitable inclined plane attached to a pendulum receives an amount of energy roughly inversely proportional to the amplitude, or to the velocity at midswing. As the amplitude increases, the energy delivered into the system becomes less. But in spite of these controls the amplitude does vary in fact, which at the present time probably accounts for more irregularities than any other factor. Dr. Jackson has shown[6] that if the daily amplitude is regularly recorded and the clock rate corrected accordingly, a number of the irregularities in the Shortt clocks at Greenwich can be explained.

If a direct amplitude control could be applied to a clock such corrections ought not to be necessary. Such a method is now being tried on a Shortt clock in the Loomis Laboratory. When the amplitude of the pendulum exceeds a predetermined amount by even 0.0001 inch, a signal is sent, via a pointed electrode which hangs in the evacuated pendulum casing, to a vacuum tube which operates a relay so that the next 30-second driving impulse is omitted. The electrode is so adjusted that about every fifth impulse is omitted in this way. It is found in practise that the device operates without the bob ever coming in contact with the electrode, so that no energy is taken from the pendulum by physical contact. The difference in amplitude between the operating and non-operating conditions is only about 10^{-6} cm. which corresponds to about 0.3 second of arc. Thus the mean amplitude can be controlled to within approximately 0.1 second of arc. In order to insure that changes in level of the whole clock mechanism will not be the direct cause of changes in amplitude through this control mechanism, the mounting for the control electrode is suspended so as to be free to swing in the same plane as the pendulum, though of course having a different period and being highly damped in order to avoid sympathetic oscillations. With this control the constancy of rate appears to be improved, but at least a year's record is needed for conclusive proof, as the variations which this device seeks to control usually only occur at intervals of several months.

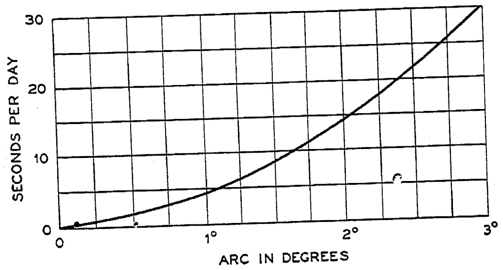


FIG. 1—CHANGE OF RATE OF PENDULUM WITH SEMI-ARC EXPRESSED IN SECONDS LOST PER DAY

The complete solution for the period of a pendulum swinging in a resisting medium, such as air, involves a factor $\sqrt{1 + kR^2}$ where R is the resistance to motion and k is a constant depending on the structure of the pendulum and the nature of the drive. Although it would appear from this that all pendulums should be operated in a high vacuum, practically it has been found[7] that the most satisfactory pressure for the Shortt clock, from the standpoint of stability of period with pressure variation, is about 15 to 20 mm. of mercury. When the pressure is very low or zero the arc, when not definitely controlled, is very unstable and causes large changes in rate, but in the neighborhood of 15 mm. of pressure there is a stabilizing action due to the air friction, and a neutralization of the following four effects of pressure variation:

1. Change in the restoring force due to buoyancy of pendulum in air.
2. Change of center of gravity due to buoyancy when pendulum is made of dissimilar materials.
3. Change of effective mass due to air carried along by the pendulum.
4. Change of friction and consequent change in arc.

The first and fourth are probably of greatest importance. With decrease in pressure the first decreases the period and the fourth increases it. At the critical pressure the two effects are about equal and opposite. In the determination of gravity,[8] where very precise pendulums are used, all of these factors must be considered very carefully.

When the amplitude control device described above is used, the pressure can be reduced to a small fraction of a millimeter, which thereby reduces the effect of friction to a minimum. The term $\sqrt{1 + kR^2}$ then becomes practically unity and the required sustaining energy can be very much reduced.

So far we have considered variations in the restoring force. Variations in the inertia term are now considered.

The expansion of the pendulum with temperature must either be reduced or compensated for. At the present time practically all clocks of high precision employ invar pendulum rods with bobs of type metal or invar. The bob rests on an expansion collar, the dimensions of which are calculated to compensate for the lengthening of the rod. Some recent studies[9] in Japan have produced a "super-invar," to use the expression given, which may have a zero or even a slightly negative coefficient of expansion. If these new alloys are as stable as invar, they may have an important use in precision clock making, as the entire pendulum could be made from a single piece of material.

With the new technique recently developed in the laboratories of the General Electric Company it is now possible to produce pieces of clear fused quartz large enough to construct the entire pendulum for a clock. Some experiments are now being conducted with such clocks in the Loomis Laboratory. Fused quartz has a temperature coefficient of expansion of about 0.5 parts in 10^6 per deg. cent. A temperature variation of one degree would therefore cause a change in rate of only 0.02 second a day due to this effect alone. In a constant temperature vault with a fused quartz pendulum a variation of 0.01 degree would only affect the rate 0.0002 second a day. Means have been developed whereby temperature can be controlled to within 0.01 degree over long periods.

There are two other major effects that should be considered in connection with pendulum length. These are (1) variations in the position of the point of oscillation, and (2) the instability of materials with time, in other words, aging. The effect of knife edge wear on the period of an ordinary pendulum is appreciable. A wearing of one thousandth of a millimeter will change the rate of a seconds pendulum nearly a tenth of a second a day. Largely for this reason, a flexible spring suspension of elinvar[10] is used in most precision clocks. But even the spring is suspected of changes in length and, what is just as bad, also of variation in stiffness which would change the point of flexure and the restoring force. Aging effects in the pendulum rod can be reduced only by finding better materials. It may be that fused quartz, or possibly some large single crystal, will be found finally to be the most suitable material to use.

The effect of knife edge wear on the period of a pendulum can in theory be almost entirely avoided by suspending the pendulum at a distance from its center of mass equal to the radius of gyration about that point. This simple principle of mechanics, has of course long been recognized,[11] and experiments are being conducted in Germany[12] and elsewhere to determine its value in a precision timekeeper. In the case of a uniform straight rod this distance should be $\frac{1}{2\sqrt{3}}$ of the total length from the center. With this condition fulfilled a change in this length due to knife edge wear of one part in a thousand would affect the period less than one part in a million.

Returning to the fundamental equation at the beginning of the paper

$$M\ddot{x} + R\dot{x} + Sx = A \sin (t + \phi)$$

we have considered briefly the terms on the left-hand side.

We will now consider the right-hand term—the driving force—and especially the quantity ϕ the phase angle, which presents perhaps the most difficult problem in the design of all timekeepers.

The impulse should be delivered to the pendulum at the time when the velocity is a maximum, that is, at the center of the swing, in order to reduce to the minimum the effect of variations in the driving force on the rate. If the impulse is applied *before* the instant of maximum velocity, the rate is momentarily increased. If the impulse is applied *after* the instant of maximum velocity the reverse is true. Of course the impulse actually is of finite duration and the desired effect is approximated by supplying the pulse symmetrically with respect to the instant of maximum velocity thereby tending to neutralize the above mentioned acceleration and retardation effects.

The necessity for this method of applying sustaining power pulses to a pendulum has been realized almost from the beginning of precision clock history but cannot be over-emphasized.

In the Shortt clock[13] the impulse to the master pendulum is controlled by means of an auxiliary or "slave" pendulum which in turn is automatically synchronized with the master pendulum to within ± 0.003 second at all times. The means by which this is accomplished can be seen from the schematic drawing Fig. 2 which shows the master and slave pendulums with all of the essential control elements, and the counting or time indicating dials. Although the master pendulum determines the rate of the system, the slave pendulum and associated mechanism does the work of releasing the driving pulses at the proper time. The slave pendulum, through a jeweled ratchet and pawl-operated count wheel, releases mechanically a weight arm once each 30 seconds at just the right time so that a small wheel on the gravity lever, rolling down a curved inclined track associated with the slave pendulum rod, imparts the sustaining impulse to it. As soon as the mechanical impulse is delivered, this arm closes an electrical contact which resets the weight arm, before the return swing of the pendulum, in readiness for the next 30 second impulse. The current which resets the slave clock drive also releases the impulse mechanism for the master clock thereby avoiding the use of *any* mechanical connection with the master pendulum other than that of the impulse wheel. As in the case of the slave drive mechanism, the impulse lever makes an electrical contact after delivering its impulse to the master pendulum, which resets the gravity arm and in addition operates the "hit and miss" synchronizer on the slave clock. The slave pendulum normally runs a little slower than the master and is speeded up when necessary by means of a spring attached to the pendu-

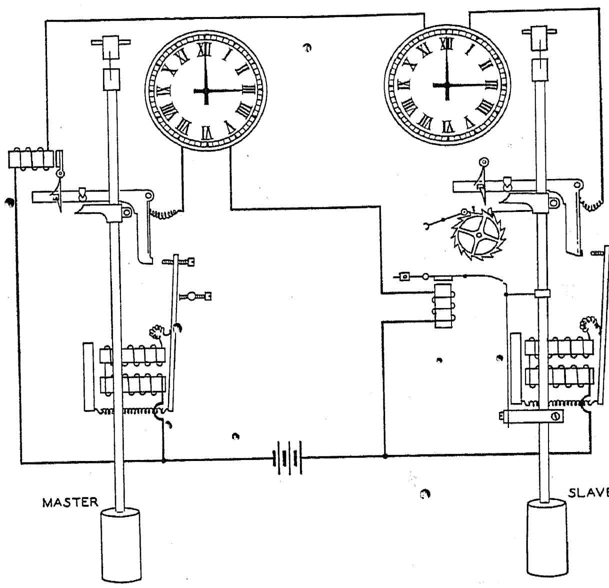


Fig. 2—Schematic Drawing of Shortt Clock Mechanism and Circuit

lum rod which is engaged by the synchronizer whenever the slave lags by more than 0.002 second behind a definite phase relation to the master.

There are reasons to believe, however, that small changes in phase do occur especially with variations in amplitude. Probably future improvements in gravity timekeepers will include improved methods for keeping the phase of the driving force more nearly constant.

It would be impossible even to survey adequately the developments leading up to the final design of any given type of clock in the limited space for this paper. It must not be assumed that the excellent performance of any timekeeping device is due entirely to any one person or company or period, but represents the combined efforts of a large number of people over an extensive period.[14,15,16] The three clocks most often recognized as precision timekeepers are the Leroy,[17] Riefler[18] and Shortt[13]. One or more of these is to be found in most of the important time laboratories and astronomical observatories throughout the world.

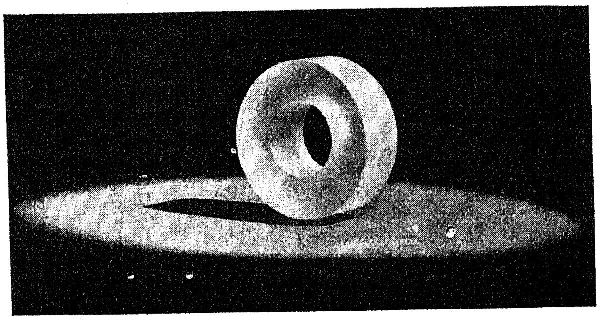


Fig. 3—Quartz Crystal with Zero Temperature Coefficient Used in Crystal Clock

Crystal Oscillators as Timekeepers

Within recent years developments have so far progressed that the constancy of rate of certain mechanical resonators, maintained in vibration by vacuum tubes, may be comparable with that of the best pendulum clocks. Such oscillators have the chief attributes of a good clock and can be used to operate or control a wide variety of time mechanisms with even greater versatility than the conventional pendulum.

The most accurate oscillators of this type known to the authors are controlled by plates of quartz crystal vibrating at a high frequency, that is, high in comparison with the frequency of a pendulum. Probably the most accurate of these at the present time are a set of four 100,000-cycle oscillators[19] that have been built by Bell Telephone Laboratories primarily for use as a primary frequency standard. These laboratories have also supplied a similar set of oscillators to the U. S. Bureau of Standards for the same purpose.

In this type of oscillator the frequency is controlled by a ring of quartz crystal about three inches in outside diameter as shown in Fig. 3. The frequency of the oscillator is the same as that of the crystal, that is, 100,000 cycles per second. One complete oscillator, including temperature and pressure controlled crystal, and shielded vacuum tube circuit, is shown in Fig. 4.

In order to operate clock mechanisms in the most direct way it is necessary to control from this high frequency another which is low enough to operate a synchronous motor. This is done by an electrical circuit known as a sub-multiple generator which controls one frequency at a definite fraction of another in absolute synchronism. Several types of circuit have been used for this, all of which are as positive in action as a set of gears. Usually the low frequency used to operate the motor is 1,000 cycles. Each pole on the motor in motion therefore corresponds to one milli-second in time and hence any variation in the motor mechanism or operation can amount at most to a fraction of a milli-second. Such a motor can be used to operate time indicating or measuring apparatus in a wide variety of forms.

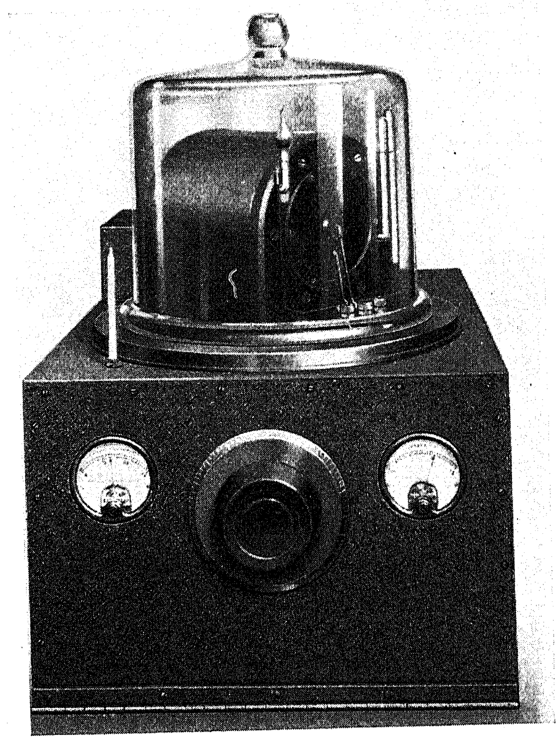


Fig. 4—Complete 100,000 Cycle Quartz Oscillator

Including temperature and pressure controlled crystal and shielded vacuum tube circuit

As a rate controlling element a crystal of quartz used in this way has a number of outstanding useful properties. As previously discussed, the rate of vibration is controlled chiefly by the effective mass and stiffness of the resonator. On account of the chemical stability of the substance Si O_2 and the physical stability of the crystal structure of quartz these may be expected to be constant in very high degree. At the present time, variations introduced by the vacuum tube circuit and the crystal mounting are such that no conclusive evidence of aging in the quartz has been obtained.

Since the elasticity is a constant for such displacements as are required (amounting to less than one part in 100,000 change of dimensions) the rate is substantially independent of amplitude. The amplitude can be controlled at nearly a constant value, however, by proper adjustment of the vacuum tube circuit.

The temperature coefficient of frequency can be made as near to zero as required at any given operating temperature by properly adjusting the shape of the resonator. Crystals that have been in continuous operation for three years have not shown any appreciable change in this adjustment.

Crystals are not affected by gravity or magnetic fields and can be shielded readily from electrostatic fields. They can be mounted so as to be relatively immune to vibrations which always seem to be present in the earth, especially near traveled roads and manufacturing centers. This may be found to be an important factor in the location of accurate clocks in cities or near earthquake zones. The crystals referred to above are mounted on the seventh floor near a much traveled thoroughfare in New York City.

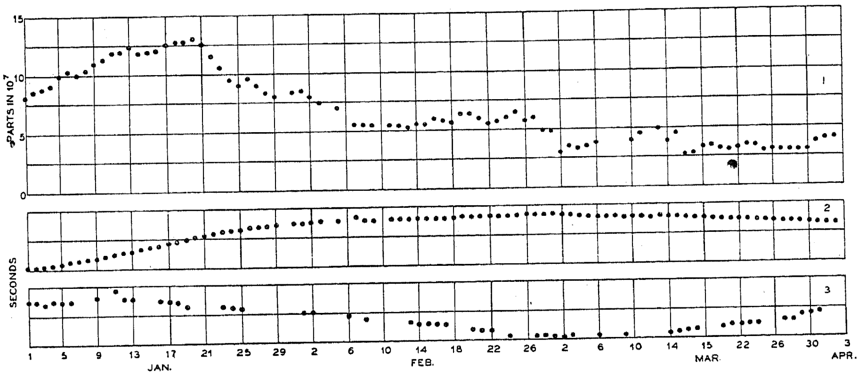


Fig. 5—Performance Data for Crystal Clocks

Curve 1. Rate comparison between crystals 1 and 2 expressed as parts in ten million

Curve 2. Integral of curve 1, showing running comparison between crystals expressed in seconds

Curve 3. Indicated time of one crystal clock vs. corrected time signals from U. S. Naval Observatory

The electro mechanical coupling to a crystal for imparting the sustaining energy to it makes use of its piezo activity. As far as has been determined there is no increase in decrement directly due to this means of coupling, although, of course, properties of the electrical circuit, such as damping, may be manifested in the crystal through this coupling. The logarithmic decrement of a 100,000-cycle low coefficient crystal mounted in air at atmospheric pressure and coupled into the electrical driving circuit is about 0.00012.

Probably the largest rate disturbing factors in the crystal clock at the present time are due to effects of the phase ϕ of the applied driving force. This factor varies slightly with circuit variables, aging of vacuum tubes and power voltage variations. Developments now in progress indicate, however, that the effects of these variables can be greatly reduced, which should make a marked improvement in performance.

The performance, as clocks, of a pair of crystals is shown for a three-month period in the graph Fig. 5. The comparison rate curve shows the variations in relative rates between the crystals while the time comparison curve shows the variation in indicated time (integrated rates). The third curve gives the rating of one of the crystals against corrected Naval Observatory time signals for the same three-month period.

While the crystal clock assembly is more costly than a pendulum clock and requires more elaborate associated electrical equipment, we believe that for many purposes it is decidedly superior because of its great versatility. For example, its rate can be adjusted at will over a sufficient range to allow for such aging effects as occur. Also, the phase, or indicated time, can be adjusted with extreme accuracy. This can be done most advantageously in the electrical circuit by means of a phase shifter which operates continuously through 360 elec. deg., or any fraction or multiple thereof. If used in the 100,000-cycle circuit, one complete turn of the dial corresponds to a final adjustment of one hundred thousandth of a second, which obviously is greater accuracy than needed in any ordinary time mechanism.

The accuracy of inter-comparison of crystal clocks may be very high. An absolute comparison accurate to better than one-hundred thousandth of a second can be maintained continuously, while under special conditions, short time comparisons accurate to one part in 10^{10} can be made.[19] This high accuracy of comparison is due chiefly to the large number of vibrations per second, 200,000 times greater than with a "seconds" pendulum. Even greater accuracy of comparison could be obtained by the use of higher frequency crystals, but for reasons of convenience and greater freedom from the effect of external circuits and the mounting, the lower frequency is preferred.

One application of considerable interest is that two clock mechanisms can be operated from the same crystal control in this way so that one keeps true mean solar time when the other keeps true sidereal time.[20] It can be shown readily that any ratio of rates can be obtained for such a purpose accurate to at least one part in 10^{10} without the use of an unduly complicated mechanism. Lord Grimthorpe has shown why a pendulum cannot be used for this purpose,[21] but his reasons do not apply to the crystal control method.

By means of a shutter of the type shown in Fig. 6, and a photoelectric cell, signals of extreme accuracy can be controlled which could be used for radio transmission of time signals or for any purpose requiring great accuracy. When this disk is rotated at ten revolutions per second by a crystal controlled motor the current from the photo cell is 1,000 cycles modulated, in the manner indicated in oscillogram No. 1, at 100 cycles per second. By marking one 0.01 second element, as for example by partially blocking one sector, a positive 0.1 second indication is obtained. The oscillograms shown in Fig. 7 were made with the disk shown and indicate how, with the use of a suitable oscillograph, a continuous time signal can be recorded that can be read to a fraction of a milli-second. This resolution would be very useful for accurate time measurement work, for studies in radio propagation times, and for measurements of gravity. Observers requiring less resolution than this could obtain 0.01 second accuracy by the use of a rectifier and a recorder accurate to only 0.01 second. The same is true for 0.1 and 1.0 second accuracy by a suitable choice of transmitting disk and receiving equipment.

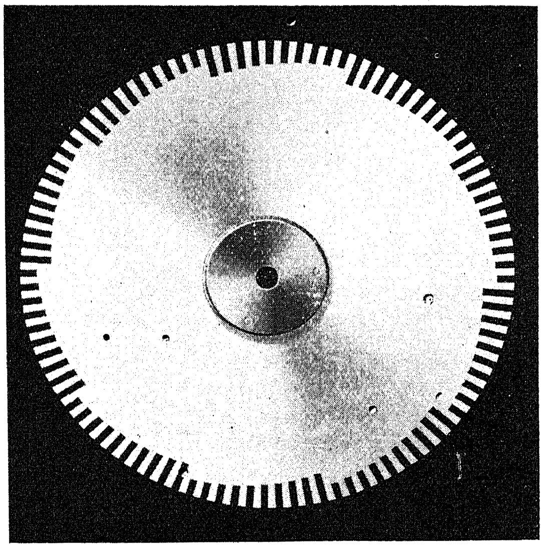
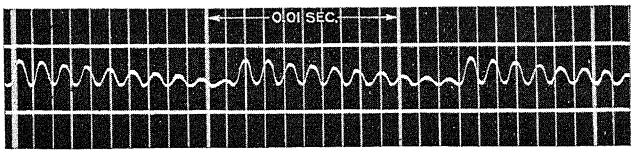


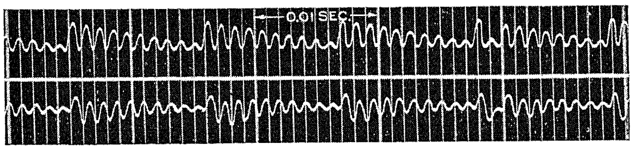
Fig. 6—Special Light Interrupter for the Production of Accurate Timing Signals by Photoelectric Cell

The second oscillogram shows the signal directly from the disk and after it had gone through a complete radio transmitter and receiver consisting of a total of eleven tandem vacuum tube stages and the associated equipment. As indicated, the total delay encountered was about 0.1 milli-second, and the distortion is not appreciable. Of course any combination of make and break signals can be controlled in this way using either a continuous or a modulated tone. If desired a series of half second dashes could be controlled, interspaced by a dot-dash code to designate each dash.

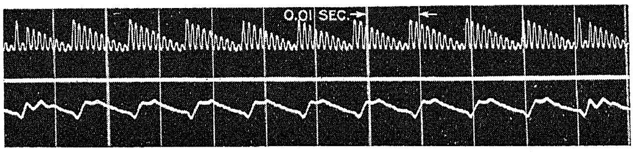
A complete time signal control mechanism based on this idea is shown diagrammatically in Fig. 8. In the method shown, light from a straight filament lamp is imaged on the slotted edge of a disk driven at 10 revolutions per second by a 1,000-cycle motor. The light entering the photo cell is modulated as indicated, at 1,000 times and 500 times per second alternately, and controls a corresponding electrical signal. The slow speed shutter, rotating at one revolution per second, causes the effective light path to alternate between the two disks once each second, and gives a corresponding variation of electrical signal. This signal can be used for any resolution from one second down to one milli-second and can be received through static and fading. It can be reduced to the dash-space type by the use of a simple voice-frequency filter to suppress the undesired half signal, or used as a continuous signal for oscillograph recording.



No. 1. Direct signal obtained from photo-cell



No. 2. Direct signal (above), and the same signal transmitted and received by radio (below). A method is indicated for marking every tenth of a second



No. 3. Direct signal as in No. 2 (above) and same signal rectified (below) for recording with only 0.01 second resolution

Fig. 7—Oscillogram of Time Signals Produced by Photo-Cell Method

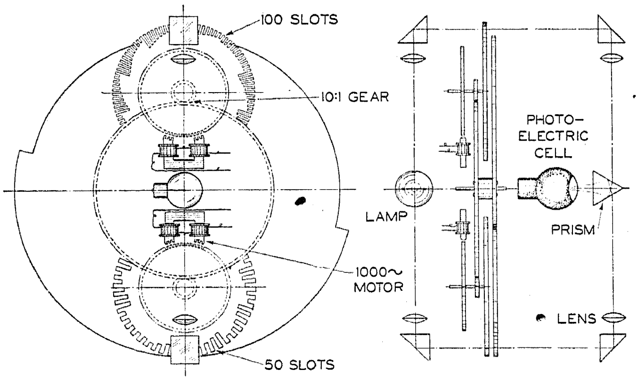


Fig. 8—Proposed Method for Production of Accurate Time Signals by Photoelectric Cell

Among the advantages of the crystal clock it should be mentioned that in order to control any number of clock mechanisms with the same precision it is only necessary to distribute the constant frequency current derived from the crystal to the separate mechanisms. This can be done on any scale economically feasible.

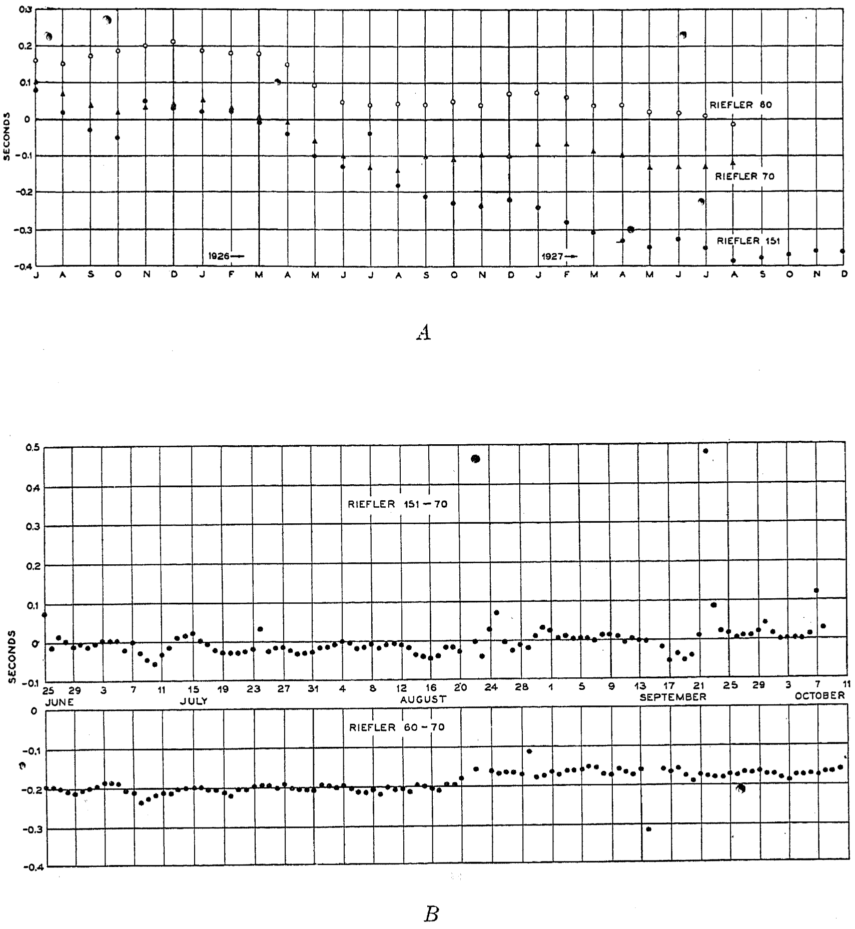


Fig. 9—U. S. Naval Observatory, Washington, D. C.

A. Observed mean daily rates of Riefler Clocks No. 60, No. 70 and No. 151
B. Relative rates of Riefler 151 *vs*. 70 and Riefler 60 *vs*. 70. (Data supplied by U. S. Naval Observatory)

PERFORMANCE OF CLOCK INSTALLATIONS

In order to indicate the performance that may be expected of precision clocks, data have been obtained from several actual clock installations and presented herewith in the form of curves. These do not show in every case, perhaps, the performance that could be obtained under ideal conditions but they do indicate the type of performance to be expected under present normal working conditions. In studying these curves, care should be taken to examine the coordinates because it was impossible to present all the data plotted to the same scale on account of the different times and the different errors involved. Also some of the data are plotted as daily rates and others as indicated time. The effect of this is that over a short period a rate curve shows greater apparent variations than a time curve and *vice versa*. In some cases the mean gaining or losing rate was removed from the data, as this has no bearing on the *constancy* of rate which alone determines the value of a clock as a timekeeper.

The essential descriptive information concerning the clocks represented accompanies each figure. Some data relative to the performance of Shortt Clocks 20, 21 and 22 in the Loomis Laboratory and the Crystal Clocks in Bell Telephone Laboratories are discussed in the following section.

EXPERIMENTS IN TIME AT LOOMIS LABORATORY

The clock installation at Tuxedo Park and some of the experimental results obtained there have been described elsewhere.[7] In brief it consists of three Shortt clocks mounted in an especially favorable location practically free from traffic and electrical disturbances, and carefully temperature controlled. The excellence of the location is enhanced by the fact that the clock vault is excavated in the solid rock of the mountain on which the laboratory stands, and the three massive masonry piers for mounting the clocks are effectively a part of this rock. One of the three Shortt master pendulums in its casing is shown in Fig. 14 mounted on its pier within the temperature controlled vault.

By means of the Loomis Chronograph[7]

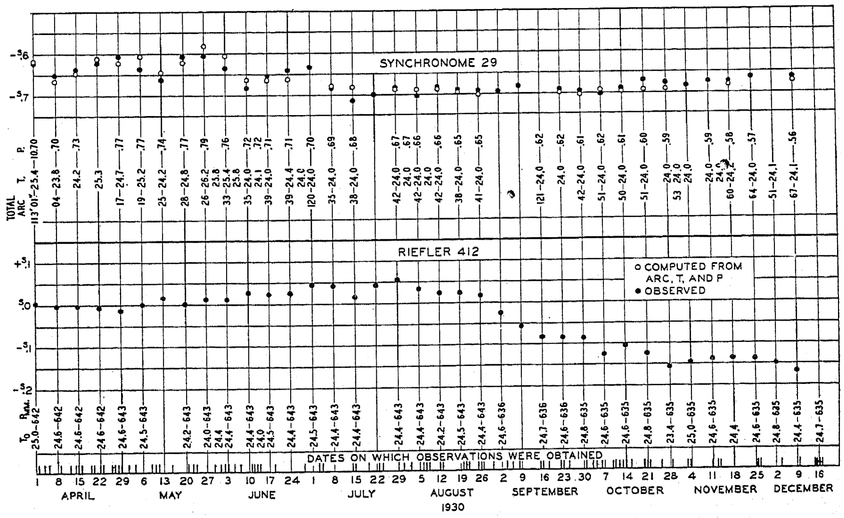


Fig. 10—Dominion Observatory, Ottawa, Canada

Observed daily rates of Riefler 412 and Shortt 29. (Data supplied by Dominion Observatory) it has been possible for the first time to obtain a running phase comparison of a number of clocks with great precision. Prof. E. W. Brown and Dr. Brouwer have shown[22] that the probable error of the mean hourly rates as measured by this chronograph is less than 0.0001 sec. With a resolution of this order, it is possible to study some effects that would be forever unsuspected with the use of any comparison equipment used heretofore.

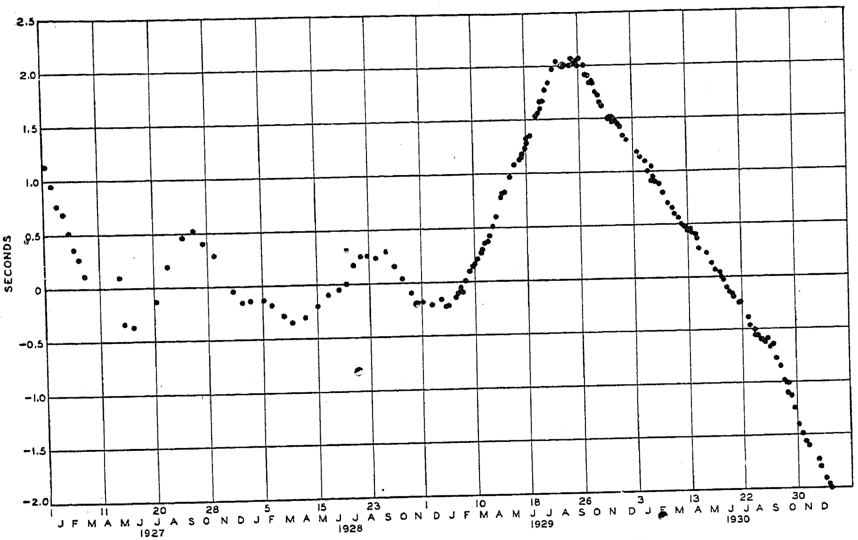


FIG. 11—ROYAL OBSERVATORY, GREENWICH, ENGLAND

Comparison of observed correction for Shortt clock No. 11 with those computed from the following formula which was based on observations from March 1927 to December 1928. Formula: $98^s 905 + 28^s 2002t + 1^s 7542t^2 + 0^s 2524 \int (T\text{-}62^o 8)\, dt$ + Nutation (Information from Monthly Notices of R.A.S. 1928-1931 inc.)

The rate curves on Fig. 15 will illustrate a case in point. These three curves show the differences in rates between the three Shortt clocks taken in cyclic order, plotted on a very open scale over a period of about three months. Although the *mean* rates over this and much longer periods are constant to one part in ten million, there are fluctuations in rate having for the most part perfectly definite periods. These fluctuations are real and a study[22] of the data by Professor Brown showed that the periods correspond to the differences in periods of the pendulums taken in pairs. This implies that there is coupling between the pendulums and that the rate of one is *modulated* by that of the others even though the pendulums swing practically in vacuum and are mounted on separate massive piers, and that they swing in planes 120 degrees apart. This would seem to show that, massive as the piers have been made, they are not infinite in comparison with the 14-lb. pendulums, and that strains are set up by each pendulum that are felt in some degree by the others through the piers and solid bed rock.

In order to indicate the comparison of these three Shortt clocks on the same basis as the Observatory clocks previously discussed, the data of the rate curves in Fig. 15 are replotted on the same graph on the basis of indicated time. This shows that although the rate variations are quite appreciable as measured by the Loomis Chronograph, the relative performance, as clocks, is all that could be desired, and indicates that factors having small but real effects on the rate can be detected by a chronograph with this high resolving power when they would not be observed at all by ordinary methods. The three rate curves and the three time curves are plotted to the same scale as the corresponding crystal data in Fig. 5.

The Loomis Chronograph consists of the following essential parts:

1. A strip of paper about 10 inches wide moving at a uniform rate between a long grounded electrode and a comb consisting of one hundred separate equally spaced pointed electrodes.

2. A distributor having one hundred segments separately connected to the one hundred elements of the comb, operated at exactly ten revolutions per second by a synchronous motor from a source of 1,000-cycle current.

3. A source of high potential consisting of an induction coil through the primary of which a large condenser is discharged when a record is to be made.

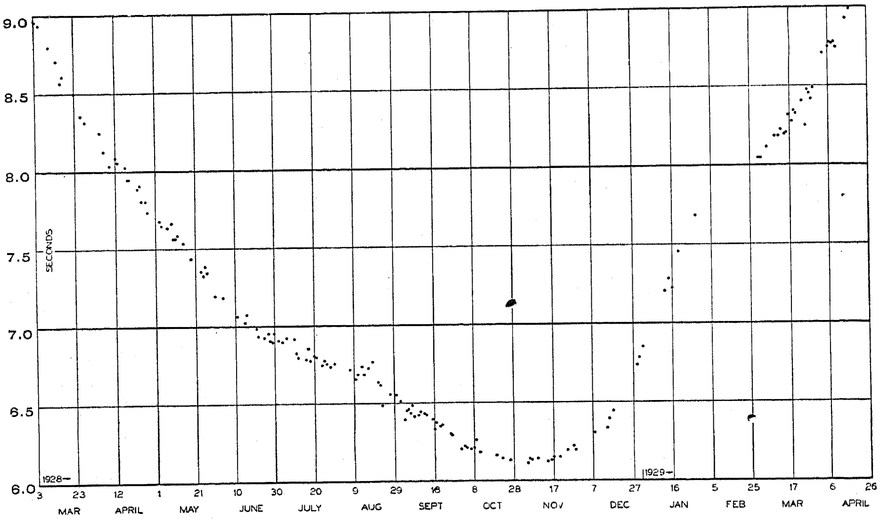


FIG. 12—PARIS OBSERVATORY, PARIS, FRANCE

Observed correction of Leroy clock No. 1185 (Information from Bulletin Horaire 1928-1930)

A small condenser on the induction coil secondary stores sufficient energy to produce an intense spark of very short duration; and

4. A single relay in the common circuit of all the time devices to be recorded on the moving paper strip.

Fig. 16 shows this chronograph in operation.

If the relay is operated every time the distributor makes some exact whole number of revolutions a series of perforations will be made in a line parallel with the edge of the paper. If in the time between sparks, the motor revolves a whole number of times plus or minus one hundredth, successive perforations will be displaced an amount corresponding to one milli-second in time.

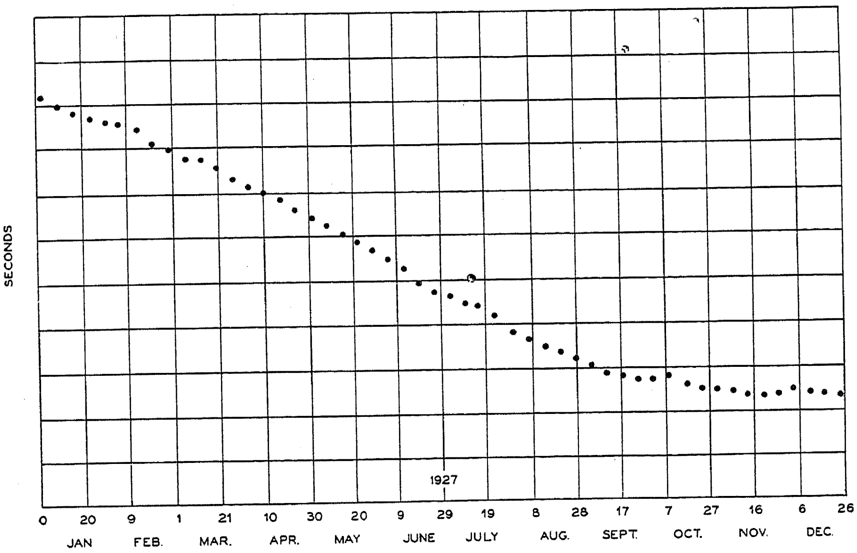


FIG. 13—ROYAL OBSERVATORY OF EDINBURGH, SCOTLAND

Observed correction of Shortt clock No. O. (Information from Proc. Royal Soc. Edinburgh Vol. XLVIII, page 161)

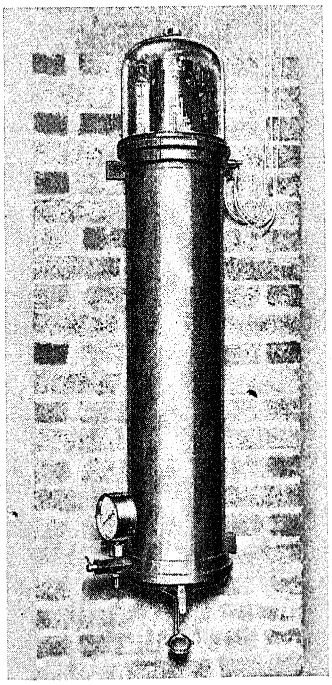


FIG. 14—MASTER PENDULUM (IN CASING) FOR ONE OF THREE SHORTT CLOCKS IN THE LOOMIS LABORATORY, TUXEDO PARK N. Y.

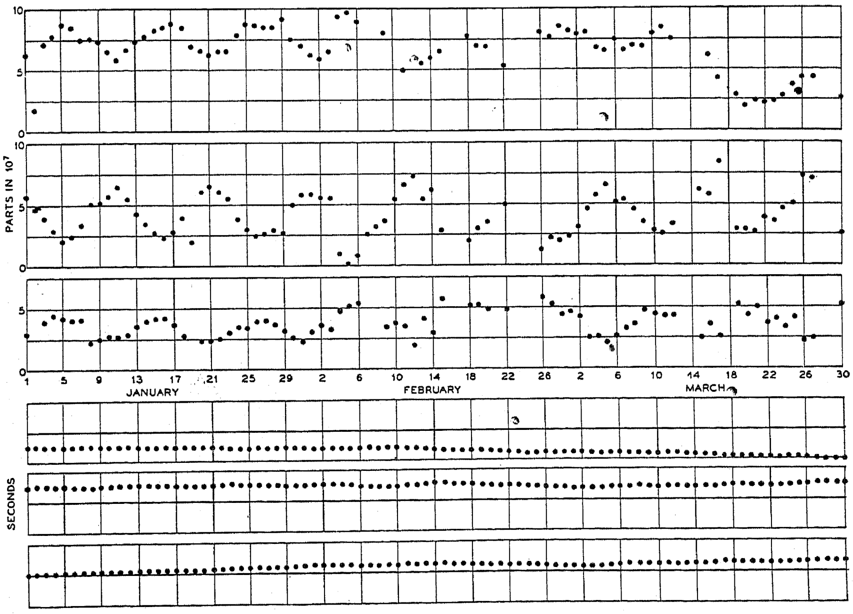


FIG. 15—INTERCOMPARISON OF THREE SHORTT CLOCKS IN LOOMIS LABORATORY

A. Rates, expressed in parts in ten million
B. Integrated rates expressed in seconds

One second in time is represented by ten complete transits of the chart, so that, in effect, the chart is 100 inches wide and the effective recording element moves at the rate of 500 ft. a minute. Thus from the slope of the line of perforations with respect to the edge of the paper the difference between the rate of the distributor and the rate of the sparks can be determined with high precision. Similarly any number of separate sets of sparks can be recorded simultaneously on the same chart, using the same actuating relay and distributor. The relative rates may be determined by measuring the difference of slopes of the rows of perforations.

This is the method used for intercomparing the three Shortt clocks and for comparing them with the crystal oscillators in New York. To accomplish this, 1,000-cycle current controlled by the Bell Laboratories' crystal is sent over a private wire to Tuxedo and there used to drive the distributor as outlined above. The resulting records are continuous comparisons of the three Shortt clocks and the crystal accurate to better than one milli-second in time.

Since the crystal does not respond to variations in gravity while the pendulum does, the difference in rates contains a term having the period of the lunar day, corresponding to the direct gravitation effect of the moon on the pendulums. Several months' record of the Loomis Chronograph have been analyzed,[22] by Professor Brown and Dr. Brouwer and a lunar term observed having the proper period and magnitude. A graph of the lunar effect derived in this way is reproduced from their paper in Fig. 17. The periodic gain and loss in indicated time is about 0.0002 second per lunar day.

The results of time studies obtained at the Loomis Laboratory have been very encouraging and it is planned to continue the researches in clocks and time measurement methods. Some aspects of this work were discussed by Professor C. V. Boys in *Nature* for October 17, 1931.

POSSIBLE FUTURE INVESTIGATIONS

No doubt there is a practical limit to the accuracy attainable in a timekeeper but as long as random changes in rate are observed which cannot be explained by uncontrollable properties of matter (such as Brownian movement) that limit has not been reached, even with the types of mechanism at present in use.

The suggestion has been made of utilizing the constant velocity of light between fixed points as a measure of period. At present the velocity of light is known to less than one part in a million.[23] Even if it were known with greater accuracy, the practical difficulties of establishing such a standard, except in a very rough way, would be insuperable. In the first place the distance between the measuring points would have to be known to one part in a million, that is, better than 0.06 inch in a mile, assuming no other error, to define a standard good to only 0.1 second a day. Considering the difficulties in measurement, an accuracy of a little better than a part in a million is about all that could be expected of such a method by any means now known.

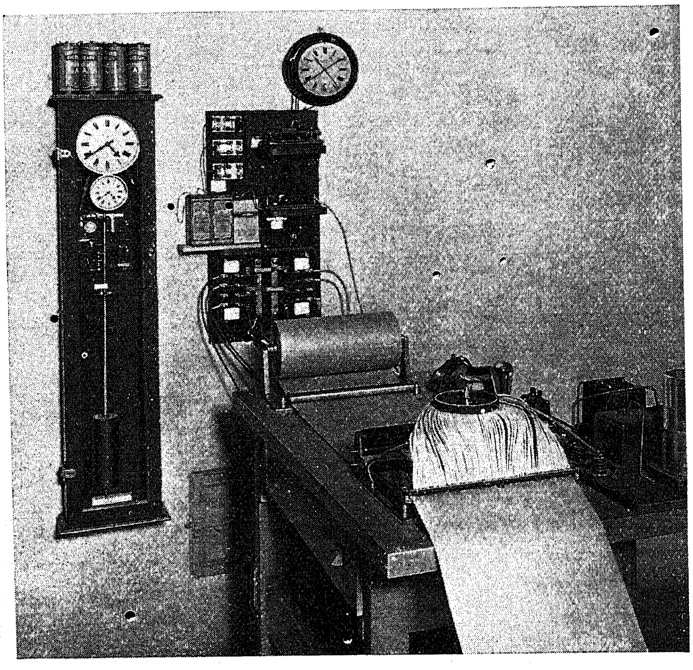


FIG. 16—LOOMIS CHRONOGRAPH AND ONE OF THREE SHORTT SLAVE CLOCKS IN THE LOOMIS LABORATORY

The extreme definition of some spectral lines indicates a very slightly damped oscillation of some sort in the atom, and some thought has been directed to means of making a frequency comparison between these vibrations and others of lower frequency that could be used to measure time as previously outlined. The effective logarithmic decrement of the "oscillator" behind the red line of cadmium can be estimated as about one tenth of that of a quartz resonator. The difficulties of utilizing this sort of thing as a time (or frequency) standard are: First, it is not known at present whether the "frequency" of an atomic disturbance has the same physical significance as vibrations-per-second and, second, even if the significance were the same, the orders of the two frequency ranges are so far separated that no means known at present could be used to effect the comparison. Besides, the energy of a spectrum line is no doubt made up of a large number of damped wave trains superposed in some fortuitous fashion as would cover the identity of separate cycles.

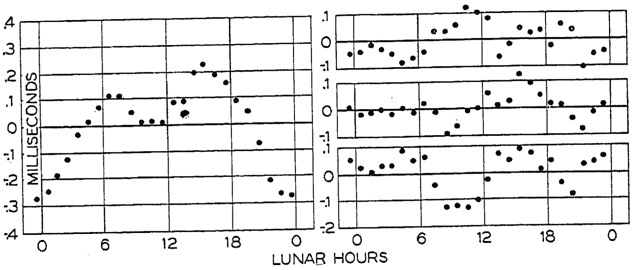


FIG. 17—ANALYSIS OF COMPARISON DATA BETWEEN CRYSTAL CLOCK AND THREE PENDULUM CLOCKS REVEALED THE PERIODIC LUNAR DAY VARIATION SHOWN IN THE LEFT HAND CURVE

Similar analyses of the comparison between the three pendulum clocks taken in pairs showed no corresponding effect

Probably the real final limit in the accuracy of timekeepers is concerned with the measure of our time standards. Some observations indicate that the rate of the earth's rotation is not constant, but that there are both systematic and apparently random changes in rate as compared with the motion of other astronomical bodies believed to be more stable.[24] For example; about 1918 a rather abrupt change in rate was observed which amounts to about one part in thirty million. This is small to be sure, but points to the improbability that the rate of any phenomenon is absolutely constant. After all, time is relative and the most nearly accurate measure of time must always be in terms of the most nearly constant motion that can be observed.

At the present time in the official observatories of the various countries the usual procedure is to plot the star observations in terms of their master clock and then to draw a smooth curve, which curve is adopted as the official clock correction. To some extent the drawing of this curve is a matter of judgment, based partly on the number and certainty of the star observations and partly on the estimated performance of the master clock in terms of their other clocks. These observatories at the present time also transmit and record radio time signals and publish periodically corrections to these signals in terms of their "standard clock error." From these data alone it is not possible for any one observatory to compare its clocks directly with the clocks in another observatory. If the observatories would publish one additional number for each of their important clocks, namely, the comparison of each clock with their official time, it would be possible to compare the rates of the clocks in such observatory directly with clocks anywhere else in the world where the radio time signals can be received. This would involve only a very small amount of additional work as the numerical data have to be obtained in any case. It is well to note in this connection that the United States Naval Observatory has adopted a procedure which is very similar to this, the only difference being that instead of publishing the corrections to each of its principal clocks, it publishes a single number which represents the correction of the weighted mean of several clocks. This is an excellent beginning and it is hoped in the interest of future clock developments and of studies in gravity and related phenomena, that others will follow.

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Discussion

For discussion of this paper see page 550.

Time Service of the U. S. Naval Observatory

BY J. FREDERICK HELLWEG[1]
Non-member

INTRODUCTION

THE U. S. Naval Observatory determines the standard time for the United States with scientific precision and broadcasts the results so as to make them available throughout a large part of the world. Since 1830 the Navy has had charge of this very important task. In the early days the demands for time service were confined principally to the mariners of our country. Scientific pursuits, increased demands of public utilities, greater business efficiency, increased speed in all activities of daily life, new inventions, all have demanded greater accuracy, and repeated expansion of the time service. At present the time service is broadcast six times a day and each broadcast is sent over a spread of about fourteen different frequencies to insure universal reception.

DESCRIPTION OF THE OBSERVATIONS WHICH ESTABLISH STANDARD TIME

The period of the earth's rotation on its axis is accepted as the standard of time measurement. The stars appear to rise in the east and after swinging across the heavens to set in the west. If a person stood on the earth's equator and measured the interval of time between two successive transits of a distant star he would have the most accurate measure of the rotation of the earth. If the same measurement was then made using the sun instead of a star the interval would be about four minutes longer due to the earth's orbital motion around the sun which continuously changes the apparent place of the sun among the stars. Therefore, as the earth rotates on its axis, this apparent motion of the sun necessitates the earth's rotating through more than 360 degrees to bring the sun again in the observer's meridian.

The stars are not within the earth's orbit, they are so far distant that their apparent positions are only very slightly affected by the earth's orbital motion. The positions of the stars are measured from a point in the sky known as the vernal equinox, and the period of the earth's rotation measured with respect to the vernal equinox is called a sidereal day. The same period measured with respect to the sun is called an apparent solar day. Owing to the variable length of the apparent solar day, astronomers had to devise a uniform measure of time and called it a mean solar day, so now we have apparent time, mean time, and sidereal time, and each time can be converted into the other by the application of the proper corrections. Because of the greater accuracy of sidereal time, that time has been universally used by astronomers.

In order to simplify the important question of time throughout the world, standard time zones have been established. All points within each zone use the same standard mean time although this may differ from their local time by as much as one-half hour. These zones represent a belt just one hour wide or fifteen degrees of longitude wide. In continental United States there are four zones:

Eastern Standard Time or 75th meridian time which is exactly five hours behind Greenwich standard mean time.

Central Standard Time or 90th meridian time, six hours behind Greenwich time.

Mountain Standard Time or 105th meridian time, seven hours behind Greenwich time.

Pacific Standard Time, or 120th meridian time, eight hours behind Greenwich time.

The Naval Observatory is thus able to furnish one time signal which provides time for all zones, not only in continental United States but for its island possessions and ships far out across the Pacific.

THE DETERMINATION OF THE STANDARD TIME

The Naval Observatory uses small quickly handled telescopes known as transit instruments in observing celestial bodies for the determination of our standard time.

These transit instruments as their name indicates are rigidly mounted so that there can be no motion either side of the meridian. The instruments are pivoted however so as to rotate from north to south. Astronomical bodies are therefore only visible through the transit instrument when on or very near the celestial meridian.

The sidereal time at which each star will cross the meridian is very accurately calculated. Stars are selected which cross the celestial meridian near the zenith of the Observatory and also whose positions are the most accurately known.

The observer operates a micrometer attachment which causes interruptions in an electric circuit as the star passes certain points in the field of the instrument. These interruptions are recorded graphically by a chronograph on the same sheet which records the standard sidereal clock ticks. By measurements of this sheet and the application of known corrections, the exact time of meridian transit is determined, according to that particular clock. The difference between this time and the previously calculated time represents the error of that clock on sidereal time. A continuous record of the standard clock corrections is maintained, determined

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[1] Captain, U. S. Navy, Superintendent, U. S. Naval Observatory.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

every clear night by observing a list of stars with one or more telescopes at the Observatory.

THE PROCEDURE USED IN BROADCASTING STANDARD TIME

Sidereal time is more convenient and more accurate for the use of the Observatory in rating the standard clocks, but it is not at all suitable for the general public.

The transmission of the time service is therefore in mean solar time and the transmitting clocks are rated to mean solar time.

Immediately before each time signal is broadcast, the error of the transmission apparatus is determined by a chronograph comparison with the standard clocks. The record sheet of this chronograph permits very accurate measurements to the fraction of a second. The continuously changing differences between sidereal and solar time and the error of the sidereal clock are also applied resulting in our mean solar time broadcast being accurate within one one-hundredth of a second.

This comparison and correction is made immediately prior to each broadcast and requires several minutes to complete.

Five minutes before the zero hour and after the transmitting clock has been corrected to mean solar time with the greatest possible accuracy, the operator throws the controlling switch which gives the Naval Observatory direct control over the Navy's high power radio stations at Arlington and at Annapolis. From then on, the operation is entirely automatic.

Every second is broadcast except certain ones omitted for identification signals, and others which act as fingers pointing definitely to the next audible second following the period of silence.

The twenty-ninth second of each minute is omitted and therefore definitely points to the 30th second of each minute. The 56th, 57th, 58th, and 59th seconds are omitted therefore definitely indicating that the next succeeding second, the 60th, is the beginning of another minute. This occurs on the 55th, 56th, 57th, and 58th minutes before the zero hour. On the 59th minute there are no audible "ticks" from the 50th to the 60th second, indicating that after a silence of nine seconds, the next audible tick is the zero hour either midnight, 3:00 a.m., noon, 4:00 p.m., 7:00 p.m., or 10:00 p.m., seventy-fifth meridian time, or Eastern Standard Time.

In order that those listening for the broadcast can distinguish the minute when they tune in for reception, the fifty-first to the fifty-fourth seconds are used as identification signals. If the 51st beat is omitted, the listener will count four more beats before the end of the minute silence occurs, the 52d, 53d, 54th, and 55th beats. This indicates that there are four more minutes to go. If the 52d beat is omitted, the listener can only count three more beats before the silent period, the 53d, 54th, and 55th beats indicating three more minutes to go. If the 53d beat is omitted, the listener can count only two more beats, indicating two more minutes to go, and if the 54th beat is omitted, the listener can count only one more beat and knows that he has only one more minute to go before the zero hour.

This standard broadcast is known all over the world and is religiously listened for in the most remote parts of the globe.

If for any reason, the broadcast is interfered with, it is repeated the following hour.

In addition to our direct control of the two high power stations at Arlington and Annapolis, the Observatory also controls by radio the retransmitting station at San Francisco, and eventually it is hoped that this service will be extended to the east.

During the time broadcasts, automatic chronographic records are made at the Observatory which show the time of the sidereal clock, the time of the transmitting clock, (mean solar time) and the actual time of broadcast by Arlington and by Annapolis.

Immediately after the completion of the time signal broadcast, this record is measured and the difference noted. This gives the time of actual emission by each station. The corrections to the standard clock are then recalculated using star sights taken both before and after the broadcast. Errors are carefully computed and a correction sheet is sent weekly to all those requiring extreme accuracy noting the error on 75th mean solar time of each broadcast of each day. In addition, immediate corrections are sent out to those requiring more rapid service.

DESCRIPTION OF THE TIME CLOCK VAULT—SIDEREAL CLOCKS—COMPARISON OF THE PERFORMANCES OF VARIOUS CLOCKS

The standard sidereal clocks are rated very accurately to sidereal time. They are maintained in a vault under constant temperature and air pressure in order to minimize the effect of changes in either temperature or pressure on the rates of the various clocks.

A new vault is now under construction. It will accommodate seven clocks each on a separate pier and so arranged that no two adjacent pendulums will swing in the same plane. The vault consists of an inner and an outer vault separated by a three-foot air space. The inner vault will be illuminated by flood lights let in through the walls from light boxes in the three-foot air space, exactly as magazines are illuminated on board naval vessels. This will permit cleaning and repairing the lighting apparatus without entering the inner vault.

The temperature control will also be installed in this air space. A periscope to be lowered through the roof of the inner and the outer vaults from the office space above ground will permit visual inspection of the clocks in the vault.

The walls of the outer vault will be continued above ground and the enclosed space will be built as an office for the time service, record rooms, etc. There will also be a sleeping room for the use of astronomers on night watches.

At present there are three clocks made by Riefler in

Germany which are about thirty years old. Also one new Shortt clock made by the Synchronome Company of England and an order has been placed for a second Shortt clock.

A study of the Riefler clock No. 60 during a period of twelve years, 1916 to 1928, showed that its performance could be predicted 10 days in advance with an average error of 0.09 second. One day predictions showed an error of about 0.009 second. The other two Riefler clocks did not perform so well.

Shortt clock No. 38 has been under observation only for a relatively short period. Based on its performance from March 15 to August 12, 1931, its performance could be predicted ten days in advance with an average error of 0.05 second and daily predictions with 0.005 second error. A study of the Shortt clocks at Greenwich Observatory over an extended period confirms these figures. Taking all three Riefler clocks together, it is not possible to predict as well as by the Shortt clock, in other words, the Shortt compares favorably with the three Rieflers combined.

For longer periods, the Shortt clock is considerably better. The Rieflers are all subject to slow progressive changes in rates amounting to a tenth or even a quarter of a second per day in a year. No such change has been evident in the running of the Shortt clock. The Greenwich Shortt clocks show evidence of such a change but to a much smaller degree than the Rieflers.

NEW DEVELOPMENTS—NEW CLOCKS—PROJECTS FOR EXTENSION OF THE TIME SERVICE

The Observatory now has under construction three new clocks, an additional Shortt clock, a crystal controlled clock, and an experimental clock. The crystal controlled clock is being constructed by the Naval Research Laboratory under the supervision of Doctor L. P. Wheeler and Mr. W. E. Bower in accordance with their design which differs from those heretofore in use. The crystal is bar shaped and about four inches long. It has a fundamental frequency of 1,000 cycles per second, and is suspended at its nodes. Condenser plates placed in proximity to its ends, are connected to the input of a vacuum tube amplifier. The output of the amplifier is used to operate a telephone unit, the acoustic output of which is directed at the middle of the crystal, thus sustaining the oscillations. It is claimed for this method that the coefficient of the coupling between the crystal and the vacuum tubes is much less than in the conventional type. The frequency of the crystal is consequently less affected by variations in the tubes.

The crystal oscillator, in addition to serving as a new standard time keeper, will be used for controlling a new automatic time signal transmission apparatus which is being developed at the Naval Observatory. This is being designed for greater accuracy and reduced labor for maintenance.

The experimental clock is being planned by the time service personnel and embodies several new ideas.

In addition the Observatory is planning a more efficient apparatus for retransmission of the Observatory's time signal via San Francisco. If this proves successful, a similar installation will be tried at Honolulu and it may be carried on to a similar one in Guam and later in Manila.

Discussion

For discussion of this paper see page 550.

Time Services of the Telegraph Companies

BY G. W. JANSON[1]

Member, A.I.E.E.

GENERAL

IN the solar time system which forms the basis of the coordination of most human activities, a specified time at a given point is understood to mean a definite position of that point in respect to the sun. Thus, the true solar time at any two places upon the earth's surface not on the same meridian is different at any given instant. In order to avoid the confusion resulting from a large number of time standards, the earth's surface has been considered as divided into time zones and a standard of time arbitrarily chosen for each such zone. The North American Continent is included in seven such zones, the width of each being approximately fifteen degrees longitude or one twenty-fourth of the earth's circumference. In each of these, a standard time is used which differs by one exact hour from that of the adjoining zones and which represents the mean solar time of a meridian passing through it.

It is essential that the indication of time by clocks and other devices within a time zone be kept alike and at a proper relationship to those of other zones and to time as indicated by the sun. This might be done at each point by frequent comparisons of the time indicating devices with determinations of true time made from observation of the sun or other heavenly bodies. Such a procedure would be quite involved and hence for general practise the observations of a selected few places are relied upon as the basis of accurate time for many other points. The results of the determinations of the selected observatories must therefore be accurately and widely communicated to the dependent places and for this a system of electrical time signals is utilized.

The distribution of time signals as described in this paper may be divided into three classes of service. The first of these, commonly known as the "noon beats," comprises the transmission during a three minute period, ending at noon Eastern Standard Time, of a series of electrical impulses at intervals of exactly one second or multiples thereof. These signals originate from the Naval Observatory at Washington and are transmitted to the Western Union and to the Postal Telegraph-Cable Company who further distribute them to telegraph and railroad offices where they are used as a comparison standard by means of which these offices may determine the error in their time indicating or recording devices.

The second service commonly known as the "jewelers beats" comprises a similar transmission throughout the day of a coded series of impulses by which the time of day may be determined to a small fraction of a second. These beats are used principally by jewelers and watchmakers for the purpose of checking and regulating time pieces. Some of this work, for example the regulation of ships' chronometers, requires a high degree of accuracy and hence a continuously available standard of correct time.

The third and largest of these services is the clock or synchronizing service. This comprises the automatic correction or resetting of clocks or other devices which independently maintain an approximately uniform motion. The Western Union Telegraph Company's synchronizing system corrects about 120,000 clocks of which 95 per cent are rented to subscribers and the remainder are in its own service or owned by individuals or firms who subscribe for the synchronizing service which hourly resets the clocks to the correct time.

NOON BEAT SERVICE

The noon beat service which forms the basis of comparison for the accuracy of all other parts of the systems described originates with the Naval Observatory at Washington. Here astronomical observations are made each night when conditions will permit and from these, the error of the three principal naval master clocks is determined. These clocks are of great accuracy, and specially installed so that their rate is not affected by temperature and other variations.

The observatory equipment includes in addition to the three master clocks, two secondary master or transmitting clocks. The rate of these clocks may be altered by magnetic means shortly before it is expected to send signals from the observatory. Adjustments are made to bring these clocks to the correct time as indicated by the master clocks and the correction for their known error.

Signals are transmitted from the Observatory several times a day for five minute periods to various radio stations and communication companies who redistribute them. The signals are always sent during the last five minutes before the exact hour and are made by closing a circuit for a brief period immediately following the end of each second except the following:

29th, 51st, 56th, 57th, 58th and 59th seconds of the 56th min.
29th, 52nd, 56th, 57th, 58th and 59th seconds of the 57th min.
29th, 53rd, 56th, 57th, 58th and 59th seconds of the 58th min.
29th, 54th, 56th, 57th, 58th and 59th seconds of the 59th min.
29th, 51st, 52nd, 53rd, 54th, 55th, 56th, 57th, 58th, and 59th seconds of the 60th min.

The signals from the observatory control a special form of telegraph repeater in the Washington office of The Western Union Telegraph Company and in the

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[1] Western Union Telegraph Company, 60 Hudson St., New York, N. Y.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

Washington office of the Postal Telegraph-Cable Company, so that daily, commencing at 11:57 a.m. Eastern Standard Time, the beats are repeated and transmitted to all of the large telegraph centers. At each of these, the incoming lines carrying the beats are similarly connected to time repeaters which repeat the signals into a number of telegraph circuits covering the territory served by that central office. In this distribution, the beat signals are connected to more than 200,000 miles of telegraph line wires.

The signals indicating the time determined as correct by the Naval Observatory are thus automatically repeated throughout the United States and made available at all points having local master clocks controlling time functions so that these devices may be corrected daily for any error which may have accrued during the preceding twenty-four hour period. All the other distributions of time described in this paper are on a local basis, that is, controlled by a local master clock serving a city or other territory not exceeding a few miles radius.

JEWELERS' BEAT SERVICE

The jewelers' beat service mentioned at the beginning of this paper is quite similar to the noon beats. Each city or local territory in which this service is given is provided with a master clock equipped with the necessary contacts and with auxiliary equipment for transmitting time beats. These signals are transmitted to circuits upon which are connected in series a number of telegraph sounders installed in the offices of the jewelers, watchmakers, and others subscribing for the service.

In general, the code followed for this service differs from that described above for the noon beat signals. The local master clocks are equipped with contacts for sending beats in accordance with a code which divides each hour into five minute periods, the first of which commences at the exact hour, with the exception of a silent minute following the exact hour. Beats or impulses are transmitted after each second except the 29th second and the last five seconds of each minute. During the last minute of each five minute period, the beats indicating the end of the 50th to 54th seconds are also omitted, making the final silent period of ten seconds duration.

CLOCK SYNCHRONIZING SERVICE

The most extensive distribution of correct time carried on by the Telegraph Company is the automatic correction of clocks. Each of these clocks is provided with an electro-magnetic device by means of which the hands may be set to a predetermined time. This mechanism is independent of the motive power moving the clock hands and is thus applicable to practically any of the general design and types of clock mechanism. A large majority of the clocks actually in service in the system described have mechanisms of the spring-driven pendulum-controlled type, provided with a local battery and an electric motor which automatically keeps the driving spring properly wound.

A number of the clock magnets is connected in series on a line wire and all such line wires serving the local territory are carried to a central or control office. Once each hour, there is sent from the Central Office a "synchronizing" impulse which operates the magnets and sets the clock hands to indicate correct time. There is in use a number of methods of distributing the synchronizing impulse so as to make all clocks conform to the correct time, and of insuring against interferencee with the service.

This paper describes the latest of these arrangements, designed for the larger cities, in which contacts on a local master clock control a machine equipped with a large number of contacts, each pair of which connects a line circuit to a source of power at the precise hour. This synchronizing machine is motor driven and as constructed at present, is capable of handling 120 line wires, each of which may connect together groups of approximately 60 clocks. The synchronizing machine is provided with many special features to safeguard the clock service and with adjusting arrangements by means of which the time as transmitted by the machine may be made different from that indicated by the master clock controlling it. This latter feature permits correction for any slight master clock error with no readjustments of that device. It is only necessary to regulate the master clock by means of weights on the pendulum when the accumulated error exceeds four seconds. In this way, frequent regulations and consequent irregularity in the operation of the master clock are eliminated.

Included with the synchronizing arrangements at the Central Office are automatic means for testing the clock circuits. These are embodied in a motor-driven test machine which is started by auxiliary clock contacts at predetermined intervals. When the machine is thus started, it connects successively to each line certain electrical testing circuits which compare that line with a simple artificial line of correct characteristics. The mechanism is so arranged that when a line under test varies considerably from the artificial line, a printed record is made on a paper tape of the designation identifying that line.

The two machines are mounted upon a 22 in. by 26 in. shelf attached to a metal cabinet 26 in. by 18 in. by 81 in. high. This cabinet houses the auxiliary apparatus required in the operation and control of the machines and is provided with a jack panel at which each synchronizing clock circuit is made available for such tests and trouble analysis as may be desired. The switchboard is provided with a volt milammeter and other equipment to facilitate such work. The completed switchboard is shown in Fig. 1.

While the above describes in general the synchronizing clock service, a more detailed description of the clock mechanism, and of the electrical circuits of the synchronizing and testing machines may be of interest.

Clock Mechanism. Fig. 2 shows the mechanism applied to a clock for resetting its hands if the clock is in error when the synchronizing pulse is received.

Regular clock mechanism connected to the shaft *G* causes that shaft to make as nearly one complete revolution per hour as practicable. The shaft is connected through a friction coupling to the double disk *Q* and hence the latter is driven by the clock mechanism but may be moved independently. To this disk is mounted the minute hand of the clock and the gear train driving the hour hand one revolution for each twelve revolutions of *Q*. A lever *D* is so arranged that when the electromagnet *A* is energized its armature *B* causes *D* to strike projections on *Q* and thus move *Q* and the clock hands controlled by it to a definite position; *i. e.*, that corresponding to the exact hour. Normally, the synchronizing lever *D* is locked in the position shown by means of the locking lever *H*, which carries a pin *S* that engages the catch *T* on *D*. This arrangement prevents any current which might be used for testing during other portions of the hour from operating the synchronizing mechanism. At three minutes to the hour, the pin *U* on the disk *Q* engages the lever *H* and moves the locking pin *S* from under the catch *T*. The mechanism is now free to operate when the synchronizing current passes through the magnet *A*. The pin *U* remains engaged with the locking lever until three minutes past the hour, at which time it allows the lever *H* to drop back in position and lock the synchronizing mechanism until the next hour.

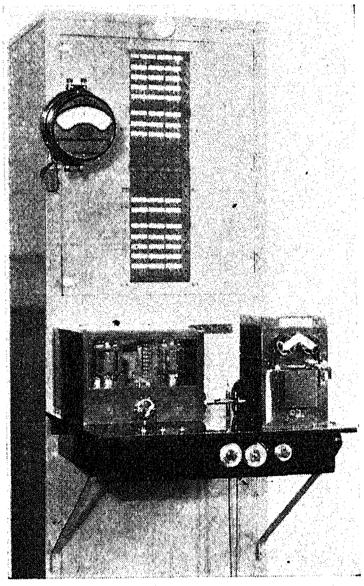

FIG. 1

A somewhat different mechanism is generally used to set the second hand of a clock and that is also illustrated in Fig. 2. The second hand is regularly attached to the heart-shaped cam *R* which is driven through a friction coupling by the clock mechanism. When the synchronizing magnet is operated, the lever *C* is pressed against this cam to advance or retard the second hand to a position indicating the beginning of a minute.

Synchronizing Machine. The clock magnets are grouped in series on line circuits which are connected to contact springs on the synchronizing machine. Each machine is provided with two cams for closing these line springs and for controlling the application of power for the transmission of the synchronizing impulse. These elements are illustrated in Fig. 3 in which *B*, *C* and B_1, C_1 represent clocks on two lines connected to the springs *LS* and LS_1 arranged for operation by the insulated cam. Around each cam are disposed ten layers each containing six sets of contact springs, the cam thus controlling contacts for a total of 60 lines. One of the cams is also arranged to operate a pair of "synchronizing contacts" shown in Fig. 3 at *SC*. Once each hour, the master clock causes the cams to revolve through one-third of a revolution. As they do so, each set of line springs is closed and then restored to its normal condition except that they rest upon a similar portion of the cam 120 deg. farther advanced. The movement of the cam also closes the spring *SC* which through the operation of the power relay *PR* connects the line wires to a source of operating current *MB*. It should be noted that the raised portion of the cam which passes under

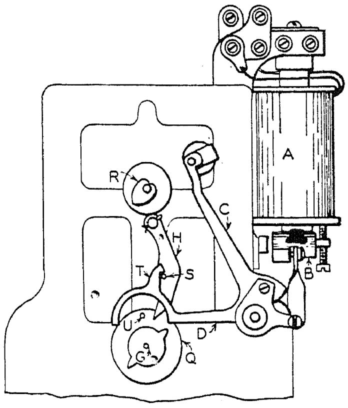

FIG. 2—CLOCK SYNCHRONIZING MECHANISM

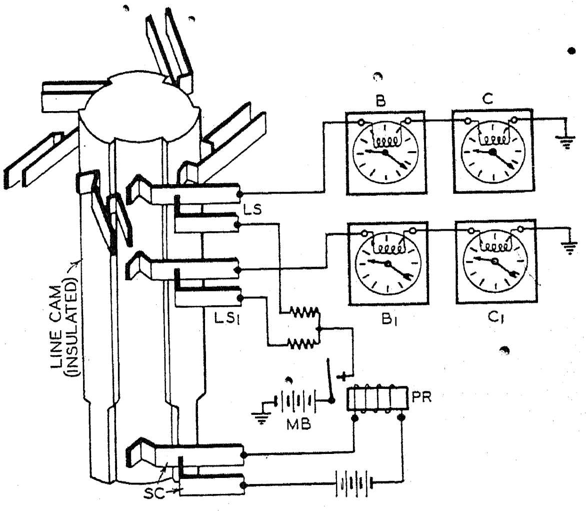

FIG. 3—ELEMENTARY CLOCK CIRCUIT

the springs *SC* is shorter than that under the line springs. This insures the line circuits being closed before power is applied and that they remain closed until the power relay has been deenergized and has removed the power connection. In practise, a number of power relays is used, each controlling the current to a group of lines varying in number from 12 to 30, depending upon the potential used.

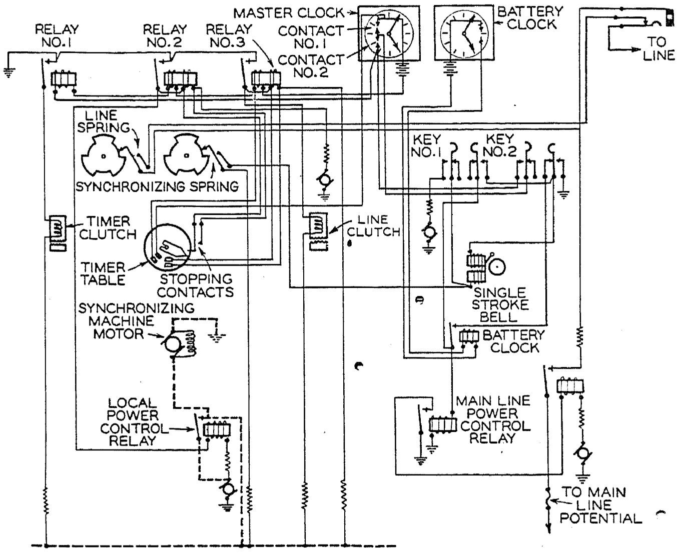

FIG. 4—ELECTRICAL CIRCUIT OF SYNCHRONIZING MACHINE

The cams are driven by an electric motor through gear trains so controlled by a master clock that once each hour the cams turn through one-third of a revolution. The time required for this movement is four seconds, and of this period approximately three-quarters of a second is used in transmitting the synchronizing impulse.

Fig. 4 shows the electrical circuits of the synchronizing machine. The master clock and battery clock shown control the machine in transmitting the impulses. At five seconds before the hour, the battery clock contacts close and remain closed until five seconds after the exact hour. These contacts through suitable relays connect the machine to the source of operating current for the clock line circuits. The master clock contact shown as Contact 2 closes at approximately 15 seconds before the exact hour and opens at five seconds before the exact hour. The closing of these contacts starts the machine in operation and sets the various control parts to a correct initial position. Through suitable clutch mechanisms, the opening of Contact 2 causes a timing arm to revolve and connect to contacts on a timer table. The timer table contacts control a "line clutch" mechanism so that during the interval between the closing of the two pairs of these contacts, the cams are turned and transmit the synchronizing signal. By manual adjustment of the position of the timer table contacts, the time relation between the connections to them and the opening of Contact 1 may be varied to compensate for errors in the master clock.

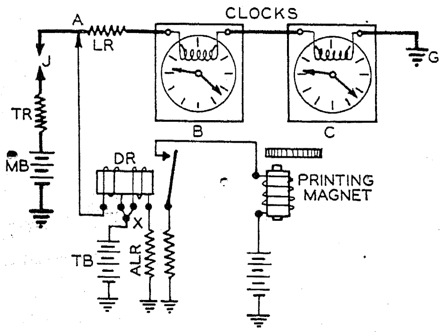

FIG. 5—ELEMENTARY TESTING MACHINE CIRCUIT

Two keys are also shown in Fig. 4. Key No. 1 when depressed operates the main power control relay, thus permitting an attendant to make tests requiring line battery. Key No. 2 and an extra set of master clock contacts (shown as Contact No. 1) are provided to facilitate comparison of signals from the synchronizing equipment with standard observatory time. Pressing Key No. 2 before the end of the 58th and 59th minute of each hour causes the entire mechanism to function in a normal manner except that the synchronizing impulse is not sent to any line but is sent to a bell, the stroke of which may be compared audibly with the corresponding signal from the Naval Observatory.

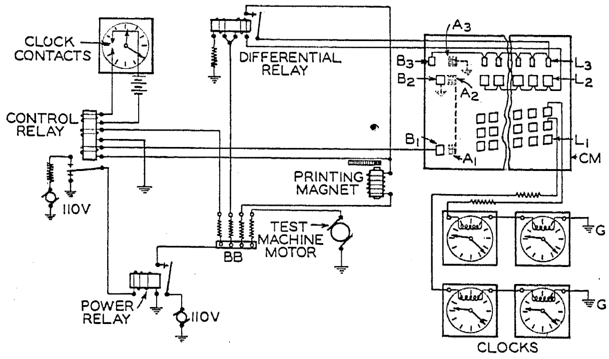

FIG. 6—CIRCUIT OF TESTING MACHINE

Testing Machine. In Fig. 5 is shown in heavy lines a schematic representation of a clock circuit and in light lines the electrical elements of the testing arrangement used in checking the quality of the clock circuit. The tap resistance *TR* is fixed at approximately one ohm per volt of operating potential of the main battery *MB*, while resistance *LR* is selected so that the current during the synchronizing impulse will be 250 milliamperes. The resistance from *A* to *G* will therefore be the same for all circuits operating from a given potential. A fault on the clock circuit would either decrease or increase this resistance value. The testing arrangement of Fig. 6 is designed to indicate when such a change from the normal resistance exists.

The center point *X* of a differentially wound relay is connected to the testing battery *TB*. One winding of the relay is connected to the line at A and the other winding is connected to the artificial line resistance ALR. Resistance ALR is adjusted to equal the nominal value which should exist for all circuits operating at the potential of the main battery MB. If the resistances in series with the two windings of the differential relay are equal, the current flowing from the battery TB divides equally and the relay will not be operated. Should the line resistance vary from that of the standard ALR as a result of a ground, open or other fault, the currents in the two relay windings will be unequal and the relay will operate the printing magnet.

Fig. 6 shows schematically the electrical circuits of the testing machine. Once each hour, the motor controlling this machine is started by the closing of the clock contacts. A brush shown at A_1 moves spirally and contacts with the line studs L_1 while brushes A_2 and A_3 contact with studs L_2 and L_3. In making these connections, each line is connected to one of the differential relay windings and thus compared with the artificial line or standard. If, as a result of an improper line condition, the relay is operated, the printing magnet causes an impression to be made upon a paper tape to indicate the position of the Brush A_1 at the time the printing magnet has operated. This identifies the line in trouble to an attendant who may then make the necessary tests to determine the nature of the fault.

Summary

The proper distribution of time information requires a continuity of accurate service. This the systems described aim to accomplish by the subdivision of the distribution into local areas each entirely independent of all others. The equipment in each area is capable of rendering an accurate time service to subscribers in that area for extended periods without assistance from other sources and by the nationwide dissemination of standard signals may be held to a negligible variation from true time. The equipment and operating methods are such that troubles occurring within the area will produce a minimum of interference with the accuracy of the distribution within that area and will be promptly indicated so that they may be remedied.

Discussion

For discussion of this paper see page 550.

Synchronous Electric Time Service

BY H. E. WARREN[1]

Associate, A.I.E.E.

BEFORE the beginning of the present century, there were a few individuals who realized the possibility that commercial alternating current might be used in some manner, by means of its recurring pulsations, to measure time; but dreams of this nature did not materialize in practical performance.

Up to 1916 there was no commercial a-c. system in existence in which generator speed was regulated with sufficient accuracy so that the current could be used for the purpose of indicating time by means of synchronous clocks. At that time two devices were brought out which in the succeeding years made possible commercial a-c. time service.

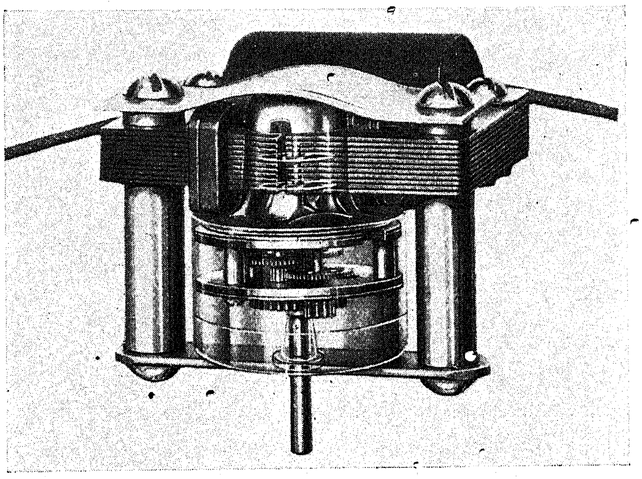

FIG. 1—TYPE B TELECHRON MOTOR AND GEAR TRAIN

The first of these devices was a self-starting synchronous motor which utilized residual magnetism to enable a rotor to start and run at synchronous speed. This type of motor differs from all the common forms of a-c. motors in that eddy currents, induced either in the mass of the rotor or in some form of winding upon the rotor, are not utilized to start the rotor from rest. In place thereof, the rotor is made of such permanent magnetic material and is so proportioned with respect to the field that remanent magnetic poles are set up in the rotor material by the a-c. magnetism of the field in such a manner as to cause the desired motion. Such a motor is sometimes known as a hysteresis motor. Fig. 1 shows a phantom view of a recent model of this motor.

The hands of a clock were connected by a gear train with the first of these self-starting synchronous motors and this clock, supplied by current from the local electric light system, was compared with standard time, at intervals, over a period of several weeks. The new motor-driven clock proved to be a very inferior timekeeper, developing daily errors as great as five or ten minutes, thus proving the need of greater accuracy in the average frequency of the power supply if it were to be used for timekeeping.

By observing these daily errors, it was easy to compute the average value of the system frequency. There are 1,440 minutes in a day and the gearing of the clock was such that, if the frequency had been exactly 60 cycles, there would have been no time error. A 1 per cent error in average frequency (0.6 cycles at 60 cycles) would mean 1 per cent error in the time of the clock, which amounts to 14.4 minutes in a day. The actual computed error in the average frequency, as determined by these observations, is shown in the upper curve of Fig. 2 which was originally published in the article "Clocks in the Field of Electric Light Appliances" in the *Proceedings* of the N.E.L.A. for 1917.

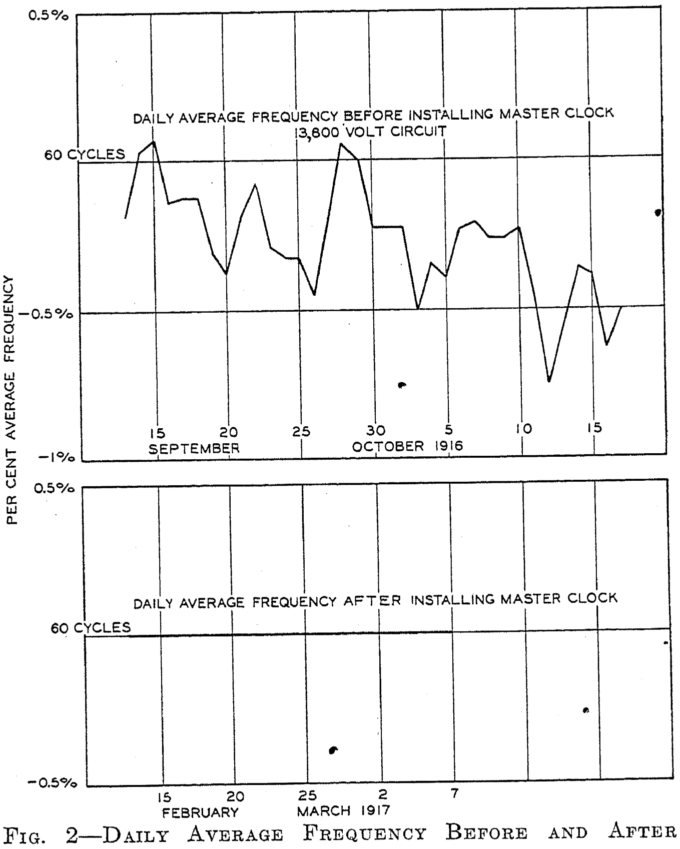

FIG. 2—DAILY AVERAGE FREQUENCY BEFORE AND AFTER INSTALLING WARREN MASTER CLOCK

The second device consisted of an indicating instrument that would show to a generating station operator, directly in terms of time, deviations of average frequency from standard. Guided by this instrument an operator could so regulate the speed of his generators as to maintain such small deviations from standard average frequency that the alternations could be used to measure time for all ordinary commercial purposes.

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[1] Warren Telechron Company, Ashland, Mass.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

Fig. 3 shows the dials of a type of master clock which is in common use. The large dial in the center has two hands, one colored black and the other gold. The black hand makes a revolution on the dial every five minutes; its rate of motion being regulated by an accurately adjusted pendulum beating seconds. The gold hand is driven by a gear train directly from one of the Telechron self-starting motors. The gear ratio is such that, when the average frequency is correct, this hand will also make one revolution in exactly five minutes.

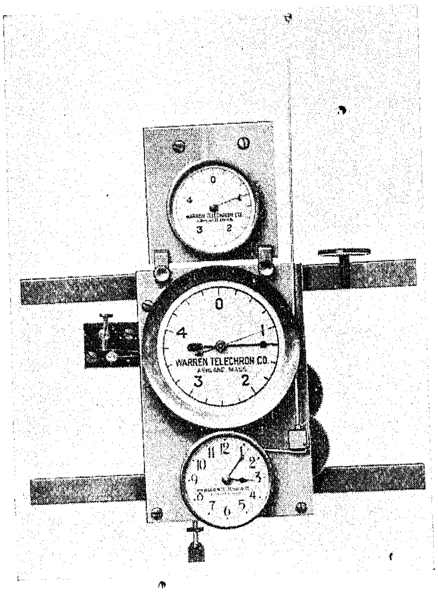

Fig. 3—Operating Dials of Type A Master Clock

The two hands are started off together. So long as the average frequency remains correct, they will keep together and there will be no visible angle between them. An error in the average frequency will cause the gold hand to move slightly faster or slower than the black hand and they will become separated. The angle of separation will be a direct visible measure of the error in the integrated alternations, which may be conveniently called the system time. If the gold hand is ahead, this will show that the system time is fast; if behind, that it is slow.

An error in the average frequency as small as 1/60th of a cycle will cause a spread of one second between the two hands of the master clock, in one hour. If such an error continues, the spread will continue to increase as time passes. The accuracy of the indication is limited only by the time keeping precision of the pendulum itself. When carefully regulated this is better than one second per day, or an absolute precision of about one part in one hundred thousand.

When the operator, at the generating station which is regulating frequency, adjusts the turbine governors from time to time so as to keep the black and gold hands of the master clock together, he automatically causes all other synchronous motor clocks on the same system to keep correct time. The lower curve of Fig. 2 shows the great improvement in the average frequency value which was obtained after the first master clock installation.

The standardization of average frequency is of great value to the power companies in the following ways:

1. By improving the service furnished to manufacturing establishments, ensuring more uniform speed of motor-driven machinery and as a general result assuring the quality of the product.

2. By facilitating interconnection with other power companies which have correspondingly standardized their frequency.

3. By enabling power companies to use synchronous motor movements in their maximum demand meters and graphic recorders, thereby obtaining more accurate and synchronized records at lower cost of maintenance.

Before master clocks were available, the inherent errors in frequency indicators were sufficient to handicap operators on interconnected systems in their efforts to equalize the frequency preparatory to synchronizing the two systems, and to lead to controversy as to the true value of the frequency after the two systems were synchronized.

Master clocks have been generally adopted by the power companies of the United States for the purpose of regulating frequency. At the present time, it is estimated that the regulation of the current to over 90 per cent of the consumers' meters in this country is guided by these master clocks.

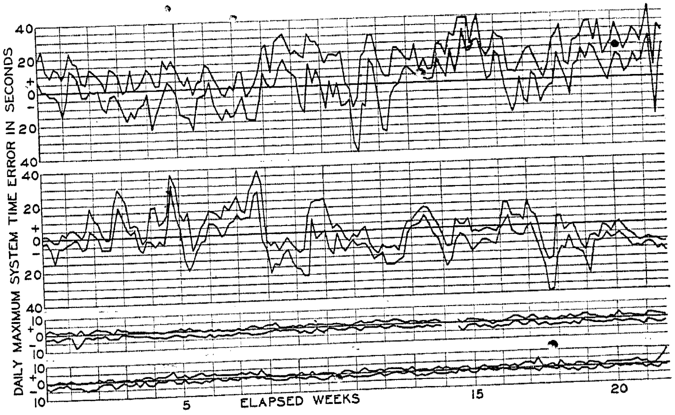

Fig. 4—Maximum Daily System Time Errors of Four Large Power Systems

The accuracy which may be expected at present from a synchronous electric clock is indicated by the records shown on Fig. 4. The records shown on this chart are taken from four typical very large interconnected systems east of the Mississippi River. The ordinates of the curve in each case are seconds deviation from standard time, and the abscissas weeks over a period of five months. The solid line through the center of each curve represents standard time, and the two points plotted for each day at the top and bottom of the band represent the maximum range of the deviations from standard for that day. The widest deviations shown are 44 seconds fast or slow, which means that a synchronous clock on these systems set absolutely accurate with standard time did not, through the period plotted, indicate incorrect time more than 44 seconds fast or slow. The records of the two systems which show the smallest departure of standard time indicate that a clock operating on these systems would never be more than 9 seconds fast or slow. There are few uses of time which justify higher accuracy, and even the worst of these records is far superior to that of previous types of commercial clocks.

Some power companies throughout the country may be giving better than the best time service indicated on this chart. Probably there are some permitting wider variations. Practically all power companies, however, are giving sufficiently accurate frequency regulation to provide far better time on synchronous clocks than on commercial clocks previously available.

All factors in power system development are tending to require continued improvements in frequency regulation. As a matter of fact the very large interconnected systems have so much momentum stored in rotating equipment (generators, motors, etc.,) that they will give, when properly adjusted at the beginning of a flat load period, almost perfect time with no adjustment of governors for speed. Governor adjustment is necessary, however, over the irregular portions of the load curve and much work is being done at present to divide the burden of this regulation on large interconnected systems between the various power companies by the use of automatic frequency control to supplement the master clocks. When perfected and applied these instruments will undoubtedly reduce the system time error to insignificant values of the order of 1 or 2 seconds.

About 1928, twelve years after alternating current regulated by means of master clocks illustrated in Fig. 3 became available, there appeared on the market the first of several new varieties of synchronous motors suitable for use in clocks. These motors may be divided into two classes; non-self-starting and self-starting.

The *non-self-starting motors,* of which there are several kinds, generally have toothed soft-steel rotors. The usual stator construction is bi-polar with teeth on each pole corresponding to the spacing of the rotor poles. In all multipolar motors of the common varieties the speed is found by dividing the number of current alternations per minute by the number of rotor or stator poles. In nearly all cases no attempt is made to introduce a rotating component into the field. A common arrangement of rotor and stator is shown in Fig. 5. On account of the difference in magnetic reluctance between rotor positions, where the poles do or do not correspond with the stator poles, a strong reactance torque exists when the rotor is at rest. This tends to prevent the rotor from starting; but when running at synchronous speed, tends to keep it at that speed.

It is somewhat difficult to spin such a rotor by hand so as to cause it to lock into synchronism with the alternating field, unless some means are provided which will permit a sudden phase shift of the rotor poles as the rotor speed passes through its synchronous value. Among the satisfactory schemes for accomplishing this result are the use of loosely coupled fly-wheels on the rotor shaft or spirally arranged rotor poles with means for the axial shifting of the rotor in the field. Mechanical launching devices are also sometimes used automatically to bring the rotor up to approximately synchronous speed when started by hand.

The *self-starting synchronous clock motors,* which have appeared within the last few years, utilize a-c. fields with rotating components brought about by the use of shading coils. Most of these motors have multipolar rotors with some form of squirrel cage winding or a copper element wherein Foucault currents are induced by the rotating field. Such motors get their starting torque, like ordinary induction motors, through the reaction of the Foucault currents in the field. Synchronous driving torque, in these varieties of self-starting motors, is produced by polar projections of the steel rotors or by a separate polar element loosely coupled to the rotor shaft. One variety of the recent self-starting synchronous motors utilizes remanent magnetism for starting and for synchronous operation.

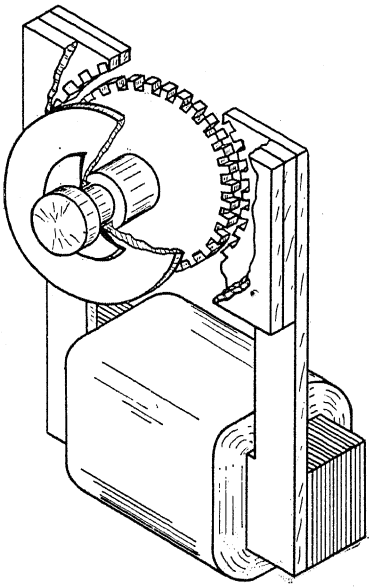

FIG. 5—NON-SELF-STARTING SYNCHRONOUS CLOCK MOTOR

Any of the rotors heretofore described may be used to drive the hands of a clock through a simple train of gears. Two forms of such gear trains are in common use, one composed entirely of spur gears and the other utilizing worm gears and worms. On account of the greater efficiency of transmission, a spur gear train is better for large clocks but for small clocks other mechanism is satisfactory. Frequently, the high-speed gears of a spur gear train are made of non-metallic material so as to run more quietly and the motor or the whole clock movement is usually mounted on cushioning material so as to reduce the a-c. hum. Some clocks are much quieter than others depending upon the rigidity of the field laminations and the design of the moving parts.

Any one of the motors mentioned is adequate to drive the hands of a clock dial 12 in. or less in diameter. The actual output required for this purpose is exceedingly small. If the clock hands are balanced and the gear train is fairly efficient, not more than one-twenty millionth of a horsepower is needed for a 12. in. dial. The power output of the various motors ranges from less than one-millionth of a horsepower to more than one-ten thousandth of a horsepower. The larger amount has been found adequate to drive tower clocks ten or twelve feet in diameter. The power output of these motors seems exceedingly small as compared with commercial motors which are used for other purposes but it is, nevertheless, fully adequate for this time keeping duty and is many times greater than the power of ordinary spring clock motors.

The electrical input to these motors is generally in the neighborhood of two watts, but as the power factor is low, the volt amperes will vary from three to six. Obviously, the efficiency of these motors is very low indeed, amounting to a fraction of 1 per cent. However, the power used is so little (17 kw-hr. per year or approximately $1.00 worth) that this feature is of minor importance.

Most synchronous motor clocks are provided with some form of indicator visible through a small hole in the dial. In clocks which are not self-starting, this indicator shows by its continuous motion that the clock is in operation. After an interruption in the current, the motion of the indicator stops permanently. In self-starting motor clocks of the Telechron brand, the indicator, back of the dial, shows a steady white target as long as the current flows. This target changes to red and so remains after an interruption in the current supply, although the clock, of course, starts and continues to run as soon as the power is restored.

Inasmuch as most kinds of synchronous motor clocks will either stop permanently or show errors after current interruptions, it is reassuring to know that system interruptions are rare in our thickly settled districts. Often, these clocks continue to show correct time within a very few seconds for many months, even for years. In order to provide against the rare stopping of the hands because of interruptions, clocks have been available for many years which have auxiliary spring movements. These movements appeared more necessary in the beginning before alternating current service had become as reliable as it is today.

The successful operation of synchronous motor clocks is quite independent of certain influences which are detrimental to other clocks such as level position, vibration, and ordinary temperature variations. Extreme temperatures near zero fahrenheit will stop some kinds of clock motors, but other varieties will operate well below zero.

Application of these small clock type synchronous motors to many devices and instruments has been going on ever since these motors became available. The first users were the power companies themselves who had many instruments and meters, especially those of the maximum demand type, which required time keeping elements. The spring clocks, which had been used for these purposes, were inaccurate, troublesome, and costly to maintain. This was especially true because they needed frequent winding and were often located at distant points. The instrument load was sometimes too great for a spring clock to handle with accuracy. As soon as the motors were substituted for the spring clocks there was a great gain in accuracy and reliability and the costly winding was eliminated. It then became possible to redesign the instruments so as to give still better performance.

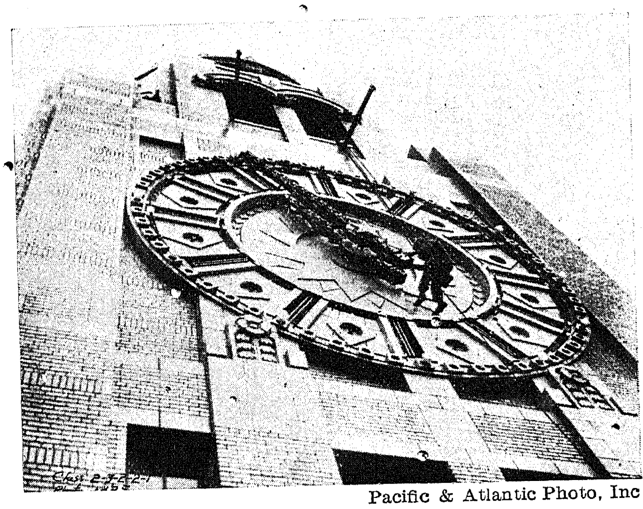
Pacific & Atlantic Photo, Inc.

FIG. 6—DIAL OF LARGE SYNCHRONOUS TOWER CLOCK

Gradually, nearly all the makers of instruments began using synchronous motors in place of spring movements until the list of such motor-driven instruments is a nearly complete catalog of devices in which the measurement of time is involved. The largest class of these motor-driven instruments is perhaps the graphic recording type. The load of driving a paper chart, especially of the strip type, is very great for a spring clock, but comparatively trifling for a synchronous motor. Moreover, the motor can handle a roll of paper so as to make a continuous record for weeks or months without regard to winding. There are also many kinds of instruments which do not make a record but serve to control or actuate, and synchronous motors have advantageously been incorporated into certain forms such as time switches, temperature controllers, traffic signals, etc.

There is a large and important class of timing devices exclusive of ordinary clocks. Among them are numerous kinds of time and cost recorders, program machines, time stamps, etc., wherein synchronous motors have gradually been replacing spring clock movements. This change has made it possible, in many cases, to eliminate local master clocks, batteries, and expensive wiring.

In building installations, where Telechron clocks, or timing devices driven by Telechron motors, are numerous it is customary for the customer to provide a so-called "resetting device" or else a duplicate source of alternating current energy to guard against the effect of interruptions in the supply of current. In all such installations, a separate wiring system is used and this is connected with the public service system through a suitable switchboard. The resetting device automatically measures the duration of an interruption and, after the current is restored, applies higher frequency or otherwise speeds up the clocks until they are again correct.

The problem of driving very large tower clocks by means of self-starting synchronous motors is much less difficult than that formerly involved in the use of conventional pendulum tower clock movements. It is necessary, of course, for the gears and shafts to be very strong because the stresses due to the great weight of the hands and the pressure of the wind are rather high. Fig. 6 shows a tower clock movement installed on the Williamsburgh Savings Bank at Brooklyn, N. Y. where each of the four dials is 26 ft. in diameter and the hands alone weigh 700 lb. per pair. The hour hand sleeves of this clock are of steel and their outside diameter is 4 inches. The power required to drive these hands during a heavy wind is less than 0.001 hp. The motors used have a very large surplus power margin.

The use of synchronous motor clocks and timing devices has increased since its conception in 1916 with extraordinary rapidity, especially during the last few years. According to estimates more than one million two hundred thousand synchronous motor clocks were sold during 1930.

There seems little doubt that, during the coming years, spring and weight driven clocks will be replaced in large measure by the new a-c. timekeepers, which have proven greatly superior in every important respect.

The author wishes to express in this paper sincere appreciation of the generous aid which has been given by numerous executives and engineers to promote the attainment of commercial time service by means of alternating current—especially those men with far sighted vision who realized the possibilities before there were any actual results.

He would also express thanks to Mr. George M. Keenan, Superintendent of the Pennsylvania-New Jersey Interconnection, who has cooperated in the preparation of this paper.

Discussion

TIME AND ITS MEASUREMENT
(Brown)

MODERN DEVELOPMENTS IN PRECISION CLOCKS
(Loomis and Marrison)

TIME SERVICE OF THE U. S. NAVAL OBSERVATORY
(Hellweg)

TIME SERVICES OF THE TELEGRAPH COMPANIES
(Janson)

SYNCHRONOUS ELECTRIC TIME SERVICE
(Warren)

L. S. Harrison: This discussion presents an important development in the particular field of institutional electric time systems—where a multiplicity of clocks is designed to influence the coordination of activities in schools, hospitals, office buildings, railroad terminals, industries, etc. It is a master clock system which hourly supervises the uniform agreement of electric secondary units.

This system which has had widespread application during the past seven years combines certain basic principles of practical timekeeping in a unique manner. The most reliable means of obtaining commercial accuracy is with a high grade 60-beat pendulum regulator. A well designed unit, properly mounted and free from vibration is capable of rating within two to three seconds per month of standard time without resetting, or an error of approximately 1 in 900,000.

The practise of all of the best known systems of electric clocks to employ, somewhere in the system layout, a 60-beat master regulator, confirms this basic necessity of time system engineering. However, the high accuracy of a master clock does not by itself, assure the successful transmission of system time to the various individual secondary time indicators because to complete its function as a time governor, a master regulator must do two things:

1. It must provide an electrical operating medium to propel secondary clocks at an accurate constant rate of advance, and
2. It must transmit to each secondary clock, periodically, a point origin of time on which to establish such constant rate.

It should be borne in mind that secondary time units are not of themselves, clocks; they cannot keep time by themselves but must depend upon an uninterrupted supply of accurately timed pulses and they, hence, inherently reflect as any other meter reflects, conditions in everything which may contribute to or affect their operation; yet these secondary clocks are the real points of utility in the time system and the medium through which the master clock delivers its service.

Further, any clock indicator which depends initially for its timekeeping quality on original setting by hand, obviously does not receive its accuracy from the master clock and cannot be said to do so.

The master clock, which is automatically self winding, transmits its own accuracy over a wiring system wholly devoted to time service transmission to the secondary timepieces both as to rate of advance and as to a point origin of time by means of a periodic time check.

The secondary units are designed electrically to sense their own agreement with system time once an hour and automatically to accept only the exact required number of driving impulses during the hour. These secondaries may be in the form of devices for signaling and recording of time as well as for indicating.

Since its introduction in 1924, this system has been placed in use by large numbers of schools, colleges, banks and office buildings, and industries throughout the United States and Canada. Three outstanding recent installations are the Pennsylvania Railroad Philadelphia Terminal Development, Cincinnati Union Terminal and the Cleveland Union Terminal Group.

A short description of this system is placed on record for reference as follows:

A description of the Hourly Supervised Electric Time System is quite readily grasped when two of its operating conditions are laid down, namely:

1. Each secondary clock (Fig. 1) individually connects itself to the supervising circuit at the 59th minute on its own indicating dial, permitting it to sense its own position with the correct system time.

2. The master clock shown in Fig. 2 provides supervising also during the last minute of the hour by a simple manipulation of the circuit. This supervision, being supplied for variable differences in error, must be handled over two of the three wires in the system, as follows:

(a) On one wire a suspension of impulses for a period of ten minutes before, so that any fast secondaries may be retarded which may have arrived at the last minute of the hour point ahead of the master clock.

(b) On the other wire extra impulses are supplied, during the last minute of the hour, in order that slow secondaries may be stepped ahead.

All units which are on time, are of course, in perfect step with the master clock and therefore do not partake of any of the effects produced by the above supervision. The third wire is a common return.

Philip Sporn: A reading of Mr. Warren's very interesting paper is likely to give the impression, particularly to one who has but a passing acquaintance with the subject, that the question of frequency control on the part of the operators of large power systems is a rather simple affair, involving, as it does not, very little more than the use of a master clock, and that such practise is very beneficial in the operation of a power system. Such, of course, is far from the case; I would, therefore, emphasize some of the dark shadows in the picture in order to give a representation more nearly in accordance with the facts. In connection with Fig. 2 of the paper there is shown a graph of daily average frequency before and after the installation of a master clock. It ought to be pointed out, however, in this connection that while a master clock permits the speeding up or slowing down of system frequency after a certain period has elapsed and the error had been allowed to accumulate, that the straight line curve does not truly represent system frequency.

Nor is the very jagged and irregular line typical of what can be done without the use of a master clock. The fact of the matter is that during the period covered by the upper curve in Fig. 2, the frequency meters available were of such a crude nature that the results shown are about all that could be expected. On the other hand, the use of a modern frequency meter with its high degree of accuracy without any master clock whatsoever could be made to give substantially as good an average time curve as is indicated by the lower curve in Fig. 2 of the paper. I think it is worth while stressing this because otherwise we may get the old familiar *post hoc, ergo propter hoc* train of reasoning started in connection with the very complicated problem of frequency control and we all know the subject is complicated enough as it is without having to encounter that.

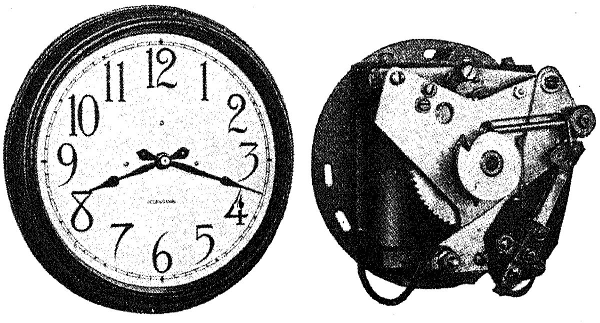

Fig. 1—International Secondary Clock and Movement

In Fig. 3 is shown a frequency chart taken over approximately 12 hours on our middle western interconnected system in June 1925. A rough examination of the chart will indicate that the *instantaneous* frequency is pretty ragged. At the same time it will be noted that at no time during the 12 hours has it been worse than ½ cycle plus or minus the normal of 60 cycles. It will also be seen that the average frequency is substantially perfect, or if it is not, over this 12-hour period, there would be no difficulty by governor manipulation, say over the 24-hour period, to bring it to perfection, as far as *average frequency* is concerned without in any way altering very much the characteristic of the frequency curve.

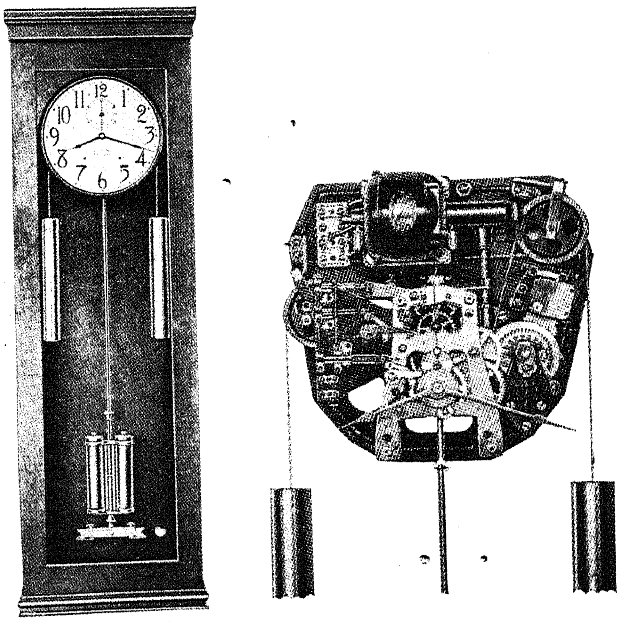

Fig. 2—International Master Clock and Movement

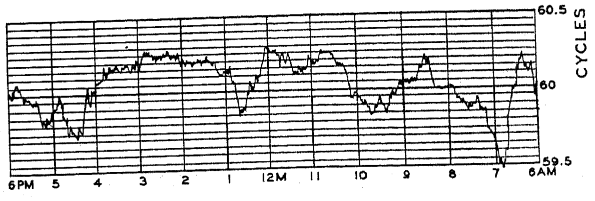

Fig. 3—Frequency Chart Taken in June 1925

Showing excellent average frequency but wide variations of instantaneous frequency

This again is shown because of the claims made in the paper by Mr. Warren that the standardization of average frequency is of great value to the power companies in the ways enumerated:

1. "By improving the service furnished to manufacturing establishments, ensuring more uniform speed of motor-driven machinery and as a general result assuring the quality of the product."

It will be seen from Fig. 3 that so far as giving good service to processes requiring accurate speed, the frequency represented by Fig. 3, although having excellent average characteristics, is decidedly unsuitable and offers no improvement over some other frequency that might have the same characteristics, but which might be floating above or below the 59-cycle line as a base.

2. "By facilitating interconnection with other power companies which have correspondingly standardized their frequency."

Again referring to Fig. 3, it will be noted that a frequency of the type shown there is not adequate in a large interconnected system although again it may be perfect from an average frequency standpoint. The attempt to regulate frequency to make satisfactory interconnection is much more complicated and brings with it a train of many other problems, some of which will be mentioned later.

3. "By enabling power companies to use synchronous motor movements in their maximum demand meters and graphic recorders, thereby obtaining more accurate and synchronized records at lower cost of maintenance."

An examination of Fig. 3 will again show that a considerable error can be introduced in the timing of devices utilizing the frequency of the type indicated there, even though the average frequency is perfect.

Mr. Warren again states that before master clocks were available the inherent errors in frequency indicators were a handicap in system operation, particularly in synchronizing different systems. It is, of course, a fact that two systems can have their *average frequency* almost perfect, and yet at the instant of synchronization be absolutely unsuited for coming together. The fact of the matter is that the average frequency has nothing to do with system operation, but that system operation was greatly facilitated and improved by the development of accurate frequency indicators. It is admitted, however, that it is highly desirable to maintain average frequency equivalent to the nominal frequency of the system as long as there is the slightest claim with regard to constancy of frequency. A master clock along with a time deviation recorder is, of course, very desirable as a means of checking the accuracy of the frequency recorders.

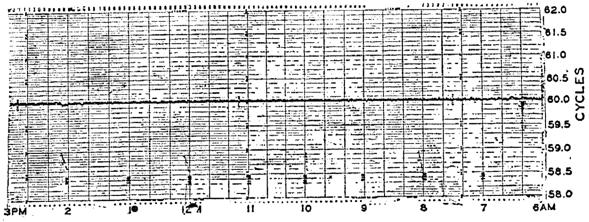

FIG. 4—FREQUENCY CHART TAKEN DURING MAY 1931

Illustrating to what extent instantaneous frequency variations can be reduced

Mr. Warren is, of course, on solid ground when he states that the factors in power system development are tending to require continued improvements in frequency regulation. These factors, however, it should be pointed out, are totally independent of and have no relation to the problem of supplying time and it is doubtful whether the advantages of being able to supply correct time would pay for the difficulties that are involved in maintaining such close frequency. He is not right, however, in saying that very large interconnected systems, because of the heavy momentum, etc., give almost perfect time with no attachment of governors for speed. In Fig. 4 there is shown a frequency chart taken on the same system which furnished the data in Fig. 3, the difference being that it was taken approximately 5 years later. It will be noted that the instantaneous frequency here is almost perfect, having a maximum deviation of approximately plus or minus 0.03 cycles. However, it was not obtained by merely bringing together a large power system and letting it run in parallel after the governors had been adjusted.

In Fig. 5, for example, there is shown a typical loading on one of the tie lines of the particular system tying together two large portions of it during a day when frequency of the order given in Fig. 4 was obtained. It will be noticed that over this particular line within a period of less than 2 hours this tie-line had its load increased from zero to about 95,000 kw., back to zero again with an actual reversal of flow, then up again to approximately 110,000 kw., and down to 20,000 kw. It is obvious that this particular line, in so far as it was intended to function as a steady deliverer of power somewhere close to its theoretical limits, was vitally handicapped by what it was doing in the way of maintaining system regulation so that a frequency shown in Fig. 4 could be maintained.

A more definite relationship between tie-line loading and time deviation on an interconnected system is shown in Fig. 6. It will be noticed that during the 2-hour period, the time deviation from true time was at no time greater than approximately 2 seconds, yet that involved carrying the tie-line through several very erratic and violent cycles of loading. Note, too, the definite relationship between time deviation and load picked up by the tie-line and furnished to the system:

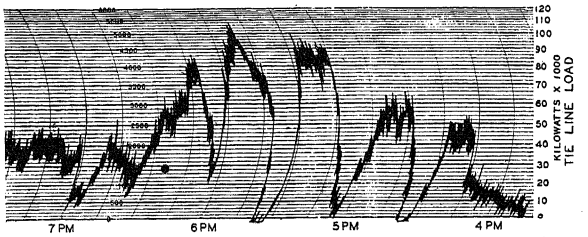

FIG. 5—TYPICAL EXAMPLE SHOWING THE EFFECT OF CLOSE FREQUENCY REGULATION ON THE LOADING OF TIE-LINE CONNECTING THE REGULATING STATION TO THE SYSTEM

For example, during the period from 5:00 p. m. to 5:20 p. m. load was coming on the system rapidly and the frequency control station was picking up this load as indicated by the tie-line chart in order to maintain frequency. At about 5:20 the upswing stopped and the regulating station backed off about 70,000 kw. in a matter of some 10 minutes. The process was again reversed at even a more rapid rate between about 5:40 and 5:50 p. m. These violent swings in load, of course, have their repercussion on the operation of the plant from which this tie-line is fed and operation under these conditions is not at all a simple matter. For that matter the entire problem of frequency regulation on an interconnected system is highly involved both from a technical and from an economic standpoint. It is important, therefore, that the impression be not allowed to remain that frequency regulation is rather simple; nor that it becomes more simple as more systems are thrown together in parallel; and that the mere installation of a master clock of the type shown in Fig. 3 of the paper will do the trick.

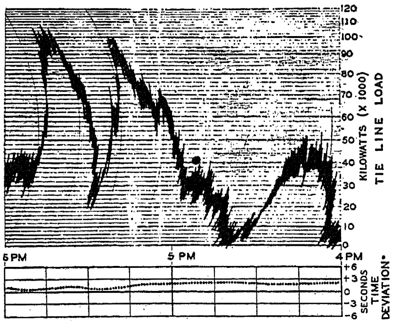

FIG. 6—TYPICAL CASE OF POWER TRANSFER ACROSS A TIE-LINE AS A RESULT OF HOLDING ACCURATE SYSTEM TIME

Time service should be a logical by-product of system operation; at the present time it appears safe to say that the ultimate method of operating large interconnected systems will be facilitated by extensive use of automatic frequency control devices and that it will be possible to obtain very accurate time service from such systems.

Considerable use has been made of synchronous motor clocks in indicating type and in round chart type demand meters, as brought out in Mr. Warren's paper and this has effected considerable economy and increased reliability in the use of these meters. There is, however, a need at the present time for a synchronous motor clock of sufficient power to drive the timing device as well as the reroll device in strip chart instruments. Some use has been made of synchronous motors in strip chart meters to date, but in practically all cases either an additional motor or a spring is used for the reroll. It is obvious that to replace these two units with single synchronous motors would simplify the construction and considerably lower maintenance. Such a device would be applicable to strip chart demand meters, wattmeters, ammeters, voltmeters, etc. In the case of voltmeters, it would probably be advisable to incorporate some type of carry-over device in order that interruption and duration of interruptions would be properly indicated on the chart.

R. J. Wensley: It is interesting to note that the first interest displayed by the central station industry in synchronous timekeeping was in the use of such methods for the integration of demand against definite time intervals. Such devices began to come into use some years before the distribution of time as a service was instituted by the power companies. As each demand interval was short, usually 15 or 30 minutes, and as the errors were not cumulative the variance from true time over the 24-hour period found in the frequency of all power systems did not affect the commercial application of such devices.

As Mr. Warren's paper is historical in character, it is perhaps in order to add a few comments to complete the historical review of the devices used in synchronous timekeeping. As the motor is the main problem in the secondary clocks, the various classes of motor and the origin of each are of some interest.

Self-starting synchronous motors suitable for clock application are of several varieties:

1. Hysteresis motors, fully synchronous.
    - (a) Smooth rotor hardened material.
    - (b) Toothed rotor softer material.
2. Reaction type, fully synchronous.
    - (a) Phase split by resistance.
    - (b) Phase split by capacitance.
3. Sub-synchronous motors.
    - (a) Two magnetic circuits and rotors.
    - (b) One magnetic circuit, two rotors.
    - (c) One magnetic circuit, one rotor.

Class 1 is the oldest as the smooth core hysteresis motor depending on the remanence of a hardened rotor was quite completely described by Nikola Tesla in 1887. The toothed core hysteresis motor depending on the reluctance of the slots for its synchronous characteristics first shows in the patent granted to Durfee, of Rochester, New York, about 1910.

Class 2 is so well known as to hardly need comment. The use of phase splitting capacitors and resistances to accelerate induction motors with salient poles to synchronous speed dates from about 1900. This type of motor has not been used to any great extent for timekeeping, though it is not at all impossible to design a motor of this type that would be suitable for some varieties of time service.

Class 3 dates back to 1890 when Elihu Thomson described a synchronous motor with an eddy current disk tending to run at high speed, while toothed magnetic structures on the rotor, acting as a magnetic escapement prevented operation above synchronous speed. Sub-synchronous motors are all of this type, differing only in the details of construction. Each of the three types listed consists of an induction element of higher speed characteristics and a magnetic escapement to determine synchronous speed. All of these types have the advantage of slower speed of rotation for a given size of motor. To counterbalance this advantage is the rather critical nature of the design due to the necessity for accurate balance between the induction and synchronous elements.

Mr. Warren mentions the difficulty he experienced in persuading the central stations to agree to frequency regulation. It is very interesting to note that the first inventor to file an application in this country on a complete timekeeping system was Mr. A. F. Poole, who, in 1914, filed on a system comprising a master clock regulator and secondary clocks with synchronous motors. Mr. Poole tried to bribe the power companies by showing a resistance connected in parallel with the secondary clocks to increase the energy consumption. It is very fortunate that the power companies did not need this added inducement even though the problem is not simple as has been quite well covered by Mr. Sporn in his discussion.

It might be well to mention at this time that the usual power system is not regulated with sufficient accuracy to permit of the use of synchronous timing of extremely short intervals. The momentary variation may amount to as much as a tenth of a per cent or more. This error would be too great to permit of the calibration of fine watches by stroboscopic methods using central station frequency as a standard. Many such problems are presented to the manufacturers and of course must be refused.

W. A. Dudley: Mr. Janson has described various time services which the telegraph companies furnish to their subscribers, and in this connection it may be well to mention another important time service which these companies provide for their own purposes.

Synchronized clocks identical with those furnished to subscribers are provided throughout the several operating rooms and offices of the telegraph companies and usually one or more of the clocks in each office is provided with cam-operated contacts which, through suitable relays, are used for the control of timing and dating stamps. The stamps are magnetically operated in equal steps to position type wheels for printing the year, month, day, meridian, hour and minute, and in some cases fractions of minutes. The principal use of the stamps is for timing of messages but they are also used for stamping employees' time cards and other supervisory purposes.

Since all of the time stamps throughout an entire telegraph system are controlled from clocks which are part of a synchronized time service as described by Mr. Janson, it may be seen that the timing of messages at all offices of the system will be on the same basis, and will thereby provide an accurate means for controlling and checking speed of service.

J. C. Wilcox: Mr. Janson's paper very ably outlines the technical characteristics of the equipment used to transmit Naval Observatory time beats to master clocks and master clock correcting signals to subscribers' clocks. As all of these subscribers' clocks are equipped with a special type of self-winding movement particularly adapted to the maintenance of a continuous correct time service, I would enlarge on that part of Mr. Janson's paper which refers to the movements, and give some further idea of their construction, use, and application.

First and foremost stands the fact that these clocks are independent operating units. They do not depend on any outside energy for operation. They are spring-driven and pendulum-operated. A motor automatically winds the main spring once each hour and maintains a constant tension on the main spring to give accurate pendulum control. Clocks running independently can be regulated within three or four seconds in twenty-four hours. To check for any variations, however, they are synchronized every hour by the Telegraph Company's master clock. If connecting line should be open for any reason and the synchronizing signal not received the clocks continue to operate continuously and within good commercial limits until the next signal brings them to time. Their outstanding feature is their ability to operate continuously and accurately regardless of outside line interruptions.

This characteristic makes them particularly desirable for use in commercial and transportation fields. In New York we find every railroad and subway system using self-winding clocks in their stations and dispatching headquarters. The largest installation is in Grand Central Terminal where over 1,000 units operate under the control of a central master clock which is checked daily with the noon time beats. The synchronizing line not only covers the terminal proper but extends as far out as Harmon, a distance of thirty miles.

The Pennsylvania Terminal has a similar system with about 250 clocks in the terminal proper at 33rd Street, and a separate synchronizing circuit running eastward to the Sunnyside freight yards at Long Island City and in the opposite direction as far west as Bristol, Pa., a distance of 67 miles.

Local subway lines are similarly equipped. The Interborough Rapid Transit System uses about 600 self-winding clocks synchronized from a master at its 59th Street power house. The Brooklyn & Manhattan Transit has approximately 100. The Hudson & Manhattan under-river tubes has 50 synchronized by a Christopher Street master clock. Approximately 100 miles of new subway system is now under construction, and a master clock with suitable relay and battery equipment has been installed in the 53rd Street power house and is available for inspection. No estimate can be made of the number of clocks the new subway will use but probably between 200 and 300.

I refer to these private installations only to point out the fields of application of this self-winding synchronized type of movement which is essentially the same as that used throughout the Telegraph Company's time service.

Herman Lemp: A symposium of time and time services would not be complete without mentioning the work of Mathias Hipp the pioneer in this art in Europe. Those who have traveled recently in Switzerland have undoubtedly seen the electric clocks on street corners, church towers, and hotels in the principal cities. These clocks have been in continuous service since about 1868 furnishing accurate time to a critical public, makers of Swiss watches and chronometers.

At the Centennial exposition in Philadelphia in 1876 Mr. Hipp received Gold Medals for his exhibit. His electric clocks are known all over Europe. The underlying fundamentals of the first central stations established to furnish accurate electric time are found in Mr. Hipp's conception of using feeble impulses but of some duration following each other every minute with reversed polarity, to actuate polarized receiving dials. These impulses have the advantage of never causing the receiver armature to advance more than *one step at a time*, even if a bad contact such as might be likened to a wire passing over a file, should give many impulses in lieu of one. To cause the minute hand to advance another step, the current will have first to travel *in opposite direction*, and as stated above is independent of the number of impulses in the same direction. Hence if atmospheric interference should cause the hand on one dial to advance between legitimate signals the current from the master clock would find its work already done and at the next minute the dial would be in step again.

Mr. Hipp was the first to invent the tuning fork escapement for a chronoscope, used in technical schools to measure the time a ball takes to fall—say three decimeters. He then invented the Hipp escapement, a toggle joint electric contact making device, which would, upon lowering of the amplitude of a free swinging pendulum, send an impulse through an electromagnet acting upon an armature attached to the pendulum and thereby restore its amplitude. This device was unaffected by strength of current and permitted the master clock to be kept going indefinitely as long as electric energy was supplied and with an accuracy of a few hundredths of a second variation per day.

It was my duty in 1878 to correct the master clock of the city of Neuchâtel every noon, by signals received from the Observatory of the same place. To ascertain deviation from observatory time Mr. Hipp invented the "coincidence clock," doing for time what a vernier scale does for linear dimensions. A pendulum next to the master clock pendulum had a period of oscillation of 60 beats in 59 seconds. It was released by current from the observatory and started to swing immediately. If the master clock pendulum was fast the coincidence pendulum lagged behind, each beat approaching nearer that of the master clock. As soon as the two beats coincided, the number of beats was read on the dial, indicating the number 1/60th of a second the master pendulum was fast. On either side of the master pendulum two others were provided which, when connected to the master pendulum, would either accelerate or retard the latter 1/60th for each beat they were connected. As a check the coincidence pendulum was reset again and released a second time from the observatory when, if the master pendulum had been properly corrected, the beats of the two pendulums would coincide right at the first swing.

Mr. Hipp was also the first to construct an astronomical pendulum of steel with mercury compensation (Invar was unknown at that time) located in a glass tube hermetically sealed and kept under constant (vacuum) pressure and temperature. This pendulum was located in a vault in the observatory while its dial was placed next to the observer at the transit instrument. This astronomical pendulum which had a maximum variation of 1/200th of a second per day was exhibited at the first electric exhibition in Paris in 1881 and I think another was sent to the Naval Observatory in Washington.

Mr. Hipp's successor Mr. Albert Favarger greatly simplified the polarized synchronized dials in 1905 and his book on electric time measurements is today considered authoritative.

John D. Crawford: Time (integrated frequency) and frequency (time rate) are similar aspects of the same physical idea. Yet these two ways of viewing the problems before us can cause very real differences of opinion. The discussion between Mr. Warren, who measures time, and Mr. Sporn, who measures frequency, illustrates this point.

In the field of precision measurement, on the other hand, it is significant that Messrs. Loomis and Marrison have pooled their experiences with clocks and with frequency standards, respectively, to the very great benefit of both. This paper is the first summary of the field to appear (in recent years at least) and the authors are to be congratulated. I know that I can speak for one organization interested in frequency measurements when I say that we are glad indeed to know how the other half lives.

In connection with Mr. Marrison's reference to the use of distance as a measure of frequency in terms of the velocity of light, a mention of the recently completed measurements of the velocity of light *in vacua* may be of interest. This series of experiments was undertaken by the late Dr. A. A. Michelson on a ranch near Santa Barbara, California. The measurement involved the determination of the time it took light to travel ten times between two sets of mirrors placed at the ends of an evacuated tube about a mile long. The distance, as measured by the United States Coast and Geodetic Survey, had a probable error of something like one part in three and one-half million. A crystal clock, similar in operating principle to the one described by Mr. Marrison, was used as the reference standard for time. Data that we have available on some 15 units of this type now in commercial service, indicate that, when working under the best conditions, it is capable of determining the average frequency to within three parts in ten million. The experimental details and the results of the measurements have not yet been formally published, but it seems as though the velocity of light will soon be known with a somewhat greater accuracy than heretofore.

If this increased accuracy is realized, it will be due as much to the high accuracy of comparison that is possible with a crystal clock as it is to its frequency stability. Due to the fact that the crystal is vibrating at 100,000 cycles per second, it is inherently better suited to precise, short-interval timing than is a pendulum clock whose bob vibrates once every second. Because of this greater accuracy of comparison, it is a relatively easy matter to determine with high precision the rotational frequency of the rotating mirror used in the experiment. This point has already been mentioned by Mr. Marrison.

So it may be that the Coast and Geodetic Survey will find it possible to follow out a suggestion made some time ago by Major William Bowie, Chief of its Division of Geodesy, and lay out precise base lines in archipelagoes, mountains, and other inaccessible places by means of a crystal clock and a beam of light. Distance would then be measured in terms of time and a standard base line, using the constant velocity of light as a transfer medium.

R. C. Thayer: The noon beat signals as sent out by the two telegraph companies serve a very valuable purpose to the railroads of this country. These time signals as sent on the telegraph companies' wires are picked up by the railroad companies and re-distributed throughout their own network of communication circuits, thus each of the railroad's stations, and the train dispatchers receive the correct time daily. At terminals and the more important offices clocks designated as "standard clocks" are installed, whose accuracy is checked daily and recorded, with which the trainmen can compare their watches. The ability to obtain accurate time is of prime importance to the railroads, and without which it would be impossible to operate the fast train schedules now in effect in this country.

H. E. Warren: In answer to the comments of Mr. Philip Sporn I would not minimize the difficulties that stand in the way of securing satisfactory frequency regulation. I would agree with him that the requirements today are much more severe than was the case in 1916 when master clocks as an aid to regulating frequency first became available.

At that time the need for extreme accuracy in frequency regulation was much less and the instruments which could be used were far inferior to those which are now available. Standardization of frequency was then considered of rather minor importance. Power companies were operated regularly, for what seemed sufficiently good reasons, at frequencies as much as two or three cycles above and below 60, although rated as giving 60-cycle service. Many companies which were attempting to operate at 60 cycles were actually delivering current as much as 1 cycle above or below that value on account of instrument errors.

This state of affairs was not of course conducive to extensive interconnection. The use of the master clocks by the power companies, whatever the reason may have been for adoption, did actually tend to standardize the average frequency at an exact 60-cycle value. Moreover the use of the master clocks with their very high inherent precision probably served both as a spur and an aid to the manufacturers and to the users of frequency meters. This is for the reason that errors in the meters could be very easily discovered by comparison with the master clock so that all uncertainty regarding the relative precision of different meters for use on the same system could be readily removed.

A condition has now been brought about, as pointed out by Mr. Sporn, where it is possible by commercial instruments to maintain frequency regulation that was utterly impossible fifteen years ago. Whether or not this result would have come about if the use of the master clocks as basic standards for the average value of the frequency had not been generally adopted remains at least open to question. Certainly the adoption by many power companies of methods of regulation which call for the maintenance of exact average frequency, primarily or secondarily for the purpose of giving time service to their customers, has been a rather important factor in bringing about the present situation, where practically all power companies have the same exact standard of average frequency.

I heartily agree with what Mr. Sporn has said about the extremely difficult problems of regulating station output on large interconnected systems so as to maintain tie-line loads within satisfactory limits and I certainly would not claim that the adoption of master clocks alone, has or could completely solve these problems.

One idea which I desire to convey in my paper is that the adoption of the master clocks by the power companies has, apart from the time service that has been made possible, been a factor and aid in helping to solve the operating problems of some of the very large power companies.

In answer to the comments of Mr. R. J. Wensley I would recognize the background that has been furnished by numerous inventors in this field, as in practically all fields where something worth while has suddenly been accomplished. It seems to me that in most cases, however, the merit of the work that has been done can be measured by the accomplishment.

Forces of Electric Origin in the Iron Arc

An Explanation of Overhead Welding

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P. W. SEAL[3] Enrolled Student, A.I.E.E.
E. P. SORDON[3] Enrolled Student, A.I.E.E.

IT will be readily admitted that it is desirable, if possible, to reduce the study of the phenomena of high-current metal arcs, particularly iron arcs, owing to their importance in welding, to a state in which they can be exactly described and measured by a suitable number of parameters, so that the effect of any change of conditions may be specified by the changes in these parameters. Owing to its intense brilliance the arc cannot be studied directly and the most convenient way of studying it is to project the magnified image on a screen by means of a lens in the manner shown for instance in Fig. 3. When this is done a series of remarkable phenomena at once becomes apparent. Considering first a low current arc of four or five amperes between electrodes of about the same size as welding wire, say about an eighth of an inch in diameter, it is seen that the two ends of the electrodes each become fused into a globule, the positive electrode being materially hotter than the negative. In some cases the globule may be exclusively on the positive electrode while the negative electrode remains black containing only a small incandescent area corresponding to the cathode spot. On each electrode there is a brilliant incandescent spot and frequently a reflection of the cathode spot may be seen in the fused surface of the drop on the end of the positive electrode which presents a characteristic glazed appearance.

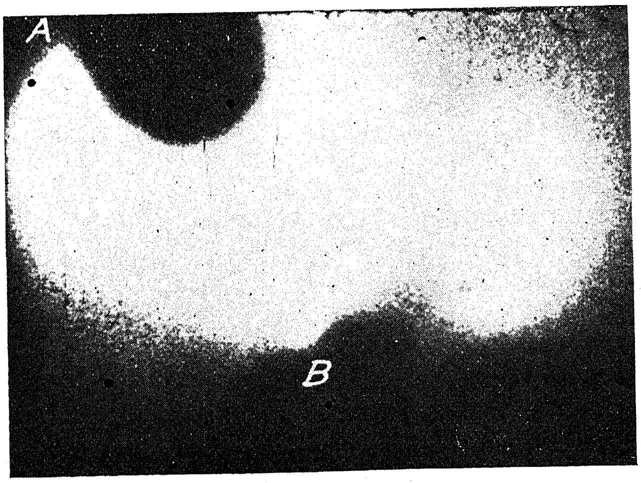

FIG. 1

The arc itself consists of a column of blue vapor surrounded by a yellow envelope of burning iron which is suppressed if the arc is struck in nitrogen or other inert gas. All this has been the subject of much study, but if an arc between electrodes of similar area but carrying a welding current of say 150 amperes is examined in the same way, entirely different phenomena become apparent. Suppose that the positive electrode is a piece of steel plate of considerable area while the negative electrode as before is a welding rod, the drop on the end of the welding rod can be seen as before but now it appears to be the seat of violent forces. It oscillates to and fro and quivers like a jelly while on its surface are to be seen what seem to be fine wave formations. Moreover it does not drop off in the same manner as in a small current of say 5 amperes when the drop leaves the electrode very much as a drop of water leaves a hose. When a powerful current is used the drop can be seen to grow to a certain diameter, usually somewhat larger than that of the rod and then there is a sharp explosion or crackling noise and the drop disappears, its motion being so fast that it is impossible to see it cross the gap between the electrodes either by the eye or by means of movie cameras of ordinary speed.

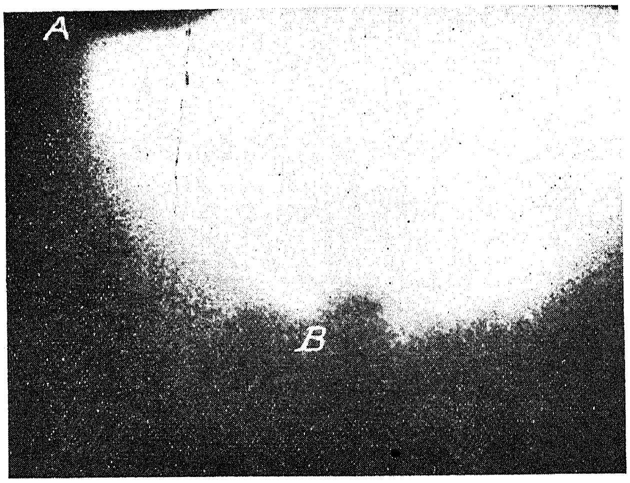

FIG. 2

In Figs. 1 and 2 are shown two succeeding pictures on a film which was given 32 exposures per second. In the first the drop can be seen clearly while in the second it has totally disappeared, (the end of the electrode being seen at *A* and the cathode spot at *B*) having therefore crossed the gap in less than this space of time (probably a great deal less). These phenomena urgently need explanation since the origin of these forces is not at all well known.

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Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

Other curious phenomena are that when welding overhead, the drop can travel in the direction opposite to that of gravity; and again, the crater or hollow space which appears to form under the welding rod requires explanation. This explanation resolves itself into two portions.

1. What is the force that causes the drop to travel across the arc space?

2. Why is it that the drop is detached with explosive violence rather than gradually? Light was thrown on the first of these questions in the following way:

1. Studying low current (five amperes) iron arcs by means of an apparatus which involved burning the arc inside a glass bell jar containing gas at atmospheric pressure it was found that, starting with a clean jar, within a few seconds of the arc being first started, a deposit began to form on the jar which gave sharp shadows of the electrode holders. This was apparently due to particles projected in straight lines which might be thought to give rise to mechanical forces on the electrodes. Within a few minutes this sharp pattern was blurred by the oxides, nitrides, etc., given off by the arc in vapor form. Investigation of these forces, whose magnitude must be far less than a gram, is difficult because the heavy arcs which it is necessary to use rapidly melt away any apparatus delicate enough to measure these small forces. Apparently the only way in which this difficulty can be overcome is to reduce the time of application of the heavy arc to a very short interval.

In order to make use of this principle the following apparatus was designed based on the principle of the ballistic galvanometer. This is shown in Fig. 3 and consists of a brass rod about 5/16-in. diameter and 8 in. long terminated at either end by an iron electrode about 3/8-in. diameter and 2 in. long. This forms the moving system which therefore consists of a double headed *T* which is suspended at the center of gravity by a bifilar suspension whose dimensions are as follows:

Length 26.6 cm. Distance apart at the bottom 2.93 cm. Distance apart at top 1.82 cm. Diameter 0076 cm. Molybdenum wire.

A galvanometer mirror is also fixed to the suspended system and reflects a spot of light on a screen in the usual manner and a pin projects from it below, dipping into a mercury cup. This forms the moving electrode. There is in addition a fixed electrode which consists of a piece of welding wire held in a clamp as shown so that it may either be brought in contact with the end of the moving electrode or moved back to any desired distance by depressing the key. The current therefore passes through the fixed electrode, the arc between it and the moving electrode, then through the moving electrode and out through the mercury cup.

To strike the arc the fixed and moving electrodes must be momentarily touched together and then drawn apart by depressing the key. This at first sight gives rise to a difficulty owing to the sensitivity of the suspension. No matter how light the contact between the fixed and moving electrode, it is sure to set the spot of light on the screen into violent oscillation which may take a long time to die away since the apparatus has no damping except that due to the pin immersed in the mercury cup. To overcome this difficulty, vanes are attached to the moving system on either side of its axis of rotation and arranged to dip into mercury troughs which could be raised or lowered by moving a lever. When the vanes are in the mercury troughs, the damping is extremely great so that contact can be made without agitating the spot of light. Having struck the arc it is possible to lower the mercury troughs and free the moving system without either shock or agitation of the spot. The method of operation of the instrument therefore consisted in striking the arc with a small current, say 5 amperes, the forces due to which are very small, then throwing on the heavy current for a short period varying from 0.1 second to 0.6 second and observing the first deflection of the spot of light. From this and the moment of inertia the impulse may be readily calculated on the familiar principles of the ballistic galvanometer. The impulse of course is defined as the force multiplied by time of application.

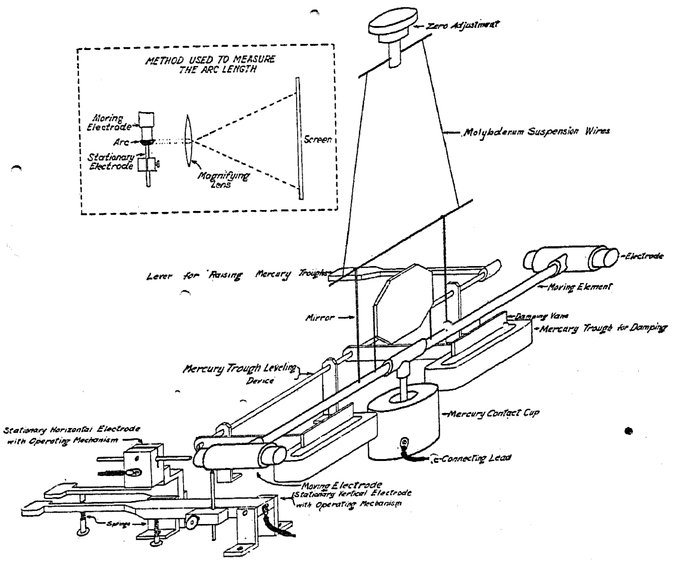

METHOD USED TO MEASURE THE ARC LENGTH
Moring Electrode
Arc
Stationary Electrode
Magnifying Lens
Screen
Zero Adjustment
Molybdenum Suspension Wires
Electrode
Lever for Raising Mercury Troughs
Moving Element
Mirror
Damping Vane
Mercury Trough for Damping
Mercury Trough Leveling Device
Mercury Contact Cup
Stationary Horizontal Electrode with Operating Mechanism
Connecting Lead
Moving Electrode
Stationary Vertical Electrode with Operating Mechanism
Springs

Fig. 3

It was also desired to study the relation between these forces and the arc length, hence the moving and fixed electrodes and the arc between them were projected on a screen as shown in the separate sketch included in Fig. 3 with a magnification of approximately 23.8 diameters which rendered the arc length readily measurable.

Examining this apparatus it is obvious that currents passing into the moving element by means of the arc and out by the mercury cup will produce certain forces in conjunction with the earth's field and the fields of other current carrying wires in the neighborhood. It is necessary therefore to find some means of estimating the magnitude of these forces. For this purpose a second electrode is provided and arranged so that an arc may be struck onto the moving electrode from below, this second electrode being vertical. This is clearly shown in Fig. 3. When an arc is struck in this position the arc forces, which the apparatus is designed to measure, will act in a vertical direction, that is in a direction in which they cannot produce any motion. Nevertheless the purely electromagnetic effects will be the same as before and hence this vertical electrode gives us a means of measuring them separately. On attempting to make these measurements, however, even with the heaviest currents, it was found that the electromagnetic effects were so small as to be quite incapable of measurement by the instrument used. By this means it was proved that the deflection obtained when the horizontal electrode was used was entirely due to the arc forces and not to these electromagnetic forces. Two different types of circuit were used to throw on the heavy arc for a short period. These are shown in Figs. 4 and 5.

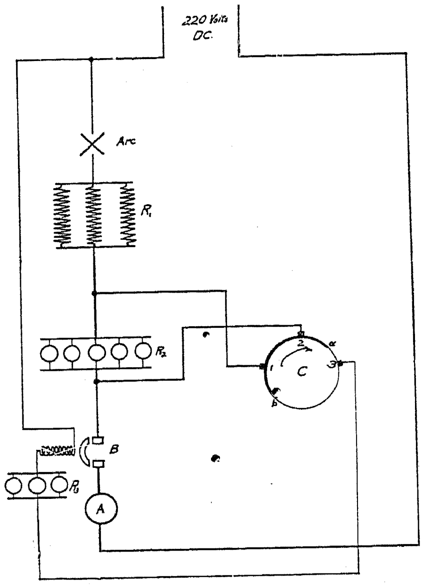


FIG. 4—DIAGRAM OF CONNECTION NO. 1

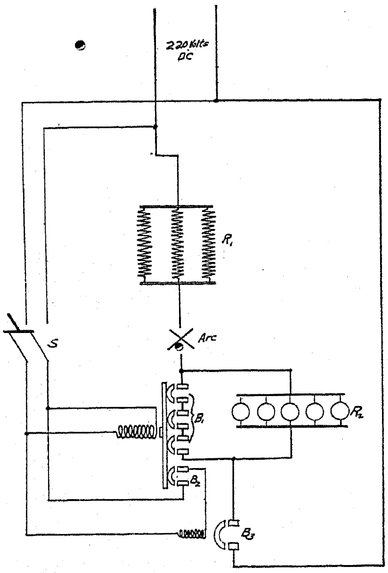


FIG. 5—DIAGRAM OF CONNECTION NO. 2

In Fig. 4 resistances R_1 and R_2 are connected in series with the arc, R_1 being of such a value as to limit the current to the value which it is desired to study, say 150 amperes while R_2 further limits it to a value of 5 amperes. A revolving contact maker C is employed which short circuits resistance R_2 between brushes 1 and 2 for a certain period depending on its speed of rotation. Between brushes 2 and 3 is connected the trip coil of a circuit breaker so that the arc circuit will be broken as soon as brushes 2 and 3 are joined. This insures that the operation of short-circuiting R_2 shall take place once only and not periodically with the rotation of the contact maker.

For very powerful currents this apparatus as constructed was not found satisfactory owing to insufficient brush area and therefore another circuit shown in Fig. 5 was employed. Here R_1 and R_2 have the same meanings as before while B_1 is a three-phase contactor, all three blades of which are placed in series, and which is fitted with an extra pair of contacts B_2 which close the trip coil circuit of the circuit breaker B_3. With this circuit the heavy arc is thrown on only for a period depending on the time lag of the circuit breaker which can be adjusted to any figure desired. This was found to give very steady and reliable results, particularly for very short time intervals.

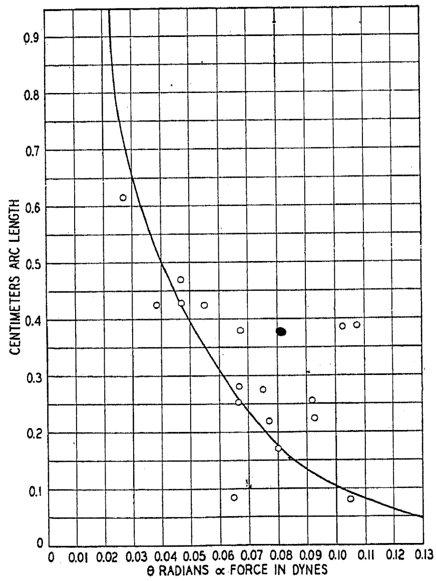


FIG. 6—ARC LENGTH *vs.* FORCE ($\alpha\theta$) FOR 120 AMPERES INDUCTANCE IN CIRCUIT TIME IS 0.1 SECOND

In using this instrument it was not found possible to regulate the arc length beforehand to the desired value before measuring the impulse. It was necessary to take a considerable number of observations with constant current and random arc lengths after which it was found that a curve could be plotted without much difficulty.

One of these curves is shown in Fig. 6 which shows the relation between the angle of deflection of the instrument in radians and the arc length. It will be seen that the points are rather widely scattered but nevertheless not so much so that a fairly satisfactory curve cannot be drawn through them. All of these instruments are rather delicate and it may be noted that the points on the curves obtained by Duffield, Burnham, and Davis referred to below are quite as widely scattered. At least one cause of these scatterings has been discovered (referred to below) and work is in progress by which it is believed it may be reduced.

A set of curves similar to Fig. 6 was observed for currents of 38, 68, 90, 120, and 154 amperes.

As may be shown easily from the theory of the ballistic instrument, the angular deflection in radians is proportional to the impulse or force multiplied by time of application. The times of application of the force corresponding to these different currents were carefully measured in the manner described below and the forces calculated from the theory of the ballistic instrument as may easily be done. (It may be noted that the moment of inertia of the moving system is 6,760 gram-(cm.)^2.) The result is shown in Fig. 7 which of course shows the smooth curves drawn through the rather widely scattered points such as those shown in Fig. 6. The most striking feature of Fig. 7 is the rapid increase of the force with decreasing arc length. Apart from this the curve for 120 amperes does not seem to be parallel to the others, at any rate for long arcs. This probably is due to the fact that arcs longer than 3 millimeters tend to take a curved form which does not lend itself to accurate measurement especially when it is remembered that this measurement has to be made within a period of 1/10 of a second.

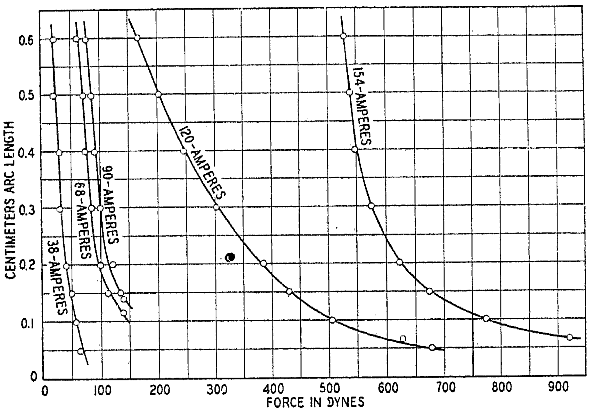


FIG. 7—ARC LENGTH *vs.* FORCE FOR VARIOUS CURRENTS BASED ON UNIT TIME

From Fig. 7 values may be taken of the force corresponding to fixed arc lengths, say 1 millimeter, and this will give 5 points for the curve of force plotted against amperes. A set of such curves is shown in Fig. 8 and it will be seen that they are approximately at least parabolas. In Fig. 9 the curves of Fig. 7 are replotted on logarithmic paper which shows that those for less than 3 millimeters are straight lines while for greater lengths some disturbing cause appears to enter which gives rise to curvature. For arcs of less than 3 millimeters therefore it is possible to obtain a simple equation. For instance for 1 millimeter $\log i = \log 17 + 0.5 \log F$ is obtained which when transposed gives

$$F = \frac{i^2}{290}.$$

For different arc lengths the constants of this equation will vary and the different values are plotted in Fig. 10 also on logarithmic paper. This gives a straight line which is parallel to other straight lines obtained by plotting arc length against force directly, these other curves being obtained simply by replotting Fig. 8 on logarithmic paper. From this curve we may ascertain how the constant in the above equation depends on the arc length and thus derive our final formula namely,

$$F = \frac{0.035\, i^2}{\sqrt{l}} \quad (F \text{ in dynes}, i \text{ in amperes}, l \text{ in millimeters}).$$

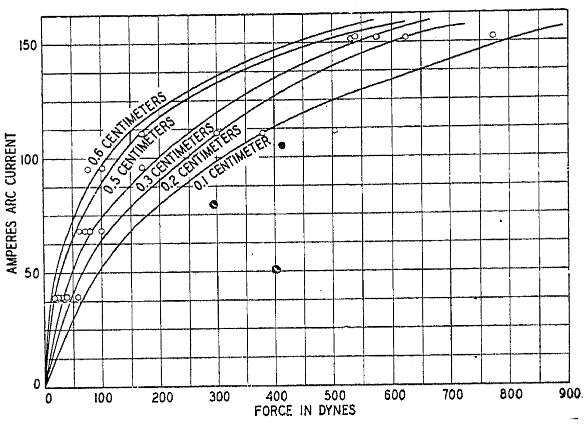


FIG. 8—CURRENT *vs.* FORCE FOR CONSTANT ARC LENGTH

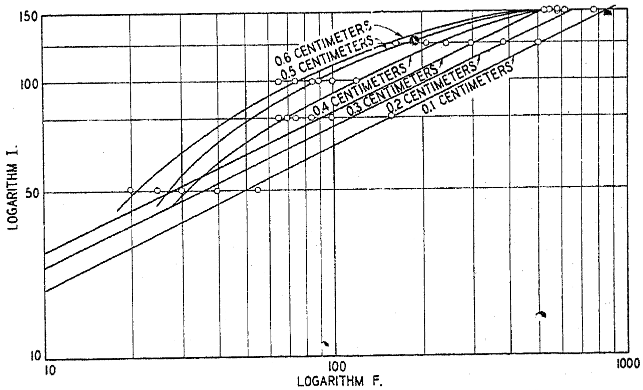


FIG. 9—LOG I *vs.* LOG F FOR CONSTANT ARC LENGTH

A study of the literature reveals that very little work indeed seems to have been done on this subject. The only papers which the writers have been able to find are a paper by Duffield, Burnham, and Davis, in the *Proceedings of the Royal Society* for 1920, v. 97, and a paper by R. Tanberg, in the *Physical Review* for 1930, v. 35.

Messrs. Duffield, Burnham, and Davis studied various materials including iron and measured the force for currents lying between 3 and 10 amperes by means of a torsion balance which must have been a very delicate experiment. The arc length corresponding to their curves is not very clearly stated but there are several references in the paper to arcs of 3 millimeters and the writers assume that this is the length of arc they employed. Their curves are reproduced in Fig. 11. Curve 1 showing the force with the moving system positive and curve 2 with it negative. Curve 3 shows what the force should have been calculated by the formula given above and it will be seen that these two sets of results are of quite the same order although they do not exactly coincide. This is hardly to be expected considering the difficulty of the experiments. The experiments of Duffield, Burnham, and Davis, and those described above can therefore be regarded as mutually confirming one another.

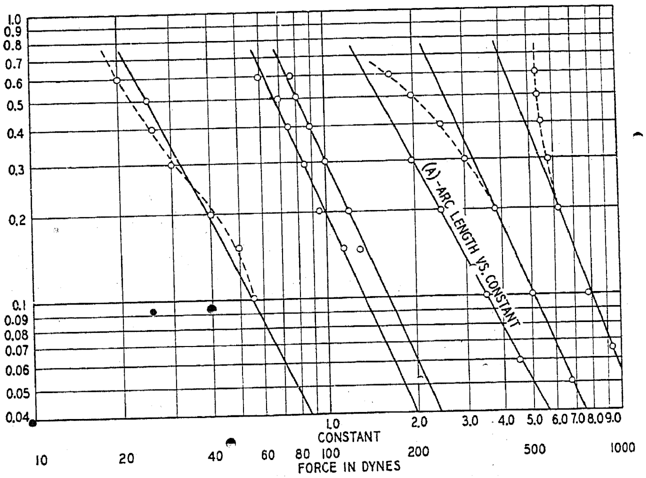


Fig. 10—(*A*) Arc Length *vs.* Constant, Other Five Curves are Arc Length *vs.* Force

Tanberg's results were taken *in vacuo* which introduces a radical change of conditions. A few of the many interesting deductions which may be made from this striking investigation are mentioned below.

Fig. 12 shows in curve 4 Tanberg's results which were taken with copper electrodes. Curve 6 shows the force which would be calculated from the above formula for arcs of 3 millimeters. This, of course, is for iron electrodes. But Duffield, Burnham, and Davis experimented both on copper and iron and found that the force in the case of the copper electrode is roughly double that obtained with an iron electrode under the same conditions. Curve 5 therefore has ordinates twice as great as curve 6 and might be supposed to be the curve which might be obtained by a series of experiments similar to the above, taken with copper electrodes at atmospheric pressure.

Nothing can be more striking than the difference between the forces obtained by Tanberg and those calculated in the manner just described and this difference can only be ascribed to the absence of atmospheric pressure in Tanberg's case. This therefore enables us to give a probable explanation as to why the force is so much greater with short arc lengths. The most reasonable hypothesis as to the origin of this force ascribes it to particles projected from the electrodes, particularly the cathode. The force measured on the electrodes therefore is a recoil due to the expulsion of these particles and the greater the number of particles expelled the greater the force. In the absence of atmospheric pressure it is clear that many more particles can be expelled than when the atmosphere is present reducing the mean free path of the particle to small proportions. This is likely to be the explanation of the much larger forces observed by Tanberg. Again at atmospheric pressure the shorter the external path between electrodes the greater the number of particles which can be expelled and therefore the greater the force which is in accordance with our observations.

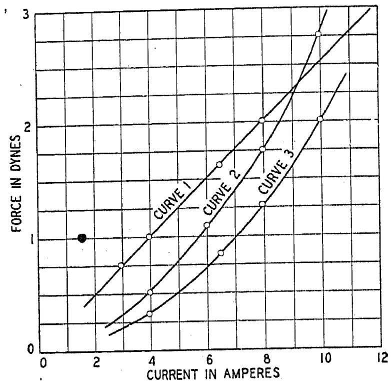


Fig. 11—Curve Sheet No. 1

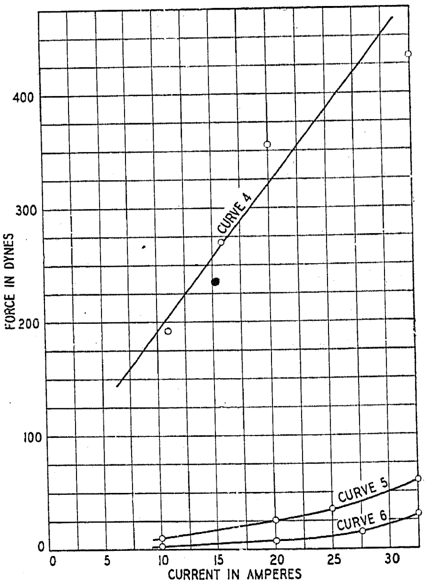
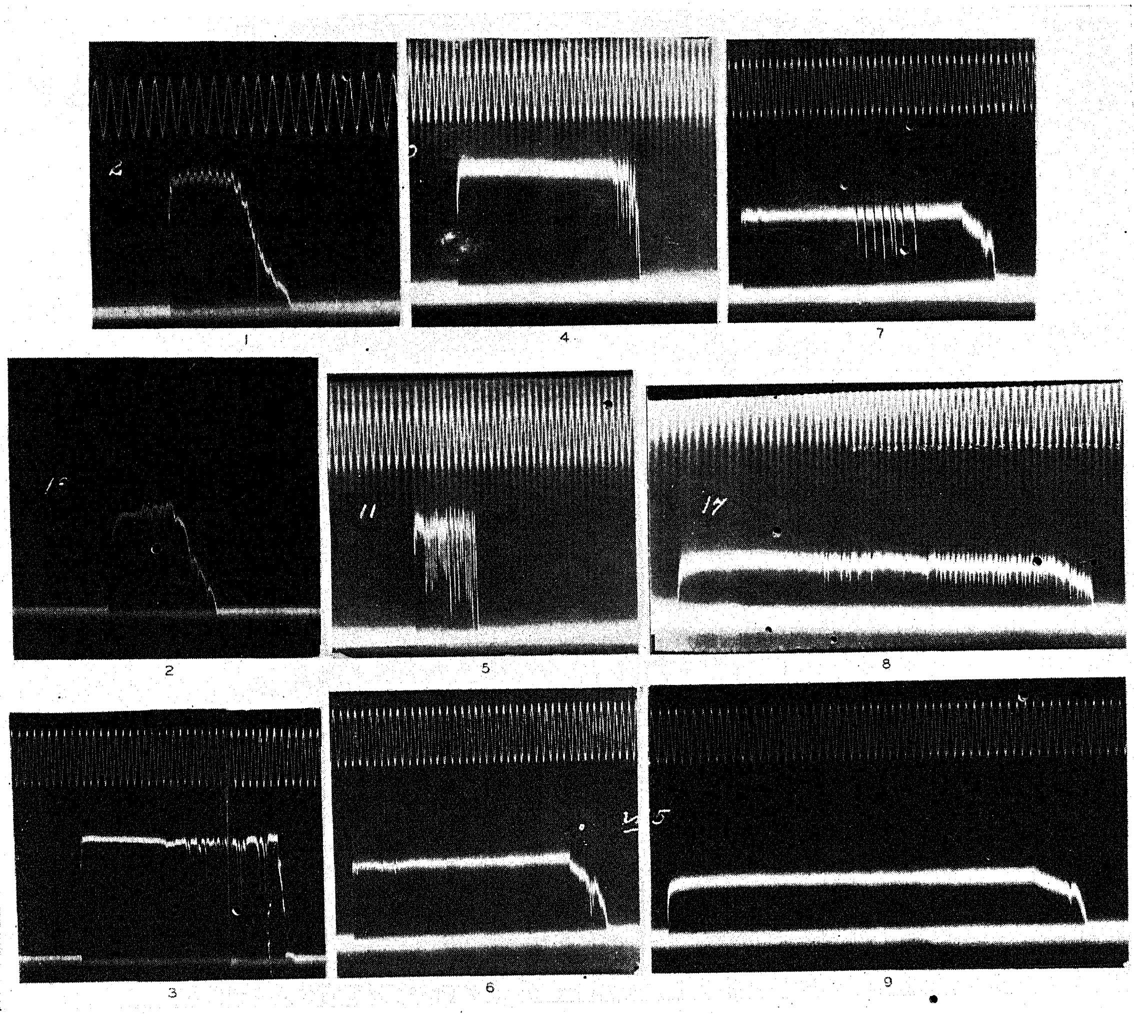


Fig. 12—Curve Sheet No. 2

It will be observed that the quantity measured by the instrument described above is the impulse due to the arc forces, that is the product of force multiplied by time of application. It is essential that the time of application should be accurately measured. This is normally done by measuring the speed of rotation of the revolving contact maker of Fig. 4, but in order to check this a number of oscillograms was taken some of which are reproduced in Fig. 13.

Where the circuit of Fig. 5 is employed there is no other convenient method of measuring time except by the oscillograph. Fig. 13, (No. 1) shows the current impulse when the circuit of Fig. 5 is used, the arc being excited with rotary converter current. The effect of the slots in the rotary converter on the current wave should be noted. In order to eliminate this, battery current was substituted. The result of this is shown in Fig. 13, (No. 2) from which it will be seen that the oscillations have by no means disappeared. For the reasons mentioned below, a heavy self-induction was connected in series during this test and it will be seen that instead of the current rising instantly as it does in No. 1 it rises in the well-known exponential form.



1 4 7

2 5 8

3 6 9

Fig. 13

When the contact maker of Fig. 4 is employed some remarkable phenomena become apparent as will be seen from numbers 3, 4, and 5. These show current impulses which correspond to the same time of closing of the contact maker. It will be seen that the times during which the current is on are entirely different in these three cases which reveal some of the reasons why the points on the curves deviate from the average. It will be seen that the current contains extremely pronounced oscillations of very high frequency. Apparently if one of these oscillations becomes large enough to reduce the current to zero, it cannot restart, so that the length of the impulse depends entirely on these oscillations, and not on the rate of rotation of the contact maker. In order to reduce these oscillations a large self-induction was placed in series with the arc circuit. The effect is shown in numbers 6 and 7 and it will be seen that the oscillations are much reduced though not entirely suppressed. Numbers 8 and 9 show a longer time period. In number 9 the oscillations are almost entirely suppressed by the self-induction but in number 8 taken under the same conditions as far as can be judged the oscillations are still pronounced.

Considering the whole series of oscillograms it will be seen that the oscillations do not start immediately the current is switched on but only after an interval which would seem to correlate them with the heating of the electrodes. This is about the only comment that can be made in general since the cause of these oscillations has not yet been discovered.

THE FORCE DETACHING THE GLOBULE

Coming to the question of the nature of the force which detaches the globule from the welding rod; about 25 years ago Carl Hering drew attention to what he called the "pinch effect" which is found in electric furnaces and a full theoretical study was published by Dr. E. F. Northrup, *Phys. Rev.*, v. 24, 1907, p. 474.

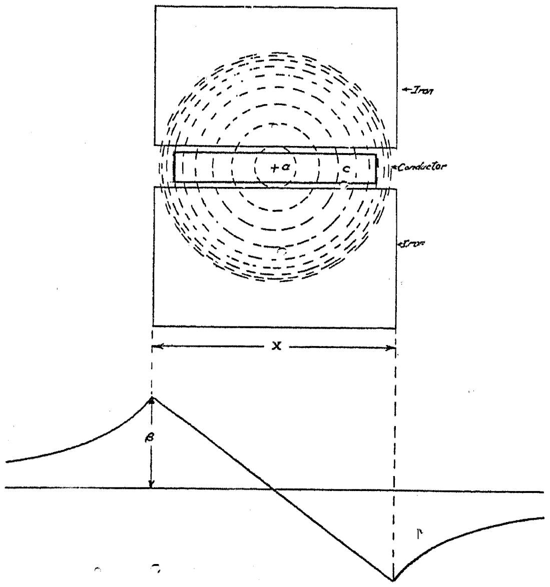


FIG. 14

In order to understand the pinch effect more clearly consider a flat conductor (Fig. 14) carrying a current of uniform current density flowing say at right angles to the paper and surrounded by magnetic material of the form shown. If we consider the central filament of current at *a* it will be surrounded by a field proportional to its magnitude which is very small, vanishingly so as the width of the filament decreases. A wider filament of current such as that between *a* and *c* will be surrounded by a field again proportional to its magnitude which is much greater. Hence the magnetic density will be zero in the middle of the strip and will rise in either direction towards the edges as shown in the curve below, whose ordinates represent *B* the magnetic density. Given this field distribution and assuming that the current is flowing upwards in the diagram, then it is clear applying the elementary rule, that the outer portions of the conductor will experience a force urging them towards the center and if the conductor is a fluid this will result in a hydrostatic pressure which becomes a maximum at the center reducing to zero at the circumference. The pressure in dynes per sq. cm. at radius r (Fig. 15) is given by the formula

$$g = \frac{I^2}{100\,\pi\,R^4}(R^2 - r^2)$$

for the case of a circular conductor of external radius R cm. carrying current I amperes. Clearly becoming a maximum when $r = 0$.

If the values corresponding to a welding rod 4 mm. in diameter carrying 300 amperes are inserted in the formula, $g = 7{,}000$ dynes per sq. cm. at the center of the rod. The value of g in the above formula gives us for a fluid conductor the hydrostatic pressure, which is equal in all directions. The total pressure in an axial direction round the circumference of a circle of radius r is

$$2\,\pi\,r\,g$$

and the total force on a plane perpendicular to the axis is

$$\int_0^R 2\,\pi\,r\,g\,d\,r = \int_0^R \left[\frac{2\,r\,I^2}{100\,R^4}(R^2 - r^2)\right] d\,r$$

Carrying out this integration we get

$$F = \text{total force} = \frac{I^2}{200}$$

The total force is therefore independent of the radius of the cylindrical conductor. If $I = 300$ amperes $F = 450$ dynes. This is approximately equal to the weight (at room temperature) of a spherical globule of iron 4.9 mm. (0.193 inches, a little over 3/16 in.) in diameter.

Hence for dimensions usually employed the force due to the pinch effect tending to separate the globule from the rod is at least equal to its weight particularly if it is remembered that due to the high temperature and the occluded gases the weight of the hot globule is probably a good deal less than that of the cold one.

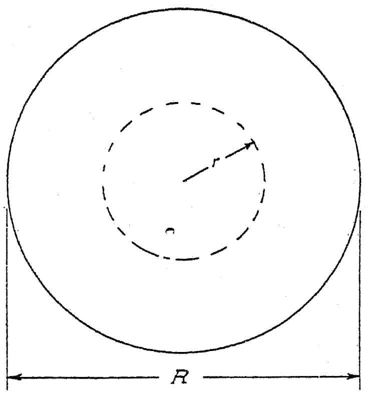


FIG. 15

Thus it is shown in general that the passage of a current through a fluid conductor, such as mercury or the molten contents of the electric furnace, causes a force of the nature of a hydrostatic pressure whose intensity is proportional to the square of the current density. Consequently if there is a constriction in the fluid path at any point so that the current density is greater there than elsewhere, the fluid will be forced out at that point and will be heaped up on either side of the constriction as shown in Fig. 16. There is then a depression in the fluid surface where the constriction occurs and a corresponding elevation at other portions, the difference of level between the two being sufficient to balance the hydrostatic pressure due to the high current density in the constricted portion. But of course a fluid cannot maintain the form shown in Fig. 16 without constant circulation hence the fluid is constantly flowing down the sides of the notch as shown in the dotted lines and being forced out of the portion having the minimum area thus setting up a circulation as shown.

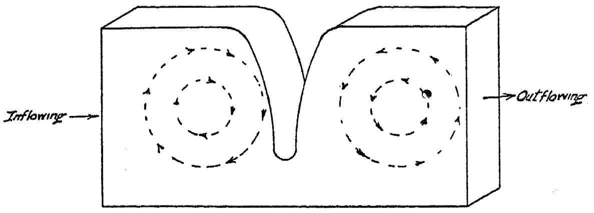


FIG. 16

It is shown above that the force starting from zero at the outer circumference of the circular conductor becomes a maximum at the center, and if a circular liquid column is arranged vertically the conducting liquid will be forced out of the center and will fall back along the outer circumference thus producing a continual fountain which remains in action as long as the current continues.

Now consider a welding rod which is arranged in a vertical position with a fluid drop on the top (see Fig. 17a) and a powerful current flowing through it leaving by an arc at the top of the fluid bead which has a diameter somewhat greater than the rod. It is not certain to what extent this bead is liquid or merely viscous. Since the diameter of the bead is larger than that of the rod, there is a constricted portion where the fluid bead joins the solid rod and at this portion the forces corresponding to the pinch effect exert their maximum power.

It was pointed out above that in the case shown in Fig. 16, the fluid in the narrowest cross section is constantly replenished by that running down the sides of the notch and that it is due to this alone that the phenomenon is a stable one, that is, that the depth of the notch has a definite value for each value of the current and may become shallower or deeper by decreasing or increasing the value of the current. But if there is a viscous fluid in which this replenishment cannot take place, then as soon as the electric forces become sufficiently powerful to overcome the viscosity of the fluid and cause the notch to begin to appear, it seems to the writers that this notch must get deeper and deeper (owing to the forcing out of the material in the smallest cross-section and to the fact that there is no means of replenishing it) until the conductor is broken in two. In other words the stability of the phenomenon depends on the free circulation of the fluid, that is on the absence of viscosity.

Now apply this to the case of the bead on the end of the welding rod. These electric forces must cause a constriction (see Fig. 17b) to form at the narrowest portion of the fluid mass, that is where it joins the solid material. This constriction forces out the material in the center of the bead the axial pressure being 7,000 dynes per sq. cm. as calculated above, and as soon as the cross section becomes smaller the pressures are increased as the square of the current density. The total force tends to sever the parts still attached, and remains constant no matter how small the area of the attached portions. Hence as soon as the constriction starts it deepens at an ever increasing rate until the bead is cut off from the rod with explosive violence in a way which can be readily observed. It may be that some of the fine wave motion which is apparently observable in the bead is due to attempts to set up a circulation such as that described in connection with Fig. 16. As soon as the globule is detached the forces measured above are ample to carry it over. For instance with a current of 300 amperes and an arc length of 0.1 millimeter the force is approximately 10,000 dynes or about 10 grams.

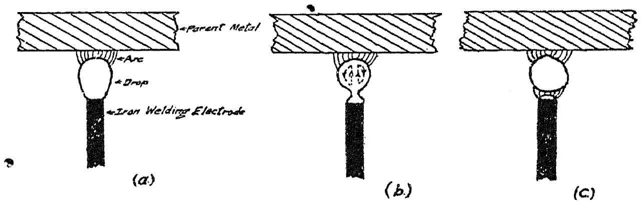


FIG. 17

Discussion

J. Slepian: Is force itself the proper measure of the capacity of the pinch effect in throwing particles from the electrode with considerable velocity? It would seem to me that impulse or work being more directly related to momentum and energy would be more appropriate units for describing the particle-throwing capacity. Hence, it should be necessary to consider either the time factor or the distance factor, that is, for how long a time can the pinch effect force act so as to give the particle momentum, or through what distance does the pinch effect force act so as to give the particle kinetic energy?

L. R. Ludwig: Mr. Creedy has attacked a rather interesting problem in his paper in attempting to describe the means by which drops are caused to travel across the arc space during welding. He has assumed that the drop first breaks contact with the welding electrode, travels across the space, and fastens itself to the work which is overhead. This, however, is not generally the case. Fig. 1, for example, is an oscillogram of the voltage and current taken during an overhead welding operation. It will be seen that the voltage drops from arc voltage to a value of about 2 volts very frequently, and it is believed that these voltage decreases correspond to a period during which the drop actually short-circuits the welding arc. In this event, it seems probable that when the drop of molten metal touches the work it freezes at the upper contact, and the surface tension will probably be sufficient to draw the drop up onto the work. It seems that Mr. Creedy has neglected to mention this possibility.

With regard to small drops, one must agree with Mr. Creedy that the vapor blast from the cathode of the arc may be an important element in the transfer of the welding material if the electrode spacing is short. If this is to serve as an explanation, however, it is essential that the welding electrode be negative and the work positive. Overhead welding is frequently done with polarities opposite to those mentioned, and this fact again seems to mean that Mr. Creedy's explanation of the transfer of material is not sufficient.

In the latter part of his paper Mr. Creedy has discussed the effect of the "pinch effect" on loosening molten drops from the welding electrode. He has calculated a force of 450 dynes, which hardly seems sufficient to explain the explosive action which he has observed when the drops leave the electrode. In order to be certain of the results, either the impulse or the momentum given to the drop should be calculated. It seems possible that the explosive action is really due to the very high temperature of the last point of contact between the drop and the electrode. The formula for the calculation of this temperature was given by Mr. Slepian before the A.I.E.E. in 1926. Thus, if it is assumed that due to the pinch effect the area common to the drop and the welding electrode is reduced to a very small value, the voltage between the welding rod and work would concentrate across the last contact point. This would give rise to a voltage drop of 2 volts across the contact (taken from Fig. 1), which according to Slepian's formula would result in a temperature of 13,000 deg. cent. at the last contact point. Although there are few data on the vapor pressure of iron at this temperature, an extrapolation would give a value of about 1,000 atmospheres. This pressure is of course exerted on a very small area, but some idea of this area may be had by calculating the amount of material vaporized as a function of the pressure, temperature, and area; finding from this and the latent heat of vaporization for iron the watts required; equating this latter result to the actual watts input and solving for the area. The result is an area of about 10^{-3} square centimeters. If the pressure given above acts on this area, the force would be about 10^6 dynes. This is much greater than the force calculated by Mr. Creedy.

Such calculations are conceded to be very rough. The calculated temperature of 13,000 deg. cent. is probably much too high, but it does seem as though the force due to this cause is of major importance in causing the transfer of welding material.

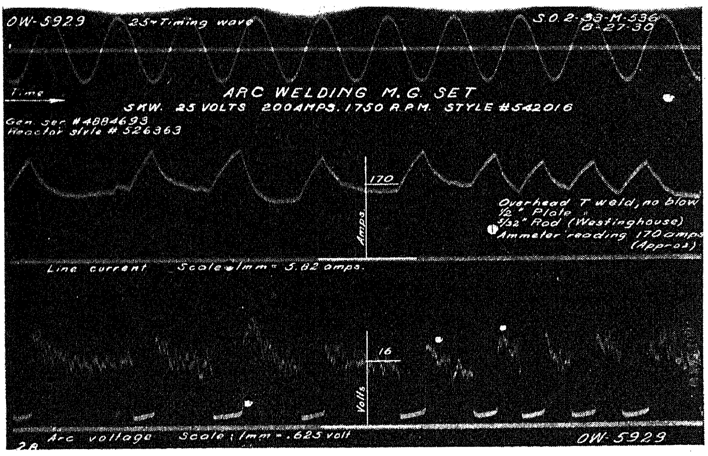


FIG. 1

F. B. Lucas and E. C. Easton: Mr. Ludwig of the W house Company states in his discussion that the tran metal in arc welding is due to a globule building up on the the electrode until it comes into contact with the plate welded then pinching off, leaving part of the molten m the plate and part on the electrode. Mr. Ludwig shows in that the voltage across the arc dropped to approximate volts at regular intervals, supposedly when metal was trans

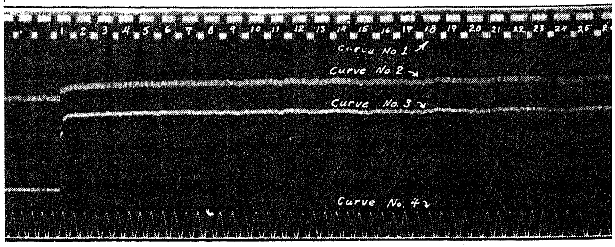


FIG. 2

We have been experimenting with electric arcs and our show that Mr. Ludwig's statements are not necessarily all cases.

We have constructed a contact maker which, when co to a moving picture camera, makes electrical contact ea an exposure is made by the movie camera. This contact was connected in a circuit with one of the oscillograph v thus recording the moving picture exposures on the oscil film. Using the above apparatus we have been able to what is happening in the arc at each instant on the oscillo

In Fig. 2 the moving picture film has been printed ab oscillograph film. The upper curve, No. 1, is the curve n the contact maker. Curve 2 is the voltage across the ar 3 is the current, and curve 4 is a 60-cycle timing wa moving pictures clearly show that a globule of metal w ferred at exposure No. 21 but there is no change in the voltage as shown by the voltage curve.

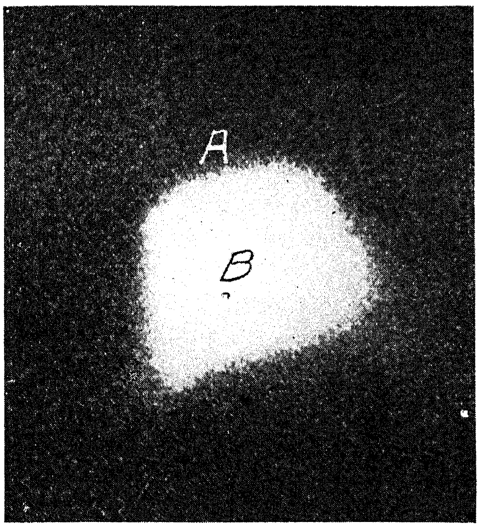


FIG. 3

Fifteen pictures such as the one shown in Fig. 2 we and they all gave the same characteristics.

Figs. 3, 4, and 5 are enlargements of exposures 20, 2 respectively. Fig. 3 shows the arc just before the globul The globule is suspended from the upper electrode as *A* and the arc is shown by the bright spot *B*.

In Fig. 4 the globule can be seen just as it starts across the arc. The globule shows up as a dark gray space at *A*. The arc in this print has become quite short.

Fig. 5 shows the arc after the globule has passed. The arc now is much longer than the arc shown in Fig. 3 due to the globule of iron passing across to the larger electrode below. It can be noticed that the arc length in Fig. 5 is just about the same as the length of the arc plus the length of the globule in Fig. 4.

These pictures show that metal can be transferred across the arc by means other than contact between the two electrodes. They also show that the transfer of metal is not necessarily accompanied by a decrease in the voltage across the arc. The voltage and current both remain approximately the same.

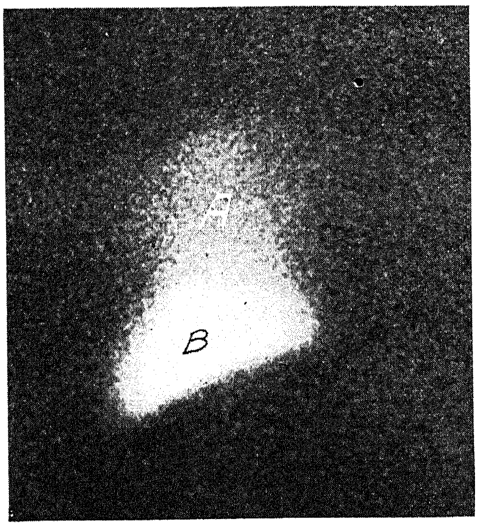


FIG. 4

G. E. Doan: So little is known about the welding arc that any data resulting from a carefully planned experiment, such as the present one, are sure to be important. For some years I have inclined toward the opinion that while the force called gravitation is the principal one in transporting arc metal in horizontal welding, that the force of first importance in overhead welding is the explosion of gases dissolved in the hot metal. If Mr. Creedy's electrodes contained gases from the beginning, I think their explosion or sudden expansion as they are heated by the arc might be responsible for at least a part of the back pressure which he has measured. These results would hold, therefore, only for electrodes of this certain gas content. The transfer by action of the "pinch effect" clearly explained by Mr. Creedy seems warranted except for one observation where it should operate but does not do so. Messrs. S. L. Hoyt and Gerald Brophy of the General Electric Laboratories observed several years ago that metal cannot be transferred upward, as in overhead welding, if the electrode is pure gas free iron, even in the hands of an expert operator. The metal merely melts and runs down the electrode somewhat as wax from a dripping candle. If the electrode contains gases, overhead welding is possible. For these reasons I consider the gas content of the electrode metal as important in determining the back pressure at the electrode and of greater moment than the pinch effect in overhead welding.

J. Leland Myer: *Pressure not necessarily due alone to particle emission from the electrodes.* The author has stated that the most reasonable hypothesis as to the origin of the force (on the electrodes) ascribes it to particles projected from the electrodes, particularly the cathode. The assumption of this same hypothesis led Tanberg[1] to the erroneous conclusion that the temperature of the cathode of a copper arc was 500,000° K. Compton[2] has shown that it is unnecessary to assign this high temperature to the cathode, as the force on the cathode can be easily accounted for by the "accommodation coefficient" of the incoming positive ions.

In addition to the original work of Duffield[3,4] referred to by the author, there is a series of interesting papers by Ratner,[5] Tyndall,[6] and Duffield,[7] wherein the exact origin of the particles and the nature of their forces are discussed. The conclusions are by no means definite. More recent work by Schmick and Seeliger[8] on the transport of material in the iron arc shows that the loss of iron at the cathode is not a linear function of time and current. This is in disagreement with the energy balance theories of Compton, Gunther-Schulze and Seeliger. It is evident that our knowledge in this general field is very meagre, and reliable data are very much needed. At any rate, it does not seem justifiable to assume that all the pressure on the electrode is caused by particle emission.

Polarity of the moving electrode not given. The author does not state whether the forces were measured on the anode or cathode of the iron arc. In view of the widely different forces measured by Beer and Tyndall[9] for the anode and cathode of a carbon arc, it would seem desirable to mention the polarity of the moving electrode in the present investigation.

Pinch effect not of sufficient magnitude to account for forces measured. The pinch effect has been applied to the arc as early as 1923 by Barlow.[10] Using Duffield's data, he found that the pinch effect accounts for less than 30 per cent of the observed force. Likewise an examination of Fig. 8 in Mr. Creedy's paper shows that the pinch effect contributes only a fraction of the forces actually measured.

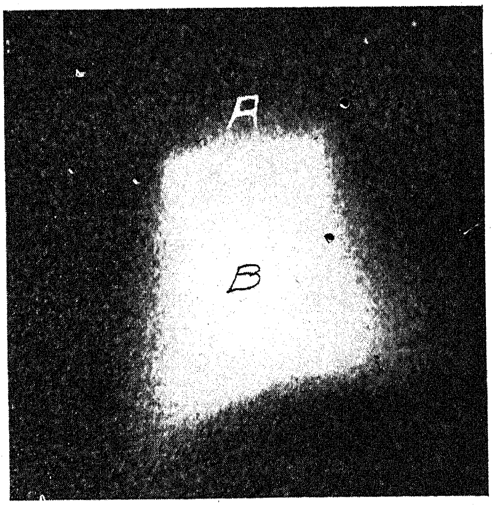


FIG. 5

For example, the pinch effect for a 100-ampere arc should exert a pressure of 50 dynes on each electrode, while the actual measurements indicate forces of 100 to 300 dynes. For 150 amperes the force of the pinch effect should be 112 dynes, while the measured forces lie between 440 and 800 dynes. The author's calculations show that the pinch effect of a 300-ampere arc gives just enough force on the average globule to counteract the action of gravity—allowing none to accelerate the globule upward. The discrepancy cited above is outstanding, and strongly suggests that there must be an additional force other than the pinch effect in overhead welding.

Gas content of the electrodes possibly an important factor. It might prove useful to know something of the quality of metal in the electrodes, particularly as concerns their gas content. The

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1. See References.

force on the electrodes might be expected to vary widely with different qualities of iron. For example, with a carbon arc, Sellerio[11] (using a torsion method also) noted that the carbon quality had a great deal more influence on the electrode repulsion forces than the arc length itself. Likewise Beer and Tyndall[9] made tests on the gas content of their carbons. On plotting time—pressure curves (maintaining constant arc length and current) they found actual pressure variations of 100:1 over a period of 12 minutes. After 12 minutes heating, the pressure reached a constant minimum value, and consistent data were then obtainable. Their final tests were carried out with degasified carbons which showed no pressure variations with time.

If gas be present in the metal electrodes, the intense heat of the arc will certainly cause gas explosions (*i. e.*, very rapid expansion of included gas upon sudden heating). It is believed that such gas explosions afford a reasonable explanation for—

1. The unusually sudden detachment of the globule, as shown in Fig. 1 and 2.
2. The sidewise spattering of metal from the molten tip of the metal electrode.
3. The excess force which must be added to the pinch effect to account for the actual forces observed.
4. The random distribution of points on the force—arc length curve of Fig. 6, as well as on similar curves of other observers.

An obvious test suggests itself, namely the repetition of these experiments with degasified iron.

F. Creedy: The discussion shows that the paper must have been badly phrased in one respect at least since not only Mr. Slepian but Mr. Myer have fallen into a serious misapprehension as to what was meant by it. It was not intended to maintain that the "pinch effect" had anything whatever to do with the passage of the globule from the electrode to the work. According to our view the function of the pinch effect is solely to separate the globule from the electrode and if there were no other forces in existence when the globule had been so separated it would fall to the ground under the action of gravitation. Our view is that the pinch effect separates the globule which is then taken across by the forces measured in the first part of the paper, and these forces, which have nothing whatever to do with the pinch effect and which are measured but not explained in the paper, are those which are responsible for carrying the globule over.

Referring to Mr. Ludwig's remarks we agree that as soon as the connection between electrode and globule is reduced to a small area a great amount of heat would be developed which would assist the explosive effect noted, but we also think that the explosive effect is partly due to the viscosity of the molten part of the electrode. Study of the mechanics of the pinch effect, as for instance described by Dr. Northrup and referred to in the paper, will show that its stability depends on the circulation of the liquid conductor. Where no circulation is possible, the reduction of area once started must go forward with greater and greater rapidity and finally with explosive violence until the conductor is completely severed. Thus the explosiveness is explained even without resorting to high temperatures.

All those who have taken oscillograms of welding must have noted that the arc is frequently short-circuited by a globule in the manner referred to by Mr. Ludwig, but the work done in our laboratories by Mr. E. C. Easton and Mr. F. B. Lucas as mentioned by them in the present discussion, has shown that it is quite possible for the globule to pass without any indication whatever in the oscillograms. Thus not all the globules short-circuit the arc and we have to explain the passage of those that do not as well as of those that do. I am obliged to Mr. Myer for the interesting references which he has given. At the same time I can not help feeling very skeptical as to the explanation of these forces as due to the explosion of occluded gases. Since the discussion took place, tests have been made with especially purified gas free iron in vacuum and forces as high as 4 grams observed. Some of the references that Mr. Myer gives refer to work done with carbon, in some cases especially degasified. It is hardly conceivable in any case that particles of carbon in the solid form should fly off in the manner supposed since the products of combustion of carbon are gaseous. This is still more true, of course, for the degasified carbons.

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An Improved A-C. Arc Welder

BY A. M. CANDY[1]
Member, A.I.E.E.

THE use of fusion welding processes has increased tremendously during the past decade having largely superseded the forge and hammer method of welding, due to the greater flexibility and continuity of the fusion processes. The two principal fusion processes are the gas (oxyacetylene) and the electric arc. This paper however will deal only with the arc process.

In lighting service both direct current and alternating current arcs have been used in the past with only detailed advantages being advanced for either because, with the electrodes used, either direct current or alternating current produces a reasonably satisfactory arc. In the welding field however the d-c. arc in the past has been used much more extensively than the a-c. arc, in spite of the fact that for a given rating the cost of the apparatus is less for an a-c. arc.

There are several reasons for this former feeling among which are:

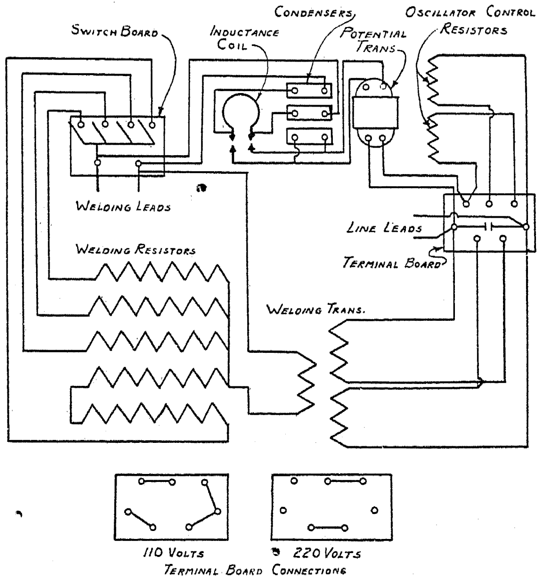

FIG. 1—100-AMPERE IMPROVED A-C. ARC WELDER

1. The open-circuit voltage of an a-c. welder is about double that of a d-c. welder thereby increasing life hazard.
2. An a-c. arc is much more difficult to strike and hold especially with bare welding wire and for vertical and overhead welding.
3. Low power factor.

The improved a-c. welder however eliminates these objections. The open-circuit (secondary) voltage is as low or lower than that of a d-c. machine. The arc is as easy to start and hold in any position under normal conditions as is a d-c. arc. In fact under certain conditions where an interfering blast (termed magnetic blow) will seriously affect the d-c. arc, the troublesome blow will be materially less when the new a-c. welder is used. The power factor at fractional loads is above that of a commercial induction motor used to drive a correspondingly rated d-c. welding generator.

The welding transformer proper shown as a part of the diagram Fig. 1 is perfectly standard in all of its design characteristics and features and is identical with standard commercial distribution transformers in construction. It is provided with a primary which can be connected in parallel or series for 220 volts or 440 volts, and with a secondary which delivers a practically constant potential of 73 volts which is used on the welding system. In this respect from the apparatus standpoint the equipment is very similar to d-c. multiple operator systems where the d-c. generator operates at a constant potential of approximately 60 volts and the current is supplied to the welding circuit through a series of control resistors for stabilizing the arc and for permitting the operator to adjust the current delivered at the arc to the value required for the work at hand. It is therefore apparent that the most interesting and novel portion of this a-c. welding equipment is that of the oscillator circuit.

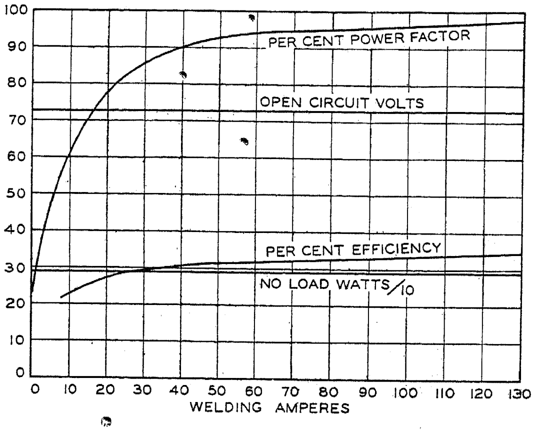

FIG. 2—PERFORMANCE CURVES OF AN IMPROVED A-C. ARC WELDER

100-ampere capacity, 110-220 volts, single phase, 50-60 cycles, 7½ kva.

Most of these and other desirable features are obtained by superimposing high-frequency oscillations on the welding circuit. The general basic idea of such use of an oscillating circuit is not new since it was

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[1] Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

(according to records) proposed in 1879. However we do not know of any previous commercial use of the idea in this country.

The scheme of connections for the welding circuits, oscillator circuits, and power input circuit is as shown by Fig. 1. The oscillator circuit comprises a small standard transformer, a condenser across the high side or secondary, two spark gaps, and an air core choke coil. The transformer is connected on the primary side to the power input for the welder, and supplies through its secondary the necessary voltage to charge the condenser and jump the spark gaps. This produces oscillation between the condenser and choke coil of a frequency determined by the relation between the capacity of the condenser and the inductance of the choke coil. This particular equipment is designed so that the frequency is below the radio broadcast band to prevent interference and as an additional assurance a condenser is connected across the power input terminals of the equipment. On the other hand the frequency selected is sufficiently high to preclude the possibility of any injurious physiological effects upon the person using the apparatus.

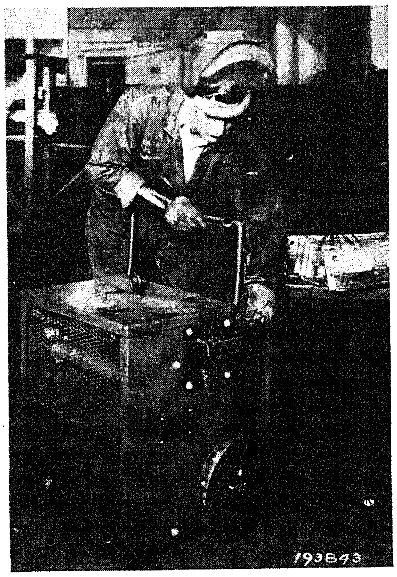

Fig. 3—Improved A-C. Arc Welder

To perform the desired functions it is necessary to have the oscillator circuit discharge between the welding electrode and the work but it is also necessary to prevent the welding circuit from supplying any energy to the oscillator, therefore two small condensers are placed in the circuit between the oscillator and the welding electrodes. These condensers block the low-frequency welding current but allow free passage of the high-frequency discharge through the welding electrode and the work. Similarly a condenser is connected across the input terminals to prevent the oscillator from feeding into the power circuit which might cause radio interference with sets in the immediate territory.

The voltage developed by the oscillator circuit is sufficiently high so that the discharge will actually start just before the electrode contacts the work. As soon as the discharge starts the ionized gases between the electrode and work produce a low resistance path for the welding current so that the arc starts very easily, in fact just as easily as a d-c. arc. Some operators actually prefer this circuit to some d-c. circuits because the arc can be started without actually making contact, thereby eliminating the sticking (freezing) of the electrode to the work.

So long as the welding arc length is maintained, the oscillator continues to discharge between the electrode and the work and eliminates the tendency for the arc to snuff out each time the welding current passes through zero. In previous designs of welding transformers these characteristics were approached by designing for an open-circuit secondary voltage of about 125 volts (compared with 73 volts for the improved transformer) and for high leakage reactance to throw the voltage and current out of phase so that when the current passed through zero the maximum voltage would be available to prevent the current from being interrupted. The old design naturally produces a very low power factor (25 to 35 per cent) compared with approximately 90 per cent for the improved transformer. (Fig. 2.)

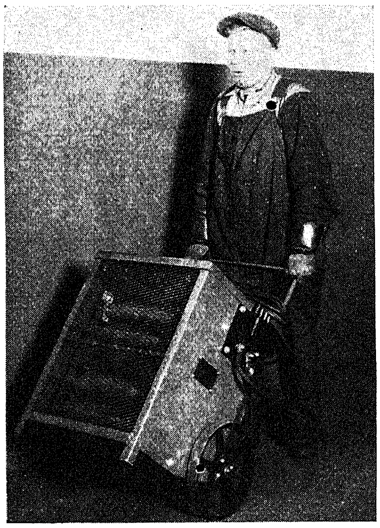

Fig. 4—Provision for Handling

The welding operator will not experience any bothersome shock from the welding circuit nor from the oscillator circuit because of the very high frequency of the oscillator circuit producing the well-known "skin-effect," and because of the low capacity of the oscillator circuit.

The mechanical construction of the new welder is both simple and sturdy, the general appearance being as indicated by Fig. 3. The welding transformer, oscillator, and current regulating resistors are assembled in an arc-welded sheet metal case, which is provided with wheels and a suitable handle for conveniently moving the unit as shown in Fig. 4.

The current regulating resistors are connected to five switches and are designed so that currents of 4, 8, 16, 32, 64 amperes, and any combination from 4 to 120 amperes in steps of 4 amperes may be obtained. It is therefore possible to adjust the current value to within 2 amperes of any desired value. The equipment is especially desirable for welding sheet steel and aluminum in the thinner gages below No. 16 in which field the d-c. arc has not been particularly satisfactory up to the present time.

The quality of the welds made with this improved a-c. welder is in every respect equal to d-c. welds. Many tests of such welds have been made and in one series of ten tests using hot rolled steel plate the average tensile strength was 52,000 lb. per sq. inch.

Because of the low no-load losses, low investment charges and low maintenance (no rotating parts) the a-c. equipment is very economical to operate especially when welding materials of 16 gage and less. It is probably destined for extensive use particularly in fields where the d-c. arc has not made much headway.

Discussion

J. B. Gibbs: For a long time inventors and designers have recognized that an a-c. welder would be very desirable, on account of its simplicity, low cost, and absence of moving parts. Several a-c. welders have been built, but they have not found wide acceptance, for the reasons given in the paper. The low power factor might perhaps have been tolerated on account of the economy of the apparatus, but the combination of danger to the operator and difficulty of operation has made the field for a-c. welders of the old type rather small.

The addition of the oscillator removes the difficulties which seemed to be inherent in a-c. welding. It is another example of the combination of well-known ideas to produce a useful result—a combination which having been perfected is of obvious advantage, but however, requiring a breath of vision and a knowledge of the art.

The new a-c. welder is essentially a transformer. The details which are added to secure portability, current control, and stability of arc are of importance but they are so simple that the whole apparatus can be built in a factory designed exclusively for the manufacturing of transformers.

The quality of the welds made with this a-c. welder is fully up to the high standard which is expected of d-c. welds. In the series of tests mentioned in the paper, the hot rolled steel plates were welded together, machined to a standard dimension, and tested to destruction. The final failure occurred sometimes in the parent metal and sometimes in the deposited metal, indicating approximately equal strength.

The interest displayed in this welder since its announcement a few months ago indicated that it meets an existing need and that it is now feasible and economical to apply electric arc welding to many uses for which, heretofore, it has been found unsuitable.

R. E. Hellmund: During the last eight or ten years a great deal of discussion has taken place regarding the relative merits of a-c. and d-c. arc welding and the possibility of using a-c. welding to a greater extent. However, up to this time the a-c. welder has not been commercially exploited to any extent in this country. This is rather surprising in view of the fact that such welders have been commercially available in Europe for almost ten years. Quite a number of them was exhibited in England about 1924 or 1925, and on the Continent both single-phase and three-phase a-c. welding were carried on commercially about that time. Shortly after this, about 1925 or 1926, one of the first a-c. welders with high-frequency "keep alive" was commercially available in France and it gave reasonably good results as far as the weld was concerned. Unfortunately it was possible for the operator to receive severe and disagreeable shocks under certain conditions, a factor which of course would not have made this welder very popular in this country and which carried with it a certain hazard, not so much from the danger of the shock itself as from the fact that the reaction from the unexpected shock might result in other accidents, such as falling, etc.

Possibly it is to be regretted that a-c. arc welders were not available earlier in this country, as they might have hastened a more general introduction of arc welding because of their somewhat lower cost, greater portability, and the absence of a rotating member and consequent reduction in attendance and maintenance. On the other hand, if we consider that arc welding in general in the absence of reliable test methods for the finished weld had to overcome many prejudices, it may be fortunate that the inferior welds likely to be obtained with the earlier types of a-c. welders were not added to the many other difficulties which had to be overcome in the introduction of arc welding. Now that an a-c. welder which is not only equal but in some respects superior to the d-c. welder is available, it will most likely open up new fields and broaden the application of arc welding.

C. J. Holslag: As we were the first ones to manufacture a-c. welders we are naturally interested in the article presented by Mr. Candy. A description of our welders was published by the American Institute of Electrical Engineers, 1919.

The open-circuit voltage of an a-c. machine is not generally double that of a d-c. welder as stated by Mr. Candy. At the present time, the use of a "shielded" arc makes it necessary to have about 70 volts open-circuit voltage in d-c. welding. This is the open-circuit voltage of the a-c. arc welders we are now manufacturing and is the same as the a-c. welders now manufactured in Europe.

Power factor and efficiency with either alternating current or direct current are closely related. In fact, the power factor depends on how much loss there is in the welding circuit. The best conditions for welding, either alternating current or direct current, require some dead resistance. With an a-c. transformer we can weld without resistance—this results in an efficient apparatus but a poor power factor. With 75 volts open circuit and 35 volts across the arc the power factor is around 50 per cent. If resistance is introduced in the circuit, the power factor will be raised. This is also true in the d-c. circuit except that the power factor is better because of the higher loss in the motor and generator. In general, with an a-c. welding transformer we have an efficiency of 80 per cent and the power factor of 50 per cent; whereas, in the d-c. circuit the total efficiency is around 50 per cent and the power factor 80 per cent. In other words, there is about the same current demand on the main line.

Mr. Candy's second point is decidedly controversial. We would also like to have Mr. Candy comment on the safety of his circuit from the operator's standpoint especially for transformers of the larger size.

A. M. Candy: Output capacity of the welding transformer has no bearing on the safety to the operator inasmuch as the voltage of the welding circuit remains the same.

Recent Developments in Design of Arc Welding Generators

BY K. L. HANSEN[1]
Associate, A.I.E.E.

THE rapid expansion of arc welding in recent years has served in an ever increasing degree to focus attention on the design and operating characteristics of arc welding generating equipments. Up until a few years ago the d-c. arc welding circuit invariably constituted a welding system rather than a welding generator. This applies even to the variable voltage individual operator sets as well as to the constant potential multiple operator equipments. Generally speaking, the so-called arc welding generator does not possess all the characteristics required for arc welding, and to compensate for this, external auxiliary devices of one kind or another are introduced. The generator to be discussed in this paper is designed so as to have inherently all the characteristics required for arc welding without the assistance of any external auxiliary apparatus whatever, and is, therefore, in every respect an "arc welding generator."

RESISTANCE CHARACTERISTICS OF ARC WELDING CIRCUIT

It is well known that the negative resistance characteristic of the arc makes it inherently unstable when operating from a constant potential circuit. To overcome this instability, the circuit must have a drooping volt-ampere characteristic; that is, the voltage must decrease with increasing current. Individual operator welding generators are, therefore, designed to have a drooping volt-ampere curve.

When there is no actual transfer of material across the arc so that its inherent resistance characteristic is the only cause for instability, the requirements are not very exacting. For example, very little more than a drooping volt-ampere characteristic is required to maintain a carbon arc. However, when there is metal transferred across the arc, as in metallic arc welding, and abrupt changes in the arc resistance are caused thereby, the conditions become much more complex and exacting.

When welding with a mild steel, bare, or lightly flux coated electrode, oscillograms show that the voltage across the arc drops to zero at more or less regular intervals. This drop of voltage takes place whenever a drop of molten metal goes across the arc and thus momentarily short-circuits the arc, at which instant the resistance of the arc is reduced to zero.

The time during which the arc remains short-circuited is exceedingly short, being on the order of one to two hundredths of a second. As compared with the length of time on short circuit, the average interval of time between drops is approximately four to five times the length of time on short circuit. The resistance of the metallic arc, therefore, varies rapidly between the value zero and such a value of resistance as to make the drop across the arc at normal welding circuit equal to the normal arc voltage.

The variations in arc resistance resulting from metal transfer across the arc are much wider and much more rapid than variations in the arc resistance caused by its inherent negative resistance characteristic. Because of the predominating character of these resistance fluctuations, the circuit to which an arc welding generator is delivering energy consists, from the standpoint of the generating equipment, of a resistance which is periodically being short-circuited at a rate of from ten to twenty times per second.

The equivalence of the metallic arc to a circuit, the resistance of which varies continuously and rapidly from a definite value to zero and back to its original value, is of great importance in the mathematical investigation of the instantaneous values of current fluctuation in welding generators. In a mathematical paper,[2] the writer made the following statement:

> It must, of course, be understood that it is not attempted to calculate the current fluctuations in the arc circuit when welding is being performed, but merely the momentary increase or decrease in current that would take place if the resistance in the main circuit is suddenly changed from one value to another; as, for example, from open circuit to short circuit. It is, however, fair to assume that any special features in design of the machine which will reduce the fluctuations when calculated in this manner will also reduce the fluctuations during the welding operation and thus help to stabilize the arc.

It has been argued against this method of treating the problem that the arc does not obey Ohm's law in so far as the voltage across the arc is not proportional to the current, but remains approximately constant over wide range of current, and that the arc should not, therefore, be considered a resistance, but an independent electromotive force. However, this objection to the method used in the writer's earlier paper does not appear to be well taken.

Regardless of whether the arc obeys Ohm's law or not, it is in its nature a resistance inasmuch as it always consumes electrical energy and voltage and is not a source of voltage. Furthermore, the reaction of the machine to abrupt and wide changes in the external resistance is, of course, precisely what it is desired to

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[1] K. L. Hansen Engg. Co., Inc., Milwaukee, Wisconsin.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

[2] *Design of Constant Current Generator for Arc Welding,* TRANS. A.I.E.E., Vol. XXXIX, Part 2, 1920.

know. Indeed, all tests that have so far been suggested for measuring the ability of welding generators to meet properly the exacting conditions of metallic arc welding consist of methods for determining the amount and duration of the momentary increase or decrease in current when the external resistance is suddenly changed from one value to another.

TREND IN DESIGN

The importance of minimizing the momentary current fluctuations, or transients as they are frequently called, is clearly discerned in the most recent modifications made in design of arc welding generating equipments.

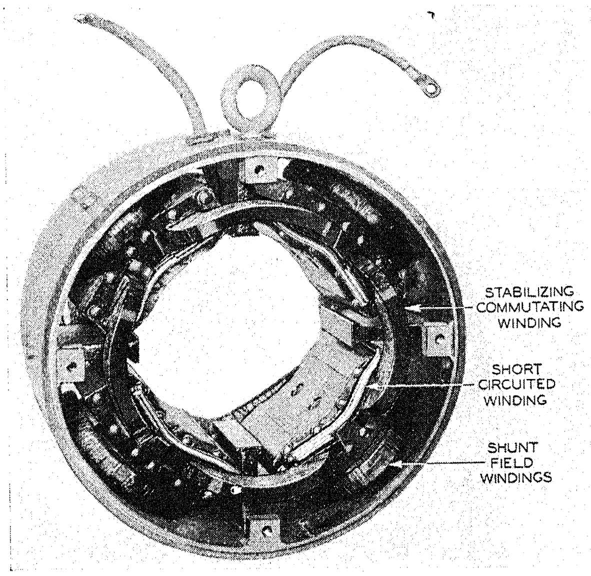

FIG. 1—COMPLETE STATOR OF 200-AMPERE HANSEN ARC WELDER

Publications which have appeared lately on the subject indicate that considerable effort has been directed toward that end. For example, in some makes of apparatus the well-known reactor, which has been used extensively to smooth out the current fluctuations, has been modified to utilize impulses from it to speed up the flux changes in the field. One such modification, known as the "flexactor," has been described by Blankenbuehler[3] and another, the "transformer reactor" by Bergman.[4] These modifications of the reactor have apparently resulted in marked improvement in operation.

In the writer's paper referred to, the desirability of eliminating the external stabilizer was strongly emphasized. Furthermore, it may be of interest to quote from this paper comments made therein on the self-excited machine in use at that time, that is, about a dozen years ago. These comments referred to the well-known system of self-excitation using a two-pole armature in a four-pole field structure and a third brush. The quotation follows:

> Since there are only two sets of main brushes, the commutator must be longer than in the four-pole machine to keep the same current density in the brushes. On the whole the material is not so economically employed as in the four-pole generator with separate excitation.
>
> Furthermore, since the series field turns generally have to be changed for different current values, either by shunting or by taps, the current adjustment is not comparable to the fine adjustment that can easily be obtained over a wide range in the separately excited machine.
>
> On the whole, the writer feels that the advantage obtained by eliminating the separate excitation is not sufficient to compensate for the disadvantages mentioned. Undoubtedly efforts will be continued in this direction, however, and if a generator can be developed possessing all the desirable features of the present separately excited machine without the need of separate excitation, the advantage of such a machine is obvious.

It probably will be conceded that an ideal welding generator is one in which:

(1) The external reactor has been eliminated, provided it has been accomplished without introducing undue complications.

(2) Current impulses in the main circuit are reflected into the field to quicken flux changes in the field and thereby make the machine more responsive to changes in the external resistance.

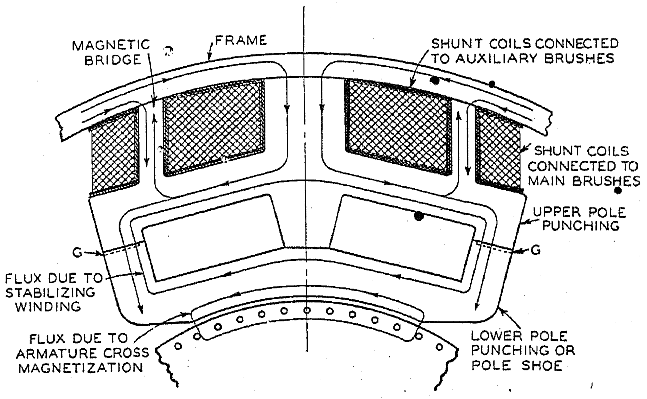
MAGNETIC BRIDGE
FRAME
SHUNT COILS CONNECTED TO AUXILIARY BRUSHES
SHUNT COILS CONNECTED TO MAIN BRUSHES
UPPER POLE PUNCHING
G
G
FLUX DUE TO STABILIZING WINDING
FLUX DUE TO ARMATURE CROSS MAGNETIZATION
LOWER POLE PUNCHING OR POLE SHOE

FIG. 2—CROSS SECTIONAL VIEW OF MAIN POLES AND SHUNT COILS OF 200-AMPERE WELDER

Stabilizing and short-circuit windings omitted

(3) The exciter has been eliminated without resorting to the two-pole armature four-pole field structure.

(4) All regulating rheostats and switches have been eliminated and still a wide range of welding currents can be obtained in fine steps which will have desirable arc characteristics over the whole range.

A generator which, in the writer's opinion, fulfils these requirements is briefly described in the following section.

COMPLETELY SELF-CONTAINED ARC WELDING GENERATOR

The first two of the objects enumerated above have been attained by a novel design of the commutating winding, shunt windings and main poles, and by adding a special winding consisting of one turn short-circuited upon itself.

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[3] *An Improved Arc Welding Generator,* TRANS. A.I.E.E., Vol. 50, June 1931, p. 657.

[4] *A New System for D-C. Arc Welding,* TRANS. A.I.E.E., Vol. 50, June 1931, p. 678.

The armature is similar to a standard d-c. armature, except that it is wound with a relatively large number of turns and correspondingly large number of commutator bars. In arc welding generators, this is desirable for several reasons.

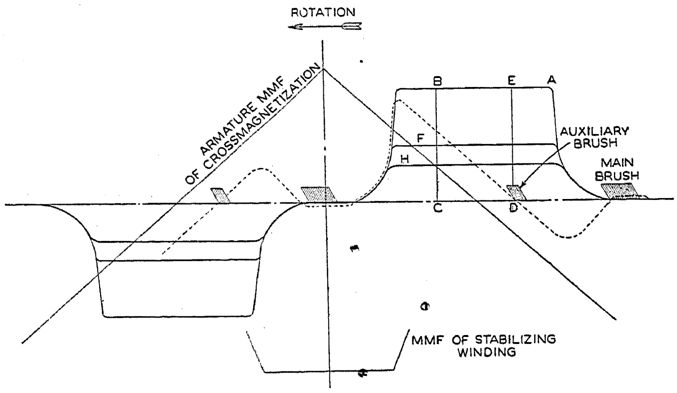
ROTATION
ARMATURE MMF OF CROSSMAGNETIZATION
B
E
A
F
H
AUXILIARY BRUSH
MAIN BRUSH
C
D
MMF OF STABILIZING WINDING

Fig. 3

It is generally recognized that with a high number of turns on the armature a comparatively great change in the terminal voltage requires only a small change in the field flux, which makes the machine more responsive to external resistance variations. Furthermore, with a high number of commutator bars, the volts per bar is low, which becomes a marked advantage in connection with the scheme of excitation to be described later.

Fig. 1 shows the field ring with all poles and field windings in place. It will be noticed that the commutating coils, instead of being wound in the usual way on the commutating poles, have been extended to and completely imbedded in the main pole flanges. The commutating winding thus extended and surrounded by laminations, functions both in capacity of stabilizing winding and commutating winding. The desired stabilizing effect is obtained without adding any windings to those ordinarily employed in commutating pole machines.

The method of assembling the pole flanges around the stabilizing winding can readily be observed from Fig. 1, assisted perhaps by a reference to Fig. 2. The pole, it will be noticed, is made up of two parts, the upper and the lower part or pole shoe. The upper part is bolted to the frame. The stabilizing coil, which has been wound and formed to the proper curvature on a press, is then laid in slots in the upper part of the poles. The pole shoe is then put in place and bolted to the upper part.

From Fig. 2 it will also be noticed that the generator has two shunt coils separated by a magnetic bridge. The function of this arrangement and of the short-circuited winding shown in Fig. 1 will be described later. The pole shoe, it will be seen, serves as a common magnetic path for the stabilizing flux and the flux due to armature cross-magnetization.

It may be pointed out here that the axial length of the back of the pole, that is, the part bolted to the frame, is considerably shorter than the axial length of the part of the pole constituting the magnetic circuit of the stabilizing winding.

Excitation and Current Control

The well-known reentrant volt-ampere curve of the shunt generator is not at all suited for arc welding. It is essential that the permanent short-circuit current be equal to or greater than the welding current. For that reason some source of excitation must be available even on short circuit when the voltage across the main brushes is zero. Reference to Fig. 3 will show how this is accomplished.

In addition to the main brushes, the machine is equipped with two sets of auxiliary brushes spaced from 50 to 60 electrical degrees ahead of the main brushes. These excitation brushes have a very high contact resistance and can without commutation difficulties handle a much higher voltage per bar than is ever reached in these generators, which, as already pointed out, is unusually low.

Line *A* represents the no-load field form. It will be seen that the auxiliary brushes take in part of the negative portion of the field form. The voltage across the auxiliary brushes on open circuit is therefore proportional to the rectangle *BCDE*. What will be the condition on some load, for example, on short circuit? Assume that saturation is negligible on short circuit so that fluxes are proportional to m.m.fs. The voltage on the shunt coil excited from the main brushes falls to zero on short circuit and the m.m.f. of the field is reduced to line *F*. In addition, there is direct demagnetizing force from the armature which reduces the

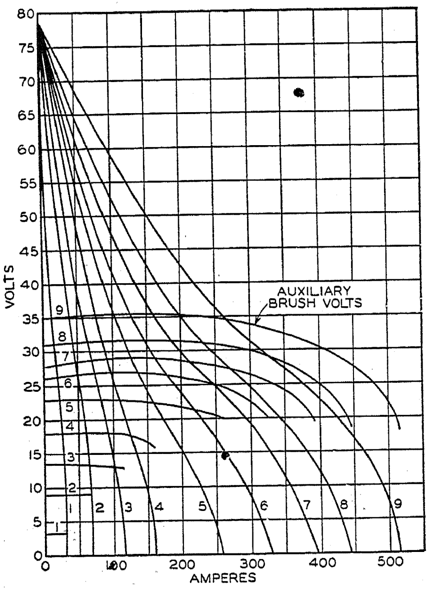

Fig. 4—Volt-Ampere Curves of 300-Ampere Hansen Arc Welder

Nos. 1, 2, 3, etc., indicate different brush positions m.m.f. on the field still further to line *H*. The remaining m.m.fs. are those due to the armature cross-magnetization and the stabilizing winding.

The distribution of flux in the air gap under the main poles is not directly affected by the m.m.f. of the stabilizing winding. It is indirectly affected by this m.m.f. when the pole shoe becomes saturated, but neglecting that for the moment, the flux distribution is the resultant of the net m.m.f. of the field and the m.m.f. of armature cross-magnetization. This is shown in Fig. 3 as a dotted line. The area included between this line and the base line, and lying above the base line, represents positive flux, and similarly the area below the base line represents negative flux. It will be noticed that on short circuit there is still a considerable positive flux between the auxiliary brushes, whereas the net positive flux between the main brushes has been considerably reduced.

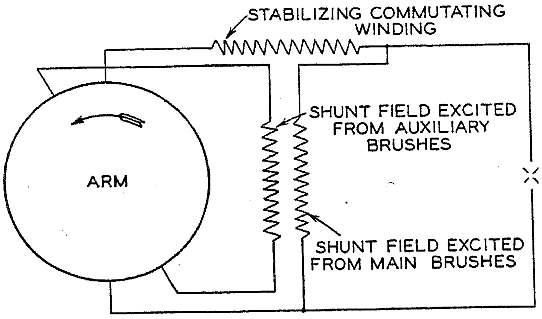
STABILIZING COMMUTATING WINDING
ARM
SHUNT FIELD EXCITED FROM AUXILIARY BRUSHES
SHUNT FIELD EXCITED FROM MAIN BRUSHES

Fig. 5

When the main and auxiliary brushes are shifted forward simultaneously, two phenomena become pronounced. The armature demagnetization is increased and the voltage on the auxiliary brushes is decreased. Both of these tend to reduce the current so that a wide variation in current can be obtained with a relatively small shift of the brushes. The brushes are shifted by means of a micrometer screw arrangement, making possible minute graduations of welding current.

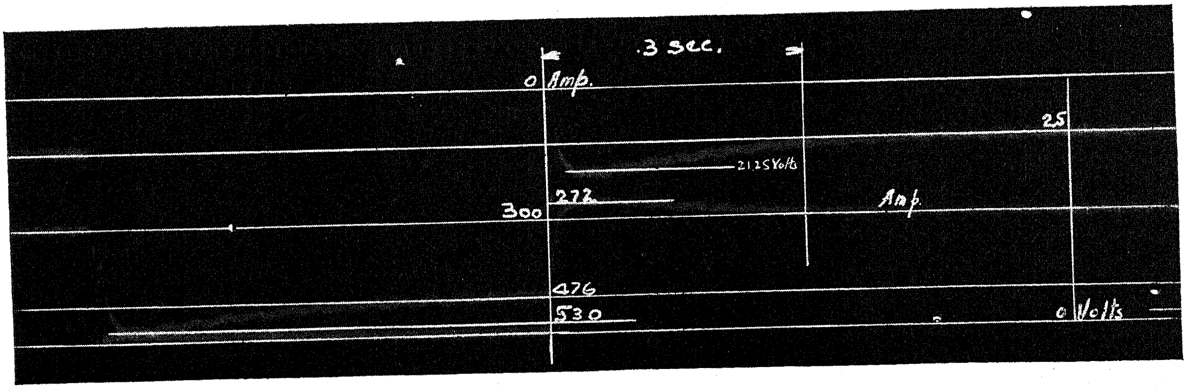

Fig. 7—Oscillogram of 300-Ampere Welder When External Resistance is Changed from Zero to Give 25 Volts Drop at 300 Amperes

Fig. 4 shows a set of volt-ampere curves of a 300-ampere welder. It will be noticed that for larger current values the voltage on the auxiliary brushes remains about constant up to about normal current for that setting and then drops off some at short circuit. However, even at short circuit there is plenty left to maintain a sufficiently large value of short-circuit current. For lower current values, the tendency for the auxiliary brush voltage to remain substantially constant down to short circuit becomes more pronounced.

It has been mentioned that when the pole shoe which carries the flux of both the stabilizing and the armature cross-magnetization becomes saturated, the air gap distribution, and consequently the compounding of the auxiliary brushes, are affected thereby. This saturation can, therefore, be modified by introducing air gaps in the path of the stabilizing flux, as, for example, at points marked *G* in Fig. 2.

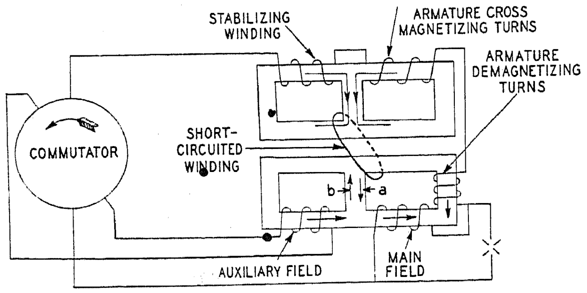

Fig. 6

Momentary Current Fluctuation and Arc Recovery

Fig. 5 is a diagram of the actual circuit of the machine, except that the one turn short-circuited winding is not shown. However, this diagram is of no assistance in forming a mental picture of the interactions between the various circuits during transient conditions, and an equivalent diagram to facilitate this has been shown in Fig. 6. It should be understood that this is not an actual diagram of the circuits, the separation of the armature cross-magnetizing and demagnetizing turns, for example, being for convenience only.

The stabilizing winding and armature cross-magnetizing turns are shown opposing each other with a common magnetic circuit, the pole shoe, between them. The armature demagnetizing turns are shown opposed to the two shunt fields and it should be observed that it is the flux within these turns which is responsible for the voltage at the main brushes. The magnetic circuit, encircled by the auxiliary field winding, is the main pole body, and the magnetic bridge is interposed between the auxiliary field and main field windings. In the absence of saturation there is no mutual induction between the stabilizing winding and the two shunt windings, so they have been shown as having separate magnetic circuits.

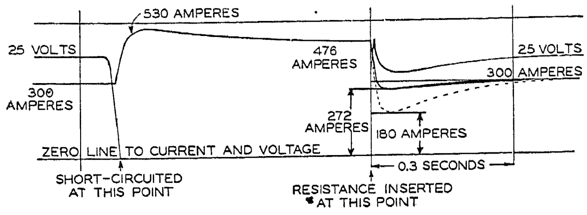

FIG. 8

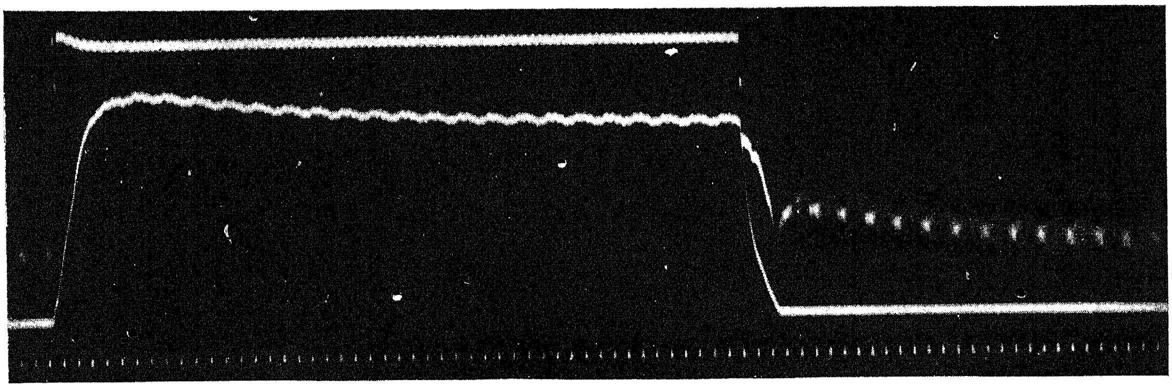

FIG. 9—OSCILLOGRAM OF CURRENT AND VOLTAGE OF 300-AMPERE WELDER IN PASSING FROM OPEN CIRCUIT TO SHORT CIRCUIT AND BACK TO OPEN CIRCUIT

The two shunt coils are excited in the same direction, and unless the main pole body is saturated, little flux passes through the magnetic bridge on open circuit. What flux there is passes through in the direction indicated by arrow *a*. On short circuit the m.m.f. of the main field winding disappears and an opposing m.m.f. of armature demagnetization appears. Now a considerable amount of flux passes across the bridge in the direction of arrow *b*; that is, instead of reducing the flux in the main pole body, which flux interlinks with the auxiliary field winding, most of it is on short circuit diverted through the bridge. The result is that the energy stored in the auxiliary field changes but very little as the terminal voltage varies up and down. The advantage of eliminating the necessity of changing the energy stored in this field when the external resistance varies is obvious.

To make the machine still faster in response to variations in the external resistance, the short-circuited winding is made to form a link between the stabilizing winding and the field windings. When the current in the stabilizing winding increases rapidly, a current will flow in the short-circuited winding, assisting in reducing the field flux. When the current in the stabilizing winding is decreasing rapidly, the transient current in the short-circuited winding assists in building up the main flux. In this method of utilizing impulses from the arc circuit to speed up flux changes in the field, there are no high potentials generated in any circuit at any time to put undue strain on the insulation.

The A.I.E.E. test of a welding generator for momentary current fluctuation and arc recovery is as follows:

Adjust generator on resistance load to obtain normal current at 25 volts. Short circuit one-half the resistance and allow current to settle. Then suddenly open the short circuit and reinsert the resistance in circuit. The current momentarily drops below normal and the time of recovery is the time required for the current to return to within 5 per cent of its original value.

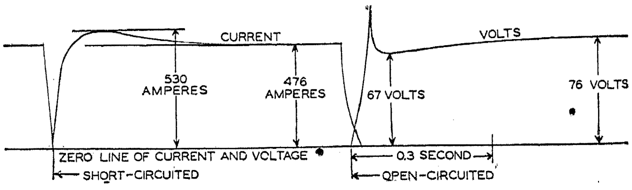

FIG. 10—OSCILLOGRAPH RECORD OF CURRENT AND VOLTAGE OF 300-AMPERE WELDER IN PASSING FROM OPEN CIRCUIT TO SHORT CIRCUIT AND BACK TO OPEN CIRCUIT

This test is of little value because the resistance variation is not wide enough to cause any noticeable depression of current below normal upon reinserting the resistance in the circuit; that is, if the machine is worth being termed an arc welding generator. The Navy specification calls for the same test for arc recovery with the exception that all of the resistance must be short-circuited and then reinserted. This test means something.

An oscillograph record of a test made according to Navy specifications at normal load on a 300-ampere welder is shown in Fig. 7. The timing is a standard 60-cycle wave. As the current and voltage scales are in opposite directions on the oscillogram, it has for convenience been replotted to scale in Fig. 8. The dotted line indicates the drop in current and the time of recovery permitted by the Navy specifications. It will be noticed that whereas the Navy permits a drop of current to 180 amperes, it actually drops to only 272 amperes.

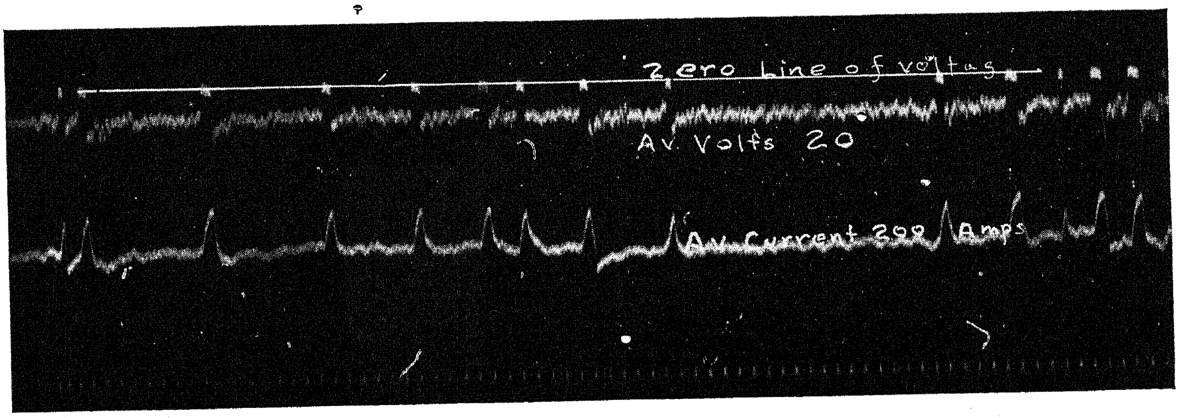

FIG. 11

Fig. 9 shows the record of current and voltage of the same machine in passing from open circuit to short circuit and back to open circuit. As in the preceding case, the voltage and current scales are in opposite directions, and for that reason voltage and current have been replotted to scale in Fig. 10.

Fig. 11 is a record of welding current and arc voltage when welding with a bare, mild steel electrode; average welding current, 200 amperes; average arc volts, 20.

Discussion

L. R. Ludwig: Mr. Hansen makes the point in his paper that the chief cause of the welding arc instability is the frequent short-circuiting of the arc by molten drops. Instability apparently is taken as meaning that the arc will extinguish, and must be manually restruck. The paper states in particular that when there is no transfer of material across the arc its inherent resistance characteristic is the only cause of instability and further, the carbon arc is cited as being very stable because there is no transfer of material. This stability of the carbon arc is more probably due to another reason and that is the thermionic nature of its cathode, which has been carefully discussed by K. T. Compton. The cathode of the usual welding arc is probably not thermionic as evidenced by its rapid motions.

The paper also states that variations in arc resistance resulting from metal transfer, are wider and more rapid than variations in arc resistance caused by its inherent negative resistance characteristic. It is difficult to see why the short-circuiting of the arc by drops will cause it to be unstable since the arc certainly restrikes with ease when the drop breaks away from the electrode. The only requirement placed on the generator by this bridging action is that its current must not rise to a high value during the short circuit. On the other hand the welding arc is subject to many rapid variations of voltage and current which the generator is called on to follow if the arc is to be stable.

One of the principal causes of instability may be variation in gas dissociation about the arc as suggested by Alexander in 1928. At any rate, there does not seem to be sufficient evidence just now for believing that short-circuiting of the arc by drops is the chief cause of instability.

The generator described by Mr. Hansen is a clever design which certainly combines into one machine all of the major requirements of a d-c. welding apparatus. The question of the desirability of such a combination is an economic one; the use of two associated pieces of apparatus would certainly have no disadvantage from a technical standpoint, but such design may have advantage from a standpoint of cost and simplicity.

The characteristic curves of Mr. Hansen's generator shown in his Fig. 4, can hardly be accepted as ideal. The stability of an arc is increased if the change in terminal voltage is made as great as possible for a given change of current. It is for this reason that an arc is more stable on a high-voltage circuit than on a low-voltage one. In order to avoid high open-circuit voltage the ideal welding generator characteristic would be quite steep in the region corresponding to about 25 volts, and flat in the vicinity of the open-circuit voltage, which is almost the opposite of the curves given by Mr. Hansen. Transient characteristic curves of course are of utmost importance, but they cannot be steeper than the so-called static curves; that is, those taken point by point.

R. E. Hellmund: Mr. Hansen has discussed in his paper an exceedingly interesting arrangement of an arc welding generator, which by skillful design can most likely be made to operate satisfactorily. I doubt, however, that the definition of an "ideal welding generator" as given at the beginning of Mr. Hansen's paper is correct for present-day manufacturing conditions. The latter dictate that in order to obtain the best economic results, all apparatus and arrangements, although they may accomplish special results, should be an assembly of standard parts manufactured in quantity. This is essential both for the sake of minimum manufacturing costs and of general availability of spare parts for nation-wide distribution at a minimum investment. While I can conceive that the solution offered by Mr. Hansen may work out satisfactorily for a small concern, it certainly is not the ideal solution for the large manufacturer. As far as the user of the apparatus is concerned, it is immaterial to him whether the desired results are obtained by a theoretical combination in a single machine or by a well-designed physical combination of standard parts; possibly he would prefer the latter in order to facilitate his renewal part problem.

Conditions as just described are often most discouraging to the engineers originating such arrangements as covered in the paper, but inasmuch as we are dealing with practical engineering questions we cannot overlook the economical factors mentioned in this connection. They are uncompromising and have prevented the commercial exploitation of many otherwise very ingenious arrangements. Of the many hundred schemes which have been proposed for combining the functions of several pieces of apparatus into one, very few have commercially survived; of those which have, the most outstanding in this country are the synchronous rotary converter, which is extremely simple, and the third-brush generator used in automobiles, which is also very simple and does not present great difficulties on account of the low voltage of the commutator and which, furthermore, is manufactured in large quantities for its own specific application. All this merely means that conditions have to be particularly favorable if specialized combination arrangements are to win out over combinations of standard parts.

D. W. Ver Planck: From a theoretical standpoint, at least, the Hansen generator is very interesting.

In all welding generators so far described it is necessary, in order to obtain the desired volt-ampere characteristic, to have a change in the amount of field flux linking the armature, as the terminal conditions vary from open circuit to short circuit or vice versa. The method of accomplishing this in separately excited machines is to cause the total flux to decrease toward zero by cancelling out the exciter m.m.f. with a m.m.f. derived from the welding current. In the self-excited machine described by Mr. Bergman* the flux from one pair of poles remains constant while that from the other set is reversed by means of a series field carrying welding current. Both of these methods require that the net number of flux linkages of the entire field circuit change by an amount equal to a little less than their no-load value. The Hansen generator is unique in that its main field flux is not changed in magnitude, but is diverted from the armature into the magnetic bridges between the coils. This involves little change in the auxiliary field linkages but the main field linkages are decreased to nearly zero. The total net change in flux linkages for both fields is somewhat less than *one-half* of the normal no-load linkages. This is an excellent point in the Hansen machine and indicates that equally as good transient characteristics as those of the Bergman type could be obtained by the use of a small corrective device.

The oscillogram of Fig. 9 when compared with the one given by Mr. Bergman seems to indicate that the two machines have about the same transient performance. However, closer examination shows that the short-circuit on the Hansen machine was interrupted quite slowly, six times more slowly than on the Bergman machine. Now it can be shown by analysis, and experience bears this out, that "instantaneous recovery voltage," or the minimum of the recovery voltage, is higher the more slowly the short circuit is interrupted. This emphasizes the need for making tests of this kind on a standardized basis so that a comparison of results will not tend to be misleading.

Referring to the volt-ampere characteristics of Fig. 4, the question arises as to whether or not the nearly perpendicular approach of the curves to the zero axis is desirable. It would seem that this is approaching dangerously near to an unstable condition.

The use of a third brush as it is done in the Hansen generator appears to be a somewhat questionable practise because this brush must commutate coils which are well under the poles where the generated voltages are high. The use of a high resistance brush may limit the short-circuit current in these coils, but even so one would suspect that considerable sparking with the consequent commutator blackening would result when the machine began to get dusty. It should be noted that the use of the excitation brush in the Bergman generator is quite different from this as it commutates coils which are in the neutral zone between the cross and main poles.

K. L. Hansen: Mr. Ludwig is undoubtedly correct in his statement that the stability of the carbon arc is conditioned by the thermionic nature of its cathode. The point which I desired to bring out was that if very wide and abrupt variations in arc resistance are superimposed on the resistance variations due to the inherent instability, the conditions, from the standpoint of the generating equipment, become much more severe and exacting than if the inherent instability alone has to be taken care of.

For example, let us assume that a carbon arc is so arranged that its negative electrode periodically touches the positive at a rate of 15 to 20 times per second, separating the electrodes to a distance of about one inch between strikes, and each time, at the moment of striking, a portion of the negative electrode is left deposited on the positive. Under these conditions, the resistance of the arc will vary between wide limits, and the conditions for thermionic emission from the cathode will change with each stroke. Also assume, as frequently happens in metallic arc welding, that there are forces present, such as magnetic fields, tending to disrupt the arc. The reestablishment of the arc after each extinction on short circuit would then probably not be so readily accomplished as Mr. Ludwig seems to think. At any rate, the demand upon the welding generator to maintain a uniform current under these conditions is without the slighest doubt much more severe than if it simply had to maintain a carbon arc with approximately constant distance between the points of the electrodes.

However, the only purpose for bringing this discussion into my paper was to justify my contention that the mathematical analysis of the arc welding generator can best be approached by investigating its reaction to wide and abrupt changes in the resistance of the external circuit. Mr. Ludwig, I believe, agrees with me on this point.

Before introducing the latest innovation, the magnetic bridge, in the Hansen arc welder, it had a volt-ampere characteristic substantially like the one described by Mr. Ludwig as being the ideal one. The introduction of the bridge, modifying the volt-ampere characteristic to the one shown in the paper, immeasurably improved the machine for welding. Whether or not the modification in the volt-ampere curve had anything to do with this improvement in operation, or whether it was entirely due to improvement in the transient conditions, such as stability and arc recovery, is problematical.

Mr. Hellmund states that the arrangement discussed in the paper "*can* most *likely* be made to operate satisfactorily." This and other statements in his discussion lead me to believe that he labors under the impression that the Hansen arc welder is just being introduced as an untried novelty. He may be interested to learn that the Hansen arc welder, in substantially its present form, has been on the market since 1927.

The number of machines now in operation is well on the way to the second thousand, one customer having had for the past three years 327 machines in operation in factory and field. Mr. Hellmund may have been influenced by the title of the paper. The more recent developments which the title has specific reference to are the short-circuited winding and the magnetic bridge, the latter being the latest innovation. However, as the distinctive features of excitation and stabilization had not been previously described in any technical publication, the machine was discussed as a whole.

It may be remarked that the introduction of the bridge has very markedly improved the welding characteristics of the generator, entirely overcoming occasional criticism in the operation of the earlier models. There is, therefore, every reason to believe that the machine will be even more favorably received in the future than it has been in the past. I wish to emphasize this because the justification for developing a distinct generator to

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**A New System for D-C. Arc Welding*, by S. R. Bergman, A.I.E.E. TRANS., June 1931, p. 678.

meet a specific application naturally depends on the number of units that eventually go into operation. When this number is large enough the machine obviously becomes a "standard" machine for that application, and the parts are regularly carried in stock.

In developing the Hansen arc welder, every effort was exerted to reduce the number of parts to a minimum. The reduction of the number of electric circuits or windings to less than one-half the number required when the usual auxiliary devices, such as stabilizer and exciter, are used, is an indication of how well that effort has succeeded. Furthermore, attention was concentrated on the elimination of those parts which experience has shown to be most vulnerable in operation.

The result has been to reduce the demand for spare parts, aside from brushes, and on rare occasions bearings, both of which are standard, to an astonishingly small amount. This was considered a much more satisfactory solution of the renewal problem from the user's standpoint than furnishing an excess number of standard parts which can readily be supplied from convenient distributing points, although the latter may have some attractive features from the manufacturer's standpoint.

As compared with another standard make of welding equipment, the elimination of unnecessary parts by correct design of the generator itself has made possible a reduction in weight from 35 to 47 per cent. Or, putting it the other way, the excess baggage required for the external auxiliary devices ranges from 54 to 90 per cent. Conservatively estimated, the average amount of finished material saved per machine is 500 pounds. The amount saved on the machines in operation so far is, therefore, moving towards the million pound mark.

If a large manufacturer, because of rigid adherence to standards, lacks flexibility to take advantage of a scheme which effects such savings while a small concern can do it, the reflection appears to be on the large manufacturer rather than on the scheme.

Mr. Hellmund is obviously under the impression that the experience of developing and assisting in putting on the market the generator in question must have been a discouraging one to the writer as the originator of the arrangements covered in the paper. I am sure that he will be pleased to know that quite the opposite has been the case. The experience has been a most interesting one and well worth while. The fact that economic forces are uncompromising is a help rather than a hindrance to those who wish to exploit new schemes of real merit. They are indeed the forces which assist the inventor in overcoming the inherent tendency in human beings to oppose change, or what might be termed human inertia. However, to succeed it is not sufficient that the proposed scheme be ingenious. It must have merit, that is, have a distinct advantage over the schemes already in use, and that advantage must in some form or other be an economic one.

Mr. Ver Planck evidently sees the full significance of the magnetic bridge between field coils, and I wish to express my appreciation of his comments on this point. The rather slow interruption of current on oscillogram, Fig. 9, at the instant of opening the short circuit, is evidently caused by the tenacity of the arc which is formed to hang on when the circuit is broken. On the earlier machines, with slower arc recovery, this phenomenon was not observed, and the voltage would have an instantaneous sharp rise due to the inductance of the circuit. Increased tenacity of the arc to hang on is obviously the result of improvement in design with reference to arc recovery. I do not believe that the minimum voltage following the first rapid rise would have been any lower if means had been employed to break the arc more rapidly. That is a point, however, which can be definitely settled by some additional tests.

Regarding the auxiliary brushes operating in the field, it was pointed out in the paper that the combination of low voltage between bars and high resistance brush has insured entirely satisfactory operation. In fact, five years of experience with this system of excitation have shown that the commutator will polish just slightly better where the auxiliary brushes are bearing on it in addition to the main brushes.

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